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LESSONS

IN

ELEMENTARY PHYSICS.

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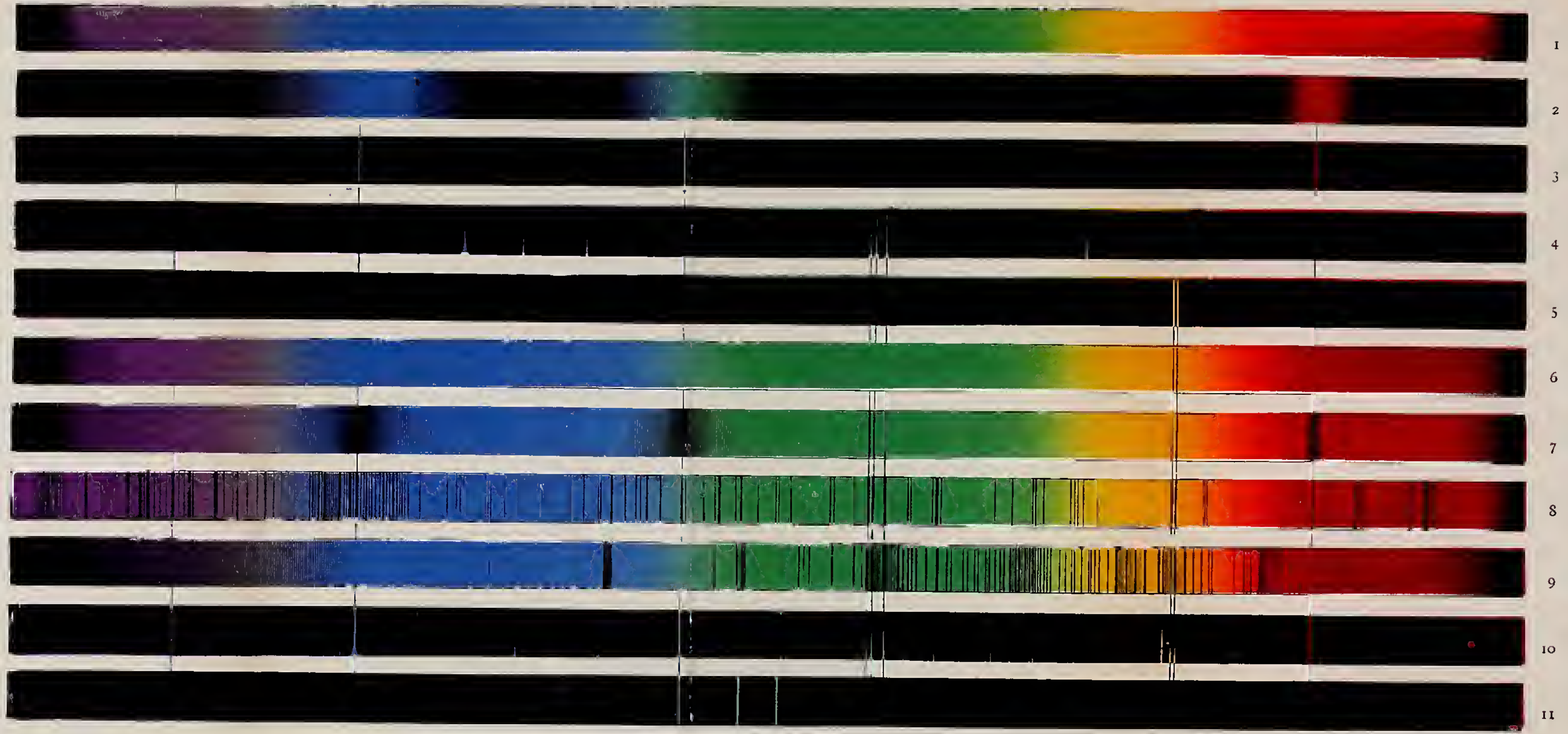
RADIATION AND ABSORPTION SPECTRA.



[*For description, see back.*

DESCRIPTION OF FRONTISPIECE.

1. Continuous Spectrum.	*Radiation.*
2. Spectrum of Hydrogen, Ordinary Pressure.	
3. Spectrum of Hydrogen, very Low Pressure.	
4. Spectrum of Magnesium Vapour, showing the Long and Short Lines.	
5. Spectrum of Sodium Vapour Low Temperature.	
6. Spectrum of Sodium Vapour, Low Temperature.	*Absorption.*
7. Spectra of the Vapours surrounding 1st Class Star, Sirius.	
8. Spectra of the Vapours surrounding 2nd Class Star, the Sun.	
9. Spectra of the Vapours surrounding 3rd Class Star, α Orionis.	
10. Spectrum of Chromosphere	*Radiation.*
11. Spectrum of Nebulæ.	

r, Low Temperature.
rounding 1st Class Star, Sirius.
rrounding 2nd Class Star, the
surrounding 3rd Class Star,

Absorption.

Radiation.

LESSONS

IN

ELEMENTARY PHYSICS.

BY

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PREFACE TO THE FIRST EDITION.

In the following pages I have endeavoured to bring before the student, in an elementary manner, the most important of those laws which regulate the phenomena of nature; but the subject is so extensive that a detailed account cannot be given in such a treatise as this.

The various branches of the subject have been so arranged that the student may perceive the connexion between them. For many particulars of this arrangement I am indebted to my friend Professor Tait.

An account of the various active agents, heat, light, electricity, &c., must always form a large portion of a work on Physics. These have been regarded as varieties of energy—the laws of energy forming, as it were, the thread upon which the various divisons of the subject are strung together. The description of these agents is not, of course, materially different from that usually given; but by this means of connecting them together, the student is constantly reminded of the paramount importance of the laws of energy.

253900

For the plate representing various spectra, which forms the frontispiece, and for that of the Kew spectroscope, I am indebted to my friend Mr. Lockyer; and I have much pleasure in thanking Mr. George Whipple, of the Kew Observatory, for many suggestions while the work was passing through the press; and also Mr. J. D. Cooper and Mr. Collings for the care they have bestowed upon the illustrations.

MANCHESTER, *October*, 1870.

PREFACE TO THE EDITION OF 1877.

IN the present edition some new matter has been introduced, more especially in the chapter which treats of Sound. I am much indebted to Professor Core, of Manchester, and to Mr. Bion Reynolds, M.A., for assistance and suggestions, and I may take this opportunity of stating that Professor Core intends shortly to publish a book of questions founded upon these Lessons.

December, 1877.

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PREFACE TO THE PRESENT EDITION.

THE chief alteration in the present edition is the introduction, near the end of the volume, of a short sketch of the more prominent practical applications of electricity which have recently been made.

I am indebted to the kindness of Mr. Kenneth Romanes for pointing out several errors, chiefly typographical, which I have now corrected.

September, 1885.

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Spectra of the Sun, Stars, and Nebulæ. *Frontispiece.*

LESSONS

IN

ELEMENTARY PHYSICS.

INTRODUCTION.

1. Definition of Physics.—As we look around on the universe in which we dwell, we are struck with a variety of objects outside of ourselves and independent of us. Some of these we see, some we hear, others we touch, or taste, or smell, while many appeal to various senses at once.

When quite young we begin to reason upon these impressions, and the constant recurrence of phenomena in a certain order gives us a well-grounded expectation that in future the same order will be observed. As night approaches, the sun appears to sink below the horizon, and so to vanish from our sight; and yet, from past experience, we have the most perfect confidence that he will reappear on the morrow. But while all classes of men in every age acquire from the necessities of life a certain knowledge of the laws which regulate the phenomena around them, this knowledge is nevertheless most superficial and imperfect. A child knows that a stone will fall to the ground, but it required a Newton to discover the law of gravitation.

It is only within the last three centuries that men have seriously set themselves to the task of acquiring a knowledge of the laws of Nature, and even now we know but a small part of these laws. Nevertheless, a great deal has been gained to the human race in that which has already been done, and the subject forms a study as elevating as it is instructive. These lessons are intended to serve as an introduction to this branch of knowledge which is called **Physics.**

2. Various Aggregations of Matter.—The student should first endeavour to realize the magnitude of the Universe or Cosmos ; and although questions of size and distance belong more particularly to astronomy, yet the results of this science may with propriety be imported into the introduction of a work on Physics.

In a clear night we see stretching across the heavens a faintly luminous band, called the **milky way** or **galaxy.** When viewed by the telescope, it is found to consist of innumerable stars, which are massed so closely together in this particular part of the heavens as to give the appearance of a gigantic whole or substance (of which the grains or particles are individual stars), occupying a particular region of space. This whole is probably the largest whole in the universe.

A ray of light, moving at the rate of nearly 200,000 miles a second, would take at least many years to move across the diameter of the milky way.

Now, each of the stars of this galaxy is an intensely hot and very large globe in size comparable to our Sun, which is in reality a star of average size. Many of these stars have, no doubt, associated with and circulating round them a number of bodies smaller than themselves. The sun has a number of **satellites** of this kind, of which our own earth is one. The sun and his satellites together form the solar system, and in like manner we may imagine each star to represent a **system.**

Descending now from the larger masses of the universe to our own earth, we meet with **substances** of various kinds, and it becomes the office of the chemist to resolve these into their components. He finds that all bodies are made up of some fifty or sixty elements, united together in various ways. Let us take, for example, a piece of table salt or chloride of sodium, and imagine that we have the power of subdividing it without limit. We have reason to think that, if we continued the process of subdivision long enough, we should at last reach a limit which could not be overpassed without altering the nature of the substance; or, in other words, we should at last reach the smallest body capable of possessing the properties of salt. This we term a **molecule.**

If we still continue the subdivision, we separate the compound molecule of salt into its two components, sodium and chlorine, forming elementary **atoms** which we do not imagine to be capable of further subdivision by any means at our disposal.

Thus, in the large or cosmical scale, we have, in the first place, clusters of starry systems; secondly, individual systems; thirdly, individual components of these systems: while in the small scale we have substances, molecules, and atoms.

3. Porosity.—Now, just as in the starry firmament there are vacant spaces between the various individual stars, so in the small scale there are probably vacant spaces between the various molecules of a body; and just as there are vacant spaces between the various components of the solar system, so there are probably vacant spaces between the various atoms that go to form the compound molecule. In other words, bodies are porous, but we must distinguish between two kinds of pores,—namely, *physical pores*, which exist in bodies with no apparent want of continuity, their existence being rendered evident by the contraction of such bodies when exposed to cold, and *sensible* or *visible pores*, which form actual cavities capable of being seen by the microscope, or made evident in some other way.

The skin of the human body is a very good example of a substance possessing sensible pores, and a piece of blotting-paper or sponge is another.

4. Three States of Matter.—Very many of the substances with which we are acquainted are capable of appearing before us in three different states. There is first of all the **solid** state, in which a body has a definite form, and endeavours to retain it; secondly, there is the **liquid** state, in which the body requires to be kept in a vessel, and adapts itself so as always to have its surface horizontal; and there is, thirdly, the **gaseous** state, in which the body cannot be held in an open vessel, but must be shut in on all sides, and always fills the vessel in which it is held. Both liquids and gases possess extreme mobility, in contradistinction to the rigidity of a solid; while a gas again is distinguished from a liquid by its incapacity of remaining in an open vessel, and having a surface.

Earth, a rock, a mountain, a table, a chair, are examples of solids; water and wine are examples of liquids; while the atmospheric air is a very good example of a gas.

5. Motion.—Having now described the various aggregations and kinds of matter, something may be said about motion.

We can only conceive of relative motion, for when a body is in motion we can only know the fact by reference to some other body which is not moving with it. Thus we know that planets are in motion because we see them continually changing their positions among the fixed stars. We know, too, that our earth is in rapid motion round the sun; and yet in a calm day, although this rapid motion of the earth as a whole is going on, there is no motion of the various parts of the terrestrial landscape among themselves.

Thus, despite the rapid motion of the whole, there may be a profound repose of the various parts. On the other hand, a body may be at rest as a whole, and yet there may be violent motions of its various parts among themselves. Let us take, for instance, any substance apparently at rest, say a block of stone. Although there is no appearance of motion in this substance, yet we have very strong reasons for supposing that its various molecules are in rapid motion of some sort among themselves, so minute and so rapid that we should not perceive it, even if we used a microscope of very great power.

In fine, no substance in the universe is at rest; the particles of all bodies are in rapid motion backwards and forwards, and the bodies themselves in rapid motion through space.

6. Force.—Let us now take a group of bodies at rest with regard to one another; this state of rest can only be changed by force. Thus, for instance, suppose we fire a gun, the previous state of rest of the bullet has now been changed by the force of the gunpowder into one of rapid motion. Or take a railway train at rest; the train is set into rapid motion through the force derived from the engine which draws it.

But as it needs force to produce motion, so does it equally need force to destroy it; the bullet from the gun will ultimately have its motion destroyed by the resistance from some hard substance against which it strikes, and the railway train will have its motion stopped by the friction caused by the break. A thing which is difficult to move is difficult to stop, and a thing which is easy to move is easy to stop, the reason being that it requires an equal and opposite application of force to set a body in motion, and to bring it again to rest.

1 We have various kinds of force in Nature, the most prominent being the force of **gravitation.** It is in virtue of this force that a body falls to the ground, and it is in virtue of this same force that the earth moves round the sun. If the attraction of gravitation were to cease, the earth would continue to move at a uniform rate in a straight line, and soon leave the sun behind it, while we in turn should be able to separate ourselves from the Earth.

2 On the small scale we have the force of **cohesion,** in virtue of which the molecules of a body keep together. If this force were taken away, everything would be reduced into small particles, and scattered about.

3 Again there is the force of **chemical attraction,** in virtue of which two different atoms cling together to form a

compound. If this force were absent, there would be no such thing as a compound substance, and we should be limited in our range to some fifty or sixty substances, most of which are metals.

Thus we see that the force of gravitation binds the larger masses of the universe together, and prevents the earth from leaving the sun. The force of cohesion binds together the various particles or molecules of the bodies which we see around us, while in virtue of chemical affinity we obtain a much greater variety of substances than we should otherwise have.

Force does not, however, always produce motion. Thus a stone, lodged on the top of a precipice, is not in motion, although in virtue of the force of gravitation of the earth, it presses or weighs upon the ground of the cliff. But this same force which causes the pressure of the stone against its support, will cause it to fall downwards over the side of the cliff, with a continually increasing velocity, when once the support is removed, and it is free to obey the attraction of the earth.

While the stone lay on the top of the cliff, the force with which the earth attracted it was counteracted by an opposite force—namely, the resistance of the support on which the stone was placed ; and when this resistance was removed, the stone began to fall, and continued to do so with increasing velocity until it reached the bottom of the cliff.

We thus see that the simplest effect of a force is the production of motion, and it is only when the force is resisted by another that we have equilibrium or repose. In the following pages, therefore, we shall commence with the case where a single force produces motion, and end with that where two or more counteracting forces produce equilibrium or repose.

CHAPTER I.

LAWS OF MOTION.

LESSON I.—DETERMINATION OF UNITS.

BEFORE proceeding further, let us fix upon our units of measurement.

7. Unit of Duration.—In the first place, with respect to duration or time, the **second** will be the most convenient unit, and being in general use nothing further need be said about it. But as regards the units of length and mass, those in use in this country are by no means well adapted for the purposes of science, in which respect the metrical system of France has decided advantages over all others. Being a decimal system, all calculations are by it rendered extremely simple, besides which it is in general use amongst the scientific men of all countries.

8. Unit of Length.—The **metre** is the foundation of the metrical system of linear measure, one metre being equal to 39·37079 English inches. In the following table the metre and its decimal derivatives on the one hand are compared with British inches on the other :—

		INCHES.
One millimetre (a thousandth part of a metre)	=	0·03937
One centimetre (a hundredth part of a metre)	=	0·39371
One decimetre (a tenth of a metre)	=	3·93708
One metre	=	39·37079
One decametre (ten metres)	=	393·70790
One hectometre (one hundred metres)	=	3937·07900
One kilometre (one thousand metres)	=	39370·79000

In the margin is a scale, representing a decimetre or tenth part of a metre, which is subdivided into centimetres and millimetres.

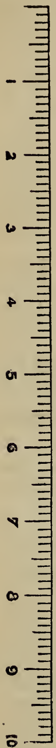

Fig. 1.

9. Unit of Superficial Extent or Surface.—The measures of surface and capacity follow easily from those of length. Of the former we have squares, of which the sides are millimetres, centimetres, decimetres, and metres ; a square metre being likewise called a centiare. We have also the square whose side is ten metres, called the are, and the square whose side is 100 metres called the hectare.

10. Unit of Capacity or Volume.—Again, with regard to measures of capacity or volume, we have the cubic millimetre, the cubic centimetre, called the millilitre, the cubic decimetre, called the litre, and the cubic metre, called the kilolitre. The relation between the measures of length, surface, and capacity is seen from the following table :—

LENGTH.	SURFACE.	CAPACITY.
(A) Millimetre	square millimetre	cubic millimetre.
(B) Centimetre	square centimetre	cubic centimetre.
(C) Decimetre	square decimetre	cubic decimetre or litre.
(D) Metre	square metre or centiare	cubic metre or kilolitre.
(E) Decametre	square decametre or are.	

If we take the first column, or that of length, we find that (B) is ten times as great as (A), (C) ten times as great as (B), and so on, each letter denoting a length ten times as great as the preceding one. Again, if we take the second column, or that of surface, we find that (B) is 100 times as great as (A), (C) 100 times as great as (B), and so on. And, finally, if we take the third column, or that of capacity, we find that (B) is 1,000 times as great as (A), (C) 1,000 times as great as (B), and so on.

Thus *ten* is the multiplier in the first column ; the *square of ten*, or 100, the multiplier in the second ; and the *cube of ten*, or 1,000, the multiplier in the third.

Keeping the table in view, the following examples will render evident the excellences of the metrical system, as compared with that in use in England.

Question I.—How many square feet are there in 150 square inches? *Answer.*—Since one foot is equal to twelve inches, one square foot is equal to 12 × 12 or 144 square inches. Hence there are $\frac{150}{144} = 1{\cdot}041\dot{6}$ square feet in 150 square inches.

Question II.—How many square centimetres are there in 150 square millimetres? *Answer.*—1·50.

Question III.—How many cubic yards are there in 93 cubic feet? *Answer.*—Since there are three feet in a linear yard there are 3 × 3 × 3, or 27 cubic feet in one cubic yard, and hence there are $\frac{93}{27}$ or $3{\cdot}\dot{4}$ cubic yards in 93 cubic feet.

Question IV.—How many litres are there in 1,789 millilitres? *Answer.*—1·789.

These examples are quite sufficient to show the superiority of the metrical system of measures.

11. Unit of Mass.—In the next place, according to this system the relation between the unit of volume and that of mass is of a very simple kind. The unit of mass is that of one cubic centimetre of pure water at the temperature of 4° centigrade, which is the point of maximum density of water. The mass of this bulk of water is called a **gramme,** and the gramme has decimal derivatives similar to those of the metre. The following table shows the relation between the French and English system of estimating masses :—

		GRAINS.
One milligramme (a thousandth part of a gramme)	=	0·015432
One centigramme (a hundredth part of a gramme)	=	0·154323
One decigramme (a tenth part of a gramme)	=	1·543235
One gramme	=	15·432349
One decagramme (10 grammes)	=	154·323488
One hectogramme (100 grammes)	=	1543·234880
One kilogramme (1,000 grammes)	=	15432·348800

12. Unit of Velocity.—Velocity, or rate of motion, is easily understood, for we have constantly before us bodies in motion as one of the most familiar experiences of life. A railway train passes, and we estimate that it is moving at the rate of forty miles an hour. We have a perfectly distinct conception of this velocity, even although the train should not travel the whole hour, or the whole forty miles. We mean that were it to go on moving at the same rate at which it was moving when we saw it, it would in the course of an hour pass over forty miles. Perhaps it begins to slacken its pace shortly after, so that its velocity is soon reduced to thirty miles an hour, then to twenty miles, then to ten miles, until it finally stops. Thus its velocity during the operation of stopping has been continually changing from the high speed of forty miles an hour downwards, and during no two seconds has it continued to move at the same rate, and yet we can say with propriety that at such an instant the train was moving at the rate of thirty miles an hour. We mean, of course, that if the train were to keep the same velocity or rate of motion it had at the given instant, it would in one hour move over thirty miles. We thus see that we mean the same velocity when we say a body is moving at the rate of thirty miles an hour, or sixty miles in two hours, or fifteen miles in half an hour, or $7\frac{1}{2}$ miles in a quarter of an hour. In fact, velocity means the whole space moved over divided by the time taken, or calling s the space, t the time, and v the velocity, then $v = \frac{s}{t}$. $= \frac{\text{metre}}{\text{second}}$

Having already fixed upon the metre as our unit of length and the second as our unit of duration, the most convenient unit of velocity will be the velocity of one metre in one second. The velocity of two metres in one second will be denoted on the scale by 2, that of three metres in one second by 3, and so on.

13. Remarks on Unit of Mass.—By its mass we mean the quantity of matter contained in a body. While we confine ourselves to bodies of the same kind, it is very easy to estimate the relative mass, for this will vary as their volume. If, for instance, we have a number of similar cubes of iron, we know at once that the united mass of two such cubes will be double that of one, of three triple, and so on. But how are we to determine the relative mass of a cube of iron, and a similar cube of lead? It may be answered by their weight, and, as we shall afterwards see, their weight is doubtless a correct representation of their mass; but we cannot accept weight as a *fundamental* method of estimating mass, for weight is due to the attraction of the Earth, and we might suppose a state of things where there was no large attracting body. Let us, for instance, imagine ourselves carried into empty space, with nothing but a cube of iron, and another of lead; then how are we to determine their relative masses? It is clear we cannot weigh them, for there is no downwards and upwards in such circumstances, there being no earth.

We reply, that *two different substances are of the same mass when the same force produces in each, after it has acted on it for one second of time, the same velocity.*

We shall find that the same force will produce at the end of one second the same velocity, if it be applied to set in motion 100 cubic metres of iron, or 69 cubic metres of lead; there is, therefore, the same amount of matter in 69 cubic metres of lead as in 100 cubic metres of iron.

As we shall afterwards find *weight* to be strictly proportional to *mass*, it is convenient to use weight as a means of estimating mass. We have therefore defined our unit of mass to be the *mass* of *matter* contained in one cubic centimetre of pure water at the temperature of 4° centigrade. This definition would, of course, hold good if there were no gravitation, in which case the water would have no weight.

14. Unit of Force.—We are now in a position to define our unit of force.

Let this be the force that will impart to unit of mass unit of velocity in unit of time, or, in other words, a force that, if applied during a second to the mass of a gramme, will produce in it a velocity of one metre in a second.

It is very easy to see that if we operate on two grammes we shall require the application of a double force in order to produce our unit velocity, for we may suppose the double mass to be made up of two separate grammes placed side by side, and one-half of the force applied to each. It will therefore take one unit of force to produce unit of velocity in the one gramme, and another unit of force to produce the same in the other, and hence we must apply two units of force.

It is not, however, equally easy to see that in order to produce *double* velocity in a mass, we must have a force twice as large as that which produces *unit* velocity in the same mass in the same time. But the truth of this statement will afterwards be perceived (Art. 23).

LESSON II.—FIRST LAW.

15. Having fixed upon our various units, let us now proceed to the laws of motion.

The first law of motion asserts that if a body be at rest it will remain so unless acted on by some external force, or if it be in motion it will move in a straight line, and with a uniform velocity, unless acted on by some external force. This law at first sight seems contrary to our every-day experience, for it obviously implies that a body once in motion will continue in motion for ever, unless acted upon by some external force ; now we know that all moving bodies on the earth's surface show a tendency to stop. A little reflection, however, will convince us that the law is true enough, but that all bodies in motion on the earth's surface are in reality acted upon by external forces, and that it is impossible to exhibit a body not so acted upon. It will be found that the more we can reduce in amount the external forces acting upon a moving body, the longer will its motion continue, so that in fact this law of motion represents the state of things under an extreme condition, which can be approached but never reached.

16. We find that friction and the resistance of the atmosphere are the two great forces tending to stop all motion at the earth's surface. To illustrate the former let us make a smooth stone slide along the ground : it will soon be brought to rest through friction ; now take the same stone to a smooth sheet of ice, and it will slide along it to a much greater distance because the friction is less.

In order to illustrate the resistance of the air, set a massive metallic top in rapid rotation in the open air, and it will come to rest in about twenty minutes; but set the same top in motion *in vacuo*, and it will remain moving for more than an hour. The resistance of the air acts very strongly upon bodies moving with great velocity; were there no air, the range of a cannon-ball would be very much increased.

The nearest approach to a perpetual motion, such as is implied in the first law of motion, is that of the earth in its orbit; any resisting medium, like the air, would have the effect of ultimately making the earth approach the sun by a sort of spiral journey, until at last it would be swallowed up by our luminary.

We have reason to believe that there is such a medium, but its tenuity is so great, that it would need a long series of ages in order to diminish sensibly the dimensions of the earth's orbit. Thus we see that the first law of motion contemplates a hypothetical state of things which does not really exist, and we shall see further on that the actual state of things may be represented by one of the laws of energy, of which the first law of motion forms an extreme case.

17. Let us now give a few examples in illustration of this law.

Example I.—A man is on horseback, and the horse starts off suddenly. In what direction will the man fall?

Answer.—He will fall backwards, for in order to cause him to change his previous state of rest, and move along with the horse, force must be applied, by the first law of motion. Now, this force can only be applied at those points at which he is in contact with the horse, so that if he be sitting loosely he will fall backwards.

Example II.—A man is on horseback, and the horse stops suddenly. In what direction will the man fall?

Answer.—This is the opposite of Example I. The man has by the first law of motion a tendency to retain that motion which he had before the horse stopped, and this can only be changed by the application of force. This force, as in the previous case, must be applied at the points where he touches the horse; if he sits loosely, he will therefore preserve his previous state of motion, and be thrown forward over the horse's head.

Again, the first law of motion serves to explain the phenomena of rotation. Thus if a disk or top be set in rapid rotation, a particle at the circumference, such as A, is at any moment moving in the direction of a tangent to the circle at that point; that is to say, in the direction of the arrow head, and if left free to itself it would in virtue of the first law of motion, continue to move in this direction A B; but it is constrained, by the cohesion of the other particles to which it is attached, continually to vary its direction.

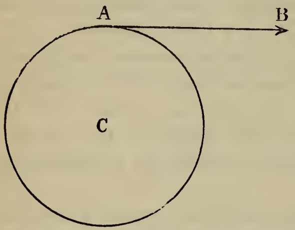

Fig. 2.

If, however, the rotation is very rapid, the force of cohesion may be insufficient to accomplish this, and the consequence will be that the particles at the circumference will leave the system, and be scattered about. In the case of a sling, the force which keeps the stone attached to the sling is intentionally withdrawn at the right moment, and the consequence is that the stone, in virtue of the first law of motion, perseveres in that path, which it was following when the central force was withdrawn.

Lesson III.—Second Law: Action of a Single Force on a Moving Body.

18. We now proceed to the second law of motion, which may be stated as follows: "If any number of forces act together upon a moving body, each force generates the same velocity as it would generate if it acted singly upon the body at rest." For the sake of clearness we may divide this statement into two, and consider—

(1) The action of a single force on a moving body;

(2) The action of several forces together upon a moving body.

Let us at present consider the action of a single force on a moving body. Suppose, for instance, that in a railway carriage which is at rest I throw up a ball with sufficient force to make it reach the roof: if I throw up the ball with the same force when the carriage is in motion, it will equally reach the roof; or if I throw the ball with a force sufficient to strike the side of the carriage with a given velocity when the carriage is at rest, and if when the carriage is in rapid motion I throw the ball with the same force, it will strike the side of the carriage with the same velocity as before.

In fact, the motion of the ball *relative to the carriage* is precisely the same in both cases; but, on the other hand, its motion *relative to the ground* is very different.

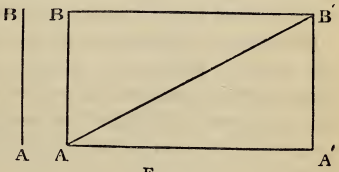

Fig. 3.

When the carriage was at rest the ball went from one side A, to another side, B, of the carriage, let us say in one second, and this was also its motion with regard to the ground. But in the moving carriage, while the ball is on its passage from one side to the other, the point A from which it started has in reality travelled over the distance AA′, so that when the ball arrives at the opposite side this has attained the position B′; thus the ball has, in reality, so far as the ground is concerned, travelled from A to B′. It has in fact travelled over the diagonal of a parallelogram of which one side represents the motion of the ball by itself, and the other the motion of the carriage by itself.

In like manner we know very well that the motion of the earth in its orbit or on its axis does not interfere with the action of forces tending to produce motion at its surface. Thus at the pole there is no motion of rotation, while at the equator there is a motion nearly equal to a mile in three seconds, and yet the same force will produce the same motion at the pole and at the equator. If I leap vertically upwards at the pole, I alight upon the place from which I sprang, and if I do the same thing at the equator the same result will follow. While I am in the air I am separated from the solid earth ; nevertheless this does not, in virtue of its rotation, move from under me at the equator at the rate of a mile in three seconds, but, in virtue of the first law of motion, I retain when in the air the same motion of rotation of the earth in which I participated when I was on its surface, so that I am still carried along with the earth ; and, in virtue of the second law of motion, my leap will be precisely the same as if the earth were at rest, or as if I had performed it at the pole.

Before proceeding further with the second law of motion, let us answer the following questions.

Question I.—A balloon at the height of two miles above the earth's surface is totally immersed in, and carried along with, a current of air, moving at the rate of sixty miles an hour. A feather is dropped over the edge of the car : will it be blown away? or will it appear to drop vertically down?

Answer.—It will appear to drop vertically down as if in a dead calm ; for since the balloon and all that it contains, including the feather, is moving along with the surrounding air, the feather after leaving the balloon will equally participate in that motion ; it will, therefore, drop calmly and slowly down, as if it were dropped in a room. In fact, the motion of the balloon and air will have no more effect upon the fall of the feather, than the motion of the earth in its orbit has upon it. But while the fall of the feather is vertically downwards as far as the balloon is concerned, it is not vertical as regards the earth.

Question II.—A ship is in rapid motion, and a stone is dropped from the top of the mast : where will it fall?

Answer.—At the bottom of the mast. For the stone, during its passage from the top of the mast, retains, in virtue of the first law of motion, the velocity which it possessed as part of the vessel ; and, by the second law of motion, gravity will act on the moving system, including the ship and stone, just as if they were at rest. The motion of the stone therefore, as regards this system, will be the same whether the system is in motion or at rest—in both cases it will fall at the bottom of the mast.

19. We have hitherto considered the case in which the motion of the body is in one direction, and a force is impressed upon it in another at right angles to the motion; let us now consider the case where both are in the same direction.

Suppose, as before, that a railway carriage is in rapid motion, and that in the carriage I throw a ball forward in the direction in which the train is moving. If this ball be impelled with the same force, it will strike the carriage with the same blow whether this be at rest or in motion; while, however, the motion of the ball as regards the carriage will be the same in both cases, its motion as regards the earth will be very different. If, when the carriage is at rest, I give an impulse to the ball that will make it move at the rate of a mile a minute, this will also represent its velocity relative to the earth; but if the carriage is also moving at the rate of a mile a minute, then the total velocity of the ball, with regard to the earth, will be the sum of the united velocities of the carriage and the ball,—that is to say, two miles in one minute.

Let us now take an example of motion in a vertical direction. Suppose, for instance, we have a moveable chamber, made by machinery to descend the vertical shaft of a mine, with the uniform velocity of 9·8 metres per second, and suppose the height of this chamber to be 4·9 metres. Were the chamber at rest, a ball dropped from the top of it would reach the bottom, through the influence of gravity, in exactly one second (Art. 21), and we shall find that the time of descent of the ball from the top of the chamber to the bottom will not be altered if we drop it when the carriage is moving downwards with the uniform velocity of 9·8 metres in a second. In this case, as well as when the carriage was at rest, the ball will reach the bottom exactly one second after it has been dropped from the top and it will also strike against the floor with the same velocity in both cases.

20. Velocity under Gravity.—We are thus prepared to acknowledge that the effect of gravity in increasing the velocity of a falling body will be the same in the same time whether that body is merely beginning to fall, or is already moving downwards with considerable velocity. Thus, if a stone be dropped from the top of a cliff, we know by experiment that after it has fallen for one second it will have acquired the velocity of 9·8 metres per second. It commences with this velocity the next second of its descent; and during this second, gravity, acting in the same manner as before, will continue to impress upon it an additional velocity of 9·8 metres per second, so that at the end of this second its whole velocity will be 19·6 metres per second. In like manner at the end of the third second its velocity will be 29·4 metres per second, so that we may express the relation between the time of descent and the velocity of a body falling from rest under the force of gravity in the following simple manner. Let t denote the time in seconds since the body began to fall, and v the velocity at the end of t seconds (unit of velocity being regarded as the velocity of one metre in a second) then $v = 9{\cdot}8t$.

Thus, let $t = 0{\cdot}25$ seconds, then $v = 9{\cdot}8 \times 0{\cdot}25 = 2{\cdot}45$; let $t = 5{\cdot}5$ seconds, then $v = 9{\cdot}8 \times 5{\cdot}5 = 53{\cdot}9$, and so on.

21. Space passed over under Gravity.—Having arrived so far, let us now trace the relation between the time occupied in descending and the space passed over in the case of a body falling from rest under the action of gravity.

We have said that at the end of the first second the body has attained the velocity of 9·8 metres per second, which means that if we could imagine gravity and every external force to cease at this instant, the body would continue, in virtue of the first law of motion, to move for ever, with the uniform velocity of 9·8 metres per second. But although it had this velocity at the end of the first second, it did not always possess it; for by the above formula (Art. 20), its velocity at the end of half a second was only 4·9 metres per second, while at the end of the first quarter of a second it was 2·45 metres, and so on.

In fact, its average or mean velocity during the first second was only 4·9 metres, which will also represent the space passed over during this time. This will be apparent if we use a graphical method of considering the subject.

First, let a body be in motion with a uniform velocity, which we may represent by AC, and let AB denote the time[1] during which the body is in motion.

Now, it will be remembered (Art. 12) that if v be the velocity, s the space and t the time, we found $v = \frac{s}{t}$, and hence $s = vt$, or the space passed over, is the product of the velocity and of the time. Hence in the annexed diagram, if the line AC denote the velocity, and AB the time, the area ABDC will represent the space passed over.

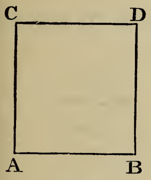

Fig. 4.

Next, let the velocity change, as in the case of a falling body. For the sake of demonstration let us suppose that this change is by fits and starts at small intervals while during each of these intervals the velocity remains constant.

Suppose, for instance, that AB (Fig. 5) denotes one second of time, and BC the velocity of 9·8 metres, which a falling body has acquired at the end of that time, and let us divide the whole time into ten equal parts—

At the end of $\frac{1}{10}$ second the velocity will be 0·98
" " $\frac{2}{10}$ " " " 1·96
" " $\frac{3}{10}$ " " " 2·94
⋮
" " $\frac{10}{10}$ (= 1) " " " 9·8

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[1] It is obvious that anything which is capable of being expressed by units can also be represented by lines. Thus if a line one inch long represents one second, a line of two inches will represent two seconds, and so on.

Suppose now that the body retains its motionless state until the end of the first tenth of a second, when it suddenly assumes the velocity 0·98, and retains this until the end

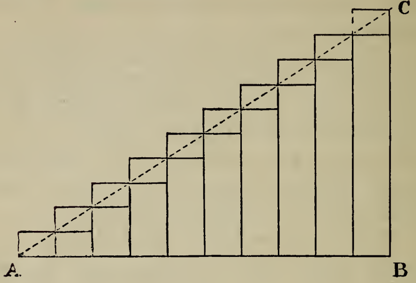

FIG. 5.

of the second tenth, when it suddenly assumes the velocity of 1·96, and so on.

Bearing in mind that the space passed over is the product of the velocity and of the time, we shall have—

	Space passed over.
During the first tenth of a second . .	·000
„ second „ . .	·098
„ third „ . .	·196
„ fourth „ . .	·294
„ fifth „ . .	·392
„ sixth „ . .	·490
„ seventh „ . .	·588
„ eighth „ . .	·686
„ ninth „ . .	·784
„ tenth „ . .	·882
In all . .	4·410

That is to say, the whole space passed over, or 4·410, will in this case be represented by the united area of the *inner* series of steps in Fig. 5, below the dotted line AC.

On the whole, this will be less than the real result, since during each tenth of a second we have supposed the body to retain the velocity it had at the beginning of the time. Suppose now we assume that during each tenth of a second the body moves with the velocity it had at the end of that interval, then the space passed over will be as follows—

	Space passed over.
During the first tenth of a second . .	·098
„ second „ . .	·196
„ third „ . .	·294
„ fourth „ . .	·392
„ fifth „ . .	·490
„ sixth „ . .	·588
„ seventh „ . .	·686
„ eighth „ . .	·784
„ ninth „ . .	·882
„ tenth „ . .	·980
In all . .	5·390

That is to say, the whole space passed over will now be represented by the united area of the *outer* series of steps in Fig 5.

It will readily be seen that this arrangement gives us a result as much above the truth as the other is below it, and that the mean of the two, or $\frac{4{\cdot}410 + 5{\cdot}390}{2} = 4{\cdot}90$ will be the true result.

Indeed we see from the figure that if we had sub-divided the second into a very great number of parts instead of into ten, the two results, of which one is above and the other below the truth, would differ from one another and from the truth only by an exceedingly small quantity, and in fact that the area of the triangle A B C represents graphically the space moved over by a falling body under gravity during the first second; and generally if AB represent the whole time of fall, and BC the velocity at the end of the time, then $\frac{AB \times BC}{2}$ will represent the space passed over.

Thus, in the present instance—

$$AB = 1 \text{ (second)}$$

$$BC = 9{\cdot}8 \text{ (metres per second),}$$

$$\text{hence area} = \frac{AB \times BC}{2} = \frac{1 \times 9{\cdot}8}{2} = 4{\cdot}9,$$

which we have seen is the space moved over during the first second.

The space which the body will pass over during the next second is easily found, for the body will commence with the velocity of 9·8 metres, and end with that of 19·6 metres per second, having during the interval a mean velocity of $\frac{9{\cdot}8 + 19{\cdot}6}{2} = 14{\cdot}7$ metres per second; this, therefore, is the space which it will describe. Thus during the first second it described 4·9 metres, and during the next second 14·7 metres: in all, at the end of the second second, it will have fallen $4{\cdot}9 + 14{\cdot}7 = 19{\cdot}6$ metres. It may be shown in like manner that at the end of the third second it will have fallen 44·1 metres from its point of rest. If we refer to the graphical representation, the rule connecting space and time becomes obvious; for in Fig. 5, if the base AB be taken to represent the whole time t from the commencement of the fall in seconds, and the vertical height BC the velocity v, at the end of this time, then the area of the triangle $(= \frac{1}{2}\, v\, t)$ will, as we have seen, represent the whole space passed over. But $v = 9{\cdot}8\, t$ (Art. 20); hence, substituting this value for v in the above expression, we have s, or whole space passed over $= \frac{1}{2} vt = \frac{1}{2} \times (9{\cdot}8\, t)\, t = 4{\cdot}9\, t^2$.

Hence if $t = 1$ (second) s (space passed over) = 4·9 metres
$t = 2$ „ s „ = 19·6 „
$t = 3$ „ s „ = 44·1 „
$t = 4$ „ s „ = 78·4 „

and so on.

This proof may be illustrated in the following manner. Let us suppose that the moveable chamber of Art. 19, being

4·9 metres in height, contains within it a man, who holds a stone loosely in his hand at the level of the inside roof of the chamber. Now let the whole arrangement be allowed to fall freely down the shaft of a mine under the influence of gravity, and at the moment when the fall begins, suppose that the man lets the stone go. Where will the stone be at the end of the first second? A little reflection will convince us that, although free, the stone will still be at the top of the carriage, for the whole arrangement is falling as fast as it can, and there is therefore no reason why the stone should gain upon the other parts.

At the end of one second, therefore, the top of the carriage will have fallen through 4·9 metres, and the stone will still be at the top.

Now let us imagine that at the end of the first second external friction is made to operate on the carriage sufficiently to stop the tendency of gravity to increase the speed. It will thus cancel, as it were, the further effect of gravity upon the carriage, leaving it to continue its descent as if there were no gravity, in which case it would, in accordance with the first law of motion, continue to move with the velocity it has already attained, that is to say, with the velocity of 9·8 metres a second. During the second second, the top of the carriage will therefore have fallen through 9·8 metres. Altogether therefore, in the two seconds it will have fallen through $4·9 + 9·8 = 14·7$ metres. But in the meantime what will have happened to the stone? Inasmuch as the stone was free within the carriage, it would not be acted upon by the friction arrangement from without, and would therefore, according to the second law of motion, obey gravity in the moving carriage, just as truly as if the carriage were at rest. During the second second, the stone will therefore have fallen, as regards the carriage, through 4·9 metres, that is to say from the top to the bottom, and it will strike the bottom precisely at the end of the second second.

We thus perceive, that while the carriage has fallen through 14·7 metres, the stone, which has all the time been perfectly free, has fallen in addition, through 4·9 metres, this being the distance from the top to the bottom of the carriage. The stone has therefore fallen in two seconds through 14·7 + 4·9 = 19·6 metres, while in one second it only fell through 4·9 metres.

We thus realize how the distance fallen through varies as the square of the time.

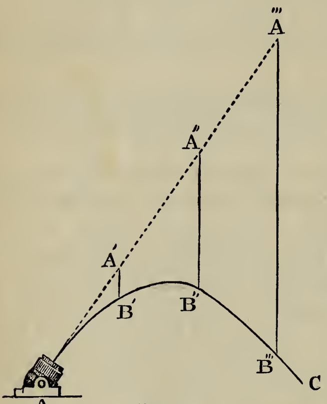

Fig. 6.

22. Oblique Motion under Gravity.—We may now easily pass to the case of oblique motion under gravity. Suppose, for instance, a projectile, such as a bombshell, to be fired off at A with a velocity which would bring it, were there no gravity, to A′ at the end of the first second, to A″ at the end of the second second, to A‴ at the end of the third second, and so on, describing equal spaces in equal times by the first law of motion. What will be its actual path under the action of gravity? Were the shell at rest, gravity would cause it to be at the end of the first second 4·9 metres below the place it would have occupied had there been no gravity, and the same will happen whatever be its velocity. Now, at the end of the first second it would have been at A′ had there been no gravity, and hence its real position at that time will be at B′, 4·9 metres below A′. In like manner at the end of the second second it would have been at A″ had there been no gravity, and hence its true position will be at B″, 19·6 metres below A″. Also at the end of the third second it would be at A‴ without gravity, and hence it will be at B‴, 44·1 metres below A‴, through the action of gravity. The true position of the projectile will therefore be a curve, bending further and further from the original line of impulse, such as we have shown in the above figure; and finally the projectile will reach the ground again at C. The curve A B C, denoting the path of a projectile, may be shown to be a parabola.

LESSON IV.—SECOND LAW: ACTION OF TWO OR MORE FORCES.

We have hitherto supposed only one force to act, and have found that its action is unaffected by the state of rest or motion of the body to which it is applied, and it is very easy to step from this to the case where two forces act at the same moment on a body.

23. Two or more Forces in the same Direction.—Let us, in the first place, suppose that the forces act in the same direction. For instance, imagine a piece of iron to fall by the action of gravity; at the end of one second it will have the velocity of 9·8 metres. Suppose, now, that at the same moment at which gravity began to act upon it, and to cause it to fall, it was acted upon by a magnet so placed as to give it in one second a downward velocity of 9·8 metres. Then, owing to the joint effect of gravity and of the magnet, it will at the end of one second, according to this law of motion, have acquired a velocity of $9·8 + 9·8 = 19·6$ metres per second, and have passed over the space of 9·8 metres instead of 4·9 metres, which it would have passed over through gravity alone.

Hence we are able to extend our definition of force (Art. 14). We thus see that if a double force be applied to the same body, it will produce a double velocity in unit of time, a triple force, a triple velocity, and so on; in fact a force may be measured by the velocity which it generates when applied for one second to a given body. Thus, suppose our unit of force is that which when applied to unit of mass produces unit of velocity in unit of time, then a force represented by *two* will produce in the same mass a velocity equal to *two* in unit of time, and so on. So that in fact, regarding both mass and velocity, we see that the magnitude of a force is represented by the product of the mass into the velocity produced in it by the action of the force in unit of time. Suppose, for instance, that a force, acting during one second, produced in mass 3, a velocity 6, then it would be equal to 3 × 6, or 18, and so on. The product of the mass into the velocity is called the **momentum,** so that a force is represented by the momentum generated by it in unit of time.

Let us now pass to the case where two forces act simultaneously on the same body, but not in the same direction.

24. Forces in Different Directions.—Thus, let a force act on a substance at A which if acting by itself would bring it to B in unit of time, and let another force act upon the same substance which if acting by itself would bring it to D in unit of time, where will the substance be at the end of one second if these two forces act simultaneously upon it? In reply, we may conceive (for the sake of illustration) that A D represents a horizontal platform that is allowed to drop over a precipice, its frame and all that it contains moving downwards in virtue of the law of gravity. Now, a substance is resting on the top of the platform at A, but in virtue of the action of gravity the point A will be found at B at the end of one second, the platform having in this time descended a space equal to 4·9 metres. But at the moment when the descent commences, let us suppose that the body at A, being made of iron, is subjected to the attraction of a magnet that would bring it to D, the other side of the platform, in one second of time. This magnet will act in the same manner while the platform falls under the influence of gravity, so that at the end of one second, owing to the joint operation of the two forces, the body will be found at C: in other words, the substance will actually have travelled in space from A to C.

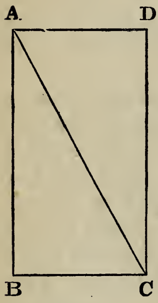

FIG. 7.

25. Parallelogram of Forces.—We thus perceive that the two forces act independently of each other, and we see generally how two or more simultaneous forces can be compounded together; but in the first place let us say a few words about the method of representing forces by straight lines.

Suppose, for instance, that a force is applied to a body at A (Fig. 8), tending to cause motion in the direction A B, and suppose that the line A B has as many units of length as the force has units of force (see definitions, Arts. 8 and 14); then A B is a correct representation of the force in question: for, in the first place, it starts from the point of application of the force, or A; secondly, it lies in the direction of motion, or A B; thirdly, its magnitude is proportional to that of the force.

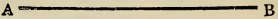

FIG. 8.

Referring now to Fig. 7, let A B denote in magnitude and direction one of two forces acting together on the body at A, and let A D represent the other; it is clear, from the example just given in Art. 24, that A C will represent in magnitude and direction the resultant force due to the joint operation of A B and A D. This proposition is known as the parallelogram of forces, and may be stated as follows:—"If two forces acting simultaneously on a particle be represented in magnitude and line of action by two lines drawn from that particle, and if these lines be made the sides of a parallelogram, then shall the diagonal of this parallelogram represent in magnitude and direction the single force which produces the effect of the two simultaneous forces.

26. Recapitulation.—We have thus seen that a force acts in the same manner upon a body in motion as if it were at rest; and secondly, that if two simultaneous forces act upon a body, each acts independently of the other, so that to find the place of the body at the end of the first second under the joint action of two forces, AB and AD, you may suppose first of all, AB to act alone to the exclusion of AD, and having found the place of the body at the end of the first second due to AB, then suppose the second force AD to act, and thus find the true place of the body.

27. Pressures.—In all this we have supposed each force to be free to produce its own motion, but forces do not always cause motion.

Thus, for instance, a piece of steel may be held by a magnet in defiance of the attraction of gravitation ; or we may take the familiar case of a heavy body resting upon the floor. Such a body is not free to obey the attraction of gravitation, and, indeed, reflection will convince us that in the great majority of instances the bodies which we see around us are not in motion through the force of gravity.

But what really happens when a heavy body rests upon the floor ? Striving to obey the force of gravitation, it presses together the particles of the floor until the resistance of these particles to further compression exactly counterbalances the force of gravity acting on the body ; and we say, in common language, that the weight of the body is supported by the floor. Thus, when the force of gravity does not produce its full motion, it causes pressure, which is measured by the resistance or opposing force, which either altogether stops or modifies the motion.

A few illustrations will make this part of our subject clear.

Question I.—A man in a carriage supports a half-hundred weight in his hand. The carriage and all that it contains are now in the act of falling over a precipice —will he still continue to feel the strain of the weight upon his arm.

Answer.—He will not. When the carriage was on solid ground, the tendency of this weight to approach the centre of the earth was resisted by his hand, and consequently he felt its pressure ; but as the whole system is now approaching the centre of the earth with the full velocity due to gravity, this resistance is no longer exercised, and the pressure no longer felt. In like manner a stone at the top of a carriage descending in this way will not, if free to fall, reach the floor ; but the whole system will fall together without relative displacement of its various parts. The student will note the difference between this example and that in Article 19.

Question II.—A weight equal to 100 kilogrammes rests upon a support, the weight of which support may be neglected. This support is not altogether prevented from falling, but, in virtue of the machinery with which it is connected, it is only allowed to acquire a velocity of 4·90 metres in one second. What will be the pressure upon the support?

Answer.—The force being measured as before by the velocity which it produces in a second, we find that the weight has really descended in virtue of the action of a force only half that of gravity, since gravity would have given it the velocity of 9·8. An upward tending force equal to 4·9, or half that of gravity, must therefore have been applied to the system. Now, as the whole force of gravity is represented by the weight of the body, the upward tending force or resistance will be represented by half the weight of the body, or by 50 kilogrammes; and this will be the measure of the pressure on the support. In like manner if the frame were so connected with machinery that it was only allowed to fall so as to reach a velocity of 1·225 metres at the end of the first second, then evidently we have an upward force tending to produce a velocity against gravity equal to 9·8—1·225, or 8·575 metres per second, and hence $\frac{8{\cdot}575}{9{\cdot}8}$, or $\frac{7}{8}$, will be the proportion of the whole weight of the body borne by the frame, and the pressure upon it will therefore be $100 \times \frac{7}{8} = 87{\cdot}5$ kilogrammes.

Thus we see that the tendency of a force such as gravity acting upon a body is to produce motion in that body; but when that tendency is altogether resisted by another opposing force, we have the statical case of two forces which are in equilibrium with one another. In like manner, too, just as we have the dynamical way of viewing the action of two forces acting simultaneously upon a body in different directions (Art. 25), so we have also a statical solution of the same problem. This will form the subject of our next lesson.

LESSON V.—FORCES STATICALLY CONSIDERED.

28. We have said that the parallelogram of forces forms a proposition in statics. In this proposition it is asserted that when two forces, acting upon a body at a point, are represented in magnitude and direction by the sides of a parallelogram drawn from that point, the diagonal shall represent the resultant of these two forces, so that the two forces will be capable of being balanced by a third force exactly equal and opposite to this diagonal, and applied to the same point.

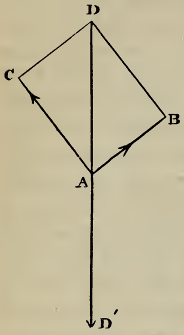

FIG. 9.

Thus, suppose we have at the point A two forces, A B = 6 and A C = 8, acting at right angles to one another. Complete the rectangle A B C D; then we know from the principles of geometry that the diagonal A D of this rectangle will have the value 10. (The figure is drawn so that A D is vertical.) This, then, will (if the parallelogram of forces be true) represent the joint action of the two forces A B and A C, and hence these two forces will be capable of being balanced by a third vertical force A D′, exactly equal and opposite to A D.

29. The truth of this statement may be proved by a very simple experiment. Suppose we have three separate strings tied together at the point A, and also two single fixed pulleys at B and C. The object of these pulleys is merely to change the direction of the force, so that, as in the figure, we have on the left hand a weight of 6 kilogrammes depending over the pulley B, and by means of the thread passing over this pulley, exercising a pressure of 6 kilogrammes, acting upon the point A, in the

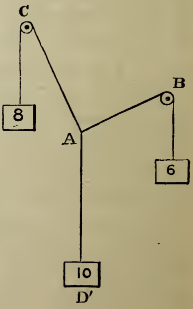

FIG. 10.

direction A B. In like manner, by means of the pulley C, and the thread which passes over it, we have a pressure of 8 kilogrammes acting upon the point A in the direction A C, and finally we have a third force, equal to 10 kilogrammes, acting downwards from A in the direction A D′.

We have thus, by means of this arrangement, produced a system of three forces acting upon A, representing in magnitude the three forces A B, A C, A D′, in the first figure; now if we allow the system to find its own equilibrium, it will be found to settle into such a position that the lines in the second figure shall be parallel to those in the first figure, in which case the forces which balance in the second figure will be represented both in magnitude and direction by the lines of the first figure, and these lines will thus form an accurate representation of the three balancing forces. In other words two forces acting at A, represented by A C and A B, will be balanced by a third force, represented by A D′. But A D is equal and opposite to A D, which is the diagonal of a parallelogram of which A C, A B are the two sides. The parallelogram of forces, therefore, holds true as a proposition in statics.

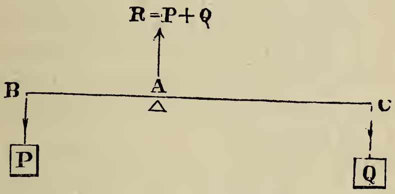

FIG. 11.

30. Continuing the consideration of forces from a statical point of view, we come to the question of parallel forces.

For instance, let A denote a fulcrum, about which a rod, BC, without weight, is free to move, and let there be two weights, P and Q, applied at B and C. We have thus two downward forces, P and Q, acting at B and C, and an upward force R, acting at the fulcrum A, representing the resistance interposed by the fulcrum to the weight of P and Q, and hence, of course, equal to the sum of these two weights. The question is, under what conditions will such a system be in equilibrium?

Now, it is found by mechanical principles that if the force P multiplied by its arm, be equal to the force Q multiplied by its arm—that is to say, if P × AB = Q × AC—then there will be equilibrium; but if P × AB is greater than Q × AC, then P will begin to move down, while Q rises: and, on the other hand, if Q × AC is the greatest, Q will begin to fall.

The product of a force into a perpendicular, dropped from the fulcrum upon its line of action, is called the **moment** of the force; and hence, as in the above case, where there is equilibrium, the moment on the one side is equal to that on the other. Numerous applications of this principle are of daily occurrence: thus, with an iron lever or crow-bar, a man, if he has sufficient leverage, or purchase as it is called, may raise a very large weight; for although his own force is small, it acts at the end of a long arm, and hence its moment is great. Pincers, nutcrackers, scissors, &c., are examples of levers where a comparatively small power at a great leverage produces a very great effect.

The principles which we have now stated and applied to two forces, P and Q, may be extended to any number of forces applied to a system which is supported on a point or fulcrum; and in all cases where there is equilibrium, we may assert that the sum of the moments tending to produce rotation in one direction is precisely equal to the sum of those tending to produce rotation in an opposite direction.

Example.—On a straight lever without weight we have on the right hand of the fulcrum two forces—namely, 8 grammes at a distance of 6 centimetres, and 12 grammes at a distance of 8 centimetres; while on the left hand we have 10 grammes at a distance of 10 centimetres: which arm will tend to fall?

On the right we have 8 × 6 = 48 and 12 × 8 = 96, in all 48 + 96 = 144, for the sum of the moments tending to depress the right arm; while on the left we have 10 × 10 = 100 for the moments tending to depress the left arm; but the former is greater than the latter, and hence the right arm will be depressed.

Lesson VI.—Third Law of Motion.

31. In the preceding pages we have exemplified the first two laws of motion. In the first of these it was asserted that a body cannot alter its state of rest or motion without the application of a force. In the third law it is asserted that this force must be applied through the agency of some external body. Thus, for instance, let us take a body which rests on the table. We say it rests, but, as already stated (Art. 5), we have reason to believe that the particles of this body are in a state of violent commotion among themselves; nevertheless these motions are not such as to alter the position of the body, which, unless acted on from without, will continue apparently in a state of rest. The forces within the body can only alter the relative position of the various particles of the body, but cannot alter the position of the body *as a whole.* In like manner, a man in a carriage cannot drive himself forward, but in order to do so he must make use of the ground, or something external to the carriage itself.

But although a body *as a whole* cannot assume motion without the application of some external force, yet owing to the action of internal forces, we may have one part of a body driven in one direction, and another driven in the opposite direction. Now, whenever this takes place, it is asserted by the third law of motion that whatever momentum may be communicated in any one direction by an internal action of this kind, precisely as much will be communicated in an exactly opposite direction.

There are, however, cases where the application of this principle is not obvious: but it only requires a little study to convince us that it holds in all. Take, for instance, a gun which is fired. At first sight it appears as if, through the exercise of internal forces, one part of the gun, namely the bullet, has been driven violently forward without there being any motion in an opposite direction. A little reflection will, however, convince us that this is not the case; for were the gun mounted on a carriage without friction, we should find that while the bullet was propelled violently forward, the gun itself was propelled in the opposite direction; and further, if we made accurate observations, we should find that the backward force was precisely equal to the forward force, so that if we multiply the mass of the bullet by its velocity of propulsion, and the mass of the gun by its velocity of recoil, the two products will be equal; or, in other words, the momentum generated in one direction is precisely equal to that generated in the other, and hence the force exerted in the one direction is precisely equal to that exerted in the other.

On this account the third law of motion is sometimes stated as follows:—

Action and reaction are equal and opposite.

To take another instance. Suppose that I drop a stone towards the ground; it soon, through the action of the gravitating earth, attains a considerable velocity. The earth has acted upon the stone, tending to pull it towards itself; and this case is, in fact, the reverse of that of the gun. In the system containing the gun and bullet, we had a repulsive force generated which propelled the bullet in one direction and the gun in another; but in the system embracing the earth and stone, we have a certain velocity of approach generated in the stone, and the stone here corresponds to the bullet of the gun, while the earth corresponds to the gun itself. We should, therefore, expect from the third law of motion that in this case, while the stone comes down to meet the earth, the earth should go up to meet the stone; and this is no doubt true, only the mass of the earth being so very much greater than that of the stone, the *velocity* with which the earth mounts upwards to the stone will be very much less than that with which the stone moves down to meet the earth. But the *momentum* of approach will be the same both for the stone and the earth.

Let us next suppose that a cannon while in the act of being fired is firmly fixed to the earth. The ball is seen to fly forward with great velocity, but the cannon, being fixed, does not appear to move. Is not the reaction destroyed in such a case as this? We answer that the reaction is not destroyed, but only rendered inappreciable; in fact, the cannon, being firmly bound to the earth, forms a part of the earth itself, so that we have the cannon-ball moving rapidly forward with a great velocity, and the earth, to which the cannon is fixed, moving backwards with an inappreciable velocity, because of its great mass, the momentum being as before, the same in both directions.

The same thing takes place when a man leaps upwards. The muscular force which he exerts is very similar to the action of the powder in the cannon, and the man, like the bullet, is propelled in one direction, while the earth is pushed downwards in the other; of course with an inappreciable velocity, on account of its enormous mass.

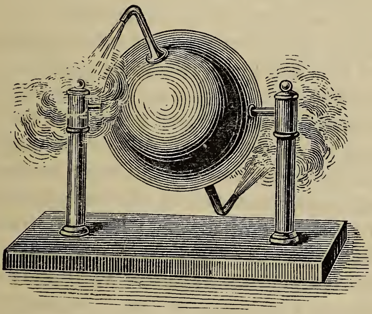

FIG. 12.

The same principle will guide us to a correct solution in a great variety of important cases. Let us take that of a bomb-shell flying along with the velocity of 200 metres a second. Suppose that it meanwhile explodes into two parts of equal weight, one of which is propelled forward in the direction in which the shell is moving, with an additional velocity of 200 metres per second. According to the third law of motion, the other half will be pushed backwards with the same velocity of 200 metres. The result of the explosion will therefore be, that one-half of the shell will be propelled forwards with the double velocity of 400 metres per second, and the other half will have lost its previous motion, and be brought to a standstill.

If the weight of the shell be 10,000 grammes, then the forward momentum before explosion was 10,000 × 200, while the momentum after explosion will be $\frac{10,000}{2} \times 400$, or precisely the same as before. We had, in fact, previous to the explosion, the whole mass moving with the original velocity, and after explosion we have the half mass moving with a velocity double of the original.

The Eolipyle, invented by Hero of Alexandria, is a very good illustration of the principle of reaction. It consists of a hollow metallic sphere capable of rotating about an axis, as in Fig. 12. It is fitted with two small appendages, open at the end, out of which steam may be made to issue. Having introduced a sufficient quantity of water into this apparatus, let it be heated by a spirit-lamp. When the temperature has risen to the boiling-point, steam will begin to issue from the openings, and in consequence of the velocity with which it issues out, the globe or boiler will be driven round in the opposite direction, on the principle of reaction.

The rocket is another illustration of the same principle, the downrush of heated gas from its lower extremity causing the rocket itself to mount upward in an opposite direction.

CHAPTER II.

THE FORCES OF NATURE.

LESSON VII.—UNIVERSAL GRAVITATION.

32. WE have already seen (Art. 6) that the forces of nature may be divided into three groups—the first embracing universal gravitation; the second, the molecular; and the third, the atomic forces; the distinction between molecular and atomic being that molecular forces denote those exerted between particles of the same substance, while atomic forces denote those exerted between particles of different substances. But of the forces connected with molecules and atoms, we have some which may be characterised as permanent, while others are temporary and evanescent. Thus a piece of iron has certain permanent properties, and certain permanent forces connected with these; besides which it may be temporarily magnetized, when it exhibits a strong attracting power for other iron, which, however, vanishes when it is again demagnetized.

In the same manner a body when electrified possesses peculiar properties, with which it parts whenever it loses its electricity. These peculiar and temporary exhibitions of force we shall consider at a subsequent stage; but in the meantime we shall confine ourselves to those forces which may be said to be permanently associated with certain bodies.

33. The most important and best-understood force belonging to matter is "universal gravitation," a force which was first clearly defined by the genius of Newton.

In order to attain a distinct perception of this force, let us begin with terrestrial gravity. It has already been stated (Art. 20) that the earth attracts a falling body with a force sufficient to generate a velocity of 9·8 metres in one second. The question naturally arises, Does it generate the very same velocity in every falling body? This was the question which Newton asked himself, and he found by experiment that the velocity was precisely the same in the case of each body.

Previous to Newton, Galileo had arrived at a similar conclusion as regards bodies of different weight; but in advocating it he met with great opposition from the schoolmen, for in those days men had hardly yet begun to question nature by experiment, and were guided almost entirely by authority. The followers of Aristotle asserted that a ten-pound weight would fall to the ground ten times as fast as a one-pound weight. Galileo, therefore, let two such weights drop from the top of the tower at Pisa, and they both reached the ground as nearly as possible together. The larger weight was, however, very slightly before the other, and it was properly remarked by Galileo that this difference was due to the resistance of the air. It is, in fact, this resistance that renders the law of equal velocities for all falling bodies difficult of perception at first sight; for if we drop a feather and a piece of lead, the feather will evidently lag behind the lead.

Now, it is obvious that the surface of the feather being very large compared to its weight, the resistance of the air will naturally influence its motion very much, and make it lag behind. We may, however, exhaust a tall jar by means of an air-pump, and contrive to drop a feather and a bullet together from the top of the jar, when the two will be found to reach the bottom at the same moment; then, if air be again admitted, the feather will lag very much behind the bullet.

34. But experiments on falling bodies are not capable of being made with very great exactness, because they pass too rapidly before our eyes. Newton, in his experiments, obviated this difficulty by taking advantage of another mode of action of the force of gravity. When a **pendulum** (see Fig. 19) is pulled aside from its vertical position, and then left free, the heavy part or **bob**, in virtue of the force of gravity, tries to settle in the lowest position, which is that of verticality; but when it regains this position it is moving with a considerable velocity, which will suffice to carry it up on the other side to a distance nearly equal to that from which it fell, and the pendulum will go on oscillating backwards and forwards on both sides of this lowest point for a very long time. Now, the time of one of these oscillations enables us to measure the force exerted by gravity on the heavy matter of the pendulum; for if there were no gravity, there would be no oscillation; if very little gravity, the oscillation would be very slow, so that the quicker the oscillation the greater the force of gravity.

By an arrangement of this nature we can measure the force of gravity with great exactness; for the pendulum will probably vibrate 10,000 times before it stops, and we can therefore ascertain the time of 10,000 vibrations with an error which bears a very small proportion to the whole time.

Now, Newton constructed a pendulum, the heavy part or bob of which was a box, into which he might put a number of different substances. In each case the same weight was put into the box, and he found that for all substances the pendulum oscillated in precisely the same time.

Let us endeavour to ascertain what this means. Since on each occasion the same *weight* was enclosed in the box of the pendulum, this means that on each occasion *the gravitating force* was the same. It does not, however, follow that the *mass* was the same in each case; for until we have found out by experiment, we have no reason to suppose that the mass of a pound of lead is the same as that of a pound of wool. But had the masses been different in Newton's experiment, the pendulum would have had a different time of oscillation according to the nature of the substance enclosed in the box, just as the box itself, if free to obey gravity, would have fallen to the ground with a velocity depending on the nature of its substance.

But Newton found the time of oscillation always the same, and hence he concluded that bodies of the same weight are also of the same mass, or that the *force* of gravity is always proportional to the *mass*.

35. Thus gravity is not like magnetism. If a pendulum with an iron bob were swung in the neighbourhood of a magnet, its time of oscillation would be very different from that of one with a leaden or brass bob; but under gravity there is no such difference. We thus see that *weight* is a correct representative of *mass*, and that the mass of a pound of feathers is equal to that of a pound of lead, half as much as that of two pounds of iron, and so on, without respect to the nature of the substance. Adopting, therefore, the gramme for the unit of mass, and for unit of force that which, when applied to unit of mass, generates in one second a velocity equal to *unity*, the force of gravity which acts on one gramme will be denoted by 9·8, inasmuch as it generates this velocity in unit of mass in one second; that which acts on 5 grammes will thus be $5 \times 9{\cdot}8 = 49$, and so on, the total moving force being measured by the momentum which it generates in one second; that is to say, by the product of the mass into the velocity.

Thus we see that the earth attracts every particle of every substance with a force proportional to the mass of the substance.

The attractive force acts in the direction in which the plumb-line hangs, or in what we call a vertical direction.

The line of direction if produced will pass very nearly through the earth's centre; but of course, the earth being round, the plumb-line at one place will differ in direction from that at another place some distance off. We thus see that the various directions in which the force of gravity acts are not in reality parallel to one another; nevertheless all plumb-lines or vertical lines within a short distance of one another are as nearly as possible parallel, seeing that it requires the distance of one mile to make a change equal to *one minute* of arc in the direction of the plumb-line.

Again, this force of attraction of the earth for substances near its surface is also dependent upon the magnitude of the earth itself; for were there no earth, there would be no attraction ; were the mass of the earth only half what it is, the attraction would only be half as great, and the mass of matter contained in a pound would then distend a spring only half as much as it does at present : on the other hand, were the earth double in mass of what it is, its size remaining the same, the attraction of gravitation at its surface would be twice as great. Thus we see that the whole force varies with the mass of the earth or *attracting* body, as well as with the mass of the *attracted* body. In fact, we may say that every particle of the earth attracts every particle of a stone or other heavy body, and, of course, by the third law of motion, action and reaction being equal and opposite, every particle of the stone attracts every particle of the earth in the opposite direction.

36. Suppose now that we could observe a stone placed twice as far from the earth's centre as it is at present, would the attraction of the earth be as great upon it as it is at present? Certainly not; for if the attraction did not diminish in some way with the distance we should be drawn to the sun rather than to the earth, since the mass of the former body is so much the greater. The attraction exerted by a heavy body must therefore vary in some manner with the distance of the body. It was reserved for Newton to find out the law of this variation. It is such that if the distance be doubled, the attraction is diminished four times ; if the distance be tripled, the attraction is diminished nine times, and so on, which is expressed by saying that the force of gravitation varies inversely as the square of the distance.

37. A proof of the truth of this assertion occurred to Newton. We cannot, indeed, place a stone so as materially to increase or diminish its distance from the earth's centre, but we have the moon upon which to make our observations.

Thus let D (Fig. 13) be the centre of the earth, and let A be the place of the moon at any moment, and C be its place one second later. When the moon was at A, it was moving in the direction A B, and would have continued to move in this line, by virtue of the first law of motion, had not the earth interfered. It would thus have been at B at the end of the first second, but by the attraction of the earth it has been pulled from B to C in one second of time. B C, in fact, corresponds to the 4·9 metres through which a body falls at the surface of the earth through the force of gravity in one second ; so that we have the following proportion :—

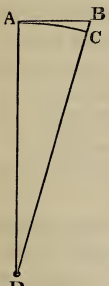

Fig. 13.

Force of earth's attraction at the moon *is to* force of same at the earth's surface *as* B C *is to* 4·9 metres.

Now A C being the arc of a circle, of which D is the centre, and A B a tangent at A, we have, by a well-known proposition in geometry, $AB^2 = 2\,CD.\,BC$ nearly, or (since A B is nearly equal to A C) $AC^2 =$ diameter of orbit $\times$ B C nearly ; hence $BC = \frac{AC^2}{\text{diameter of orbit.}}$ Now, the moon is, in round numbers, 240,000 miles from the earth's centre, or sixty times as far as the earth's surface is, and hence the circumference of the moon's orbit is 1,508,000 miles. This distance it describes in 27d. 7h. 14m., and hence the length described in one second, or A C, will be $\frac{1,508,000}{2,360,580}$ miles $=$ 0·639 miles.

Again, the diameter of the orbit, or double the radius, is 480,000 miles ; hence $BC = \frac{(0{\cdot}639)^2}{480,000}$ miles $=$ ·00137 metres.

Hence, also, the proportion previously stated becomes—

Force of earth's attraction at the moon *is to* force of same at earth's surface *as* ·00137 metres *is to* 4·9 metres, or as $\frac{1}{3,600}$ *is to* 1 ; that is to say, *as a matter of fact*, the force of the earth's attraction at the moon's surface is

$\frac{1}{3600}$ of what it is at the earth's surface. Now, the moon is sixty times farther away from the earth's centre than the earth's surface is; and hence if the law of gravitation, as announced by Newton, holds good, the earth's attraction at the moon's surface should only be $\frac{1}{(60)^2}$ of what it is at the earth's surface; now $\frac{1}{(60)^2} = \frac{1}{3,600}$: this, therefore, is the proportion which *ought to hold* if Newton's law be true; but this we have seen is also the *actual* proportion between the two attractions. The law, therefore, holds good in the case of the moon, and in like manner it might be shown to hold for the various members of the solar system, regarding the sun as the gravitating body.

38. We are thus led to the grand law of universal gravitation, which may be stated as follows:—*Every substance in the universe attracts every other substance with a force jointly proportional to the mass of the attracting and of the attracted body, and varying inversely as the square of the distance.*

To illustrate this: suppose that we have two bodies of mass equal to unity and distance from each other equal also to unity, and suppose (for the occasion) that the attraction between them is also unity.

(1) Were the one body increased six times, the whole attraction would become 6.

(2) Were both bodies increased six times, each unit of the one body would attract each unit of the other with a force equal to unity; hence the whole attraction would be 6 × 6, or 36.

(3) In like manner if one body had a mass equal to 6, and the other a mass equal to 4, the whole attraction would be 24.

(4) If in addition to the masses in (3) the distance were doubled, the attraction would now be $\frac{24}{2 \times 2} = 6$.

(5) Let the mass of the one be 9, of the other 7, and the

distance be 5, then the attraction will be $\frac{9 \times 7}{5 \times 5} = \frac{63}{25}$ and so on.

LESSON VIII.—ATWOOD'S MACHINE.

39. We cannot well make use of falling bodies to illustrate the laws of motion, because they fall too rapidly for us; but there is a machine, called Atwood's Machine, which so modifies the velocities of falling bodies as to make them suitable for our purpose. The essential part of this machine is a fixed pulley over which a fine silk thread passes. The two ends of the axle of this pulley (Fig. 14) are made to lie on the circumferences of two wheels, by which means the friction of the axle is reduced to a minimum. A thread passes over the circumference of the pulley, and to the extremities of this thread two hollow boxes of equal weight are attached ; a clock connected with the machine shows seconds, and beats in an audible and distinct manner. Besides this, we have a graduated rod to which are attached several plates and several rings ; the plates, as in the figure, will arrest either of the boxes in its descent, but the rings will allow it to pass.

40. *Experiment* A.—Suppose now that each of the boxes weighs 100 grammes, and that we put 400 grammes into the one and 450 into the other : also let us take into account the pulley in our estimation of the weight of the boxes. It is clear that the heavier box will begin to descend, and that the lighter will mount up.

The mass to be moved is in all (including the weight of the boxes) 1,050 grammes, while the force is that caused by the 50 grammes of excess in one of the boxes.

Now, a gramme being unit of mass, this force will be denoted by 50 × force of gravity = 50 × 9·8 = 490, which will therefore be the moving force. On the other hand, the whole mass to be moved is 1,050, and hence the velocity acquired in one second (represented by the moving force divided by the mass to be moved) will be $\frac{490}{1,050} = 0{\cdot}467$ nearly, while the space passed through will be half of this (Art. 21), or 0·2335 metres. Suppose now we place on our graduated rod a plate 0·2335 of a metre below the box containing the larger weight, and allow this box to fall just when the clock is beating a second, it ought to strike against the plate exactly one second after it began its descent, and just when the clock is beating the next second.

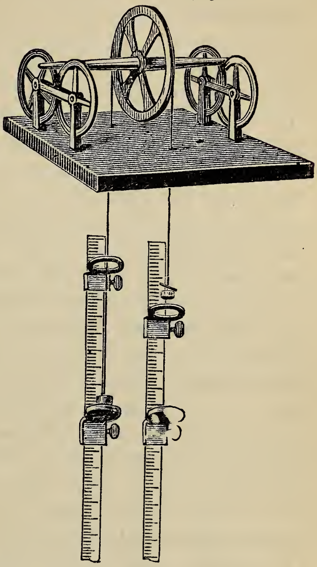

FIG. 14.

Experiment B.—Having made Experiment A, and ascertained that the result agrees with our calculations, let us now double our moving force, keeping however the same total mass. Thus let there be 375 grammes put into the one box, and 475 into the other, so that the total mass, including that of the boxes, will be 475 + 375 + 200 = 1,050, or the same as before, while the excess of weight will be 100 grammes, and the moving force 100 × 9·8 = 980, being double of that in experiment A. The velocity acquired in one second will therefore be $\frac{980}{1,050} = 0{\cdot}934$ metres nearly, while the space passed through will be half of this, or 0·467 metres.

Let us therefore place a plate so as to arrest the box with the larger weight when it has passed through 0·467 metres, and we shall find as before that it will strike against the plate exactly one second after it began its descent.

Let us now realize what we have proved by means of these two experiments. In Experiment A we had a mass equal to 1,050, and a moving force equal to 50 × 9·8, and we obtained as the result the velocity of 0·467 metres in one second.

In Experiment B we have the same mass, namely, 1,050, and a double moving force, or 100 × 9·8, and we obtained in one second a velocity equal to 0·934 metres.

We thus see that *while the mass remains the same, the velocity generated in unit of time varies as the force.*

41. *Experiment* C.—In this experiment let us keep the same force we had in Experiment A,—namely, that produced by an excess of 50 grammes, but let us diminish the whole mass by one-half. This will be done by putting 137·5 grammes into the one box, and 187·5 grammes into the other, for the excess will be 50 grammes, the same as before, while the whole mass, including that of the boxes, will be 137·5 + 187·5 + 200 = 525, or the half of 1,050, which was the mass in Experiment A. The velocity acquired in one second will here be $\frac{50 \times 9{\cdot}8}{525} = 0{\cdot}934$, and the space passed over in one second half of this, or 0·467 metres.

Placing the stage at this distance below the heavier box, it will be found to reach it at the expiration of exactly one second.

Comparing together Eperiments A and C, we find that in both we had the same moving force, or that due to an excess of 50 grammes; but in Experiment A we had a mass of 1,050, while in Experiment C the mass was only half of this. The result obtained was a velocity in Experiment C twice as great as that in Experiment A. We thus see that *while the moving force remains the same, the velocity generated in unit of time varies inversely as the mass.* Taking the results of all these experiments, we perceive that the correct measure of a force is that already given, namely, the momentum, or product of mass into velocity generated in unit of time.

42. *Experiment* D.—Let us arrange the boxes as in Experiment A, only let the excess of weight, namely, 50 grammes, be in the shape of a bar which lies across the box (Fig. 14).

Also let us place a ring instead of a plate at a distance of 0·233 metres below the point from which the heavier box begins to descend. It will as before reach the ring exactly at the expiration of one second, and by means of the ring it will be relieved of its surplus weight of 50 grammes, which, being in the shape of a bar, will be caught by the circumference of the ring, while the box itself will descend through its centre. The box will now be moving with the velocity of 0·467 metres in one second, and since there is no longer any moving force it will, by the first law of motion, continue to move at this rate. Let us therefore place a plate 0·467 metres below the ring, and it will be found that the box will reach this plate exactly one second after it reached the ring, and exactly two seconds after it first started.

Experiment E.—Everything being the same as in Experiment D, let us place the plate, not 0·467 metres, but twice this, or 0·934 metres, below the ring. Since the body, after passing the ring, moves with a uniform motion, it will be found to reach the plate in exactly two seconds after it passed the ring, or three seconds after it began to fall.

Comparing together Experiments D and E, we find that they are an illustration of the truth of the first law of motion, and that the box, being relieved by the ring of all moving force, and therefore being in a condition to illustrate the first law of motion, passes through the space of 0·467 metres during the first second, and through the same space during the next, thus describing equal spaces in equal times.

43. *Experiment* F.—In Experiment A we obtained a velocity of 0·467 metres, when a moving force due to the weight of 50 grammes operated for one second on the mass of 1,050 grammes. Suppose now we allow the same force to operate on the same mass for two seconds instead of one, and place the ring so as to take off the moving force when it has operated exactly two seconds on this mass. The ring will in this case be put at four times the distance at which the stage was put in Experiment A, or 0·2335 × 4 = 0·934 metres from the starting-point.

As the heavier box will reach this ring in exactly two seconds, it will then be moving with the velocity $\frac{490}{1,050} \times 2 = 0·934$ metres; and the moving force being left behind on the ring, the box will, by the first law of motion, continue to move uniformly with this velocity. If, therefore, we place a stage 0·934 metres below the ring, the box ought to reach this stage exactly one second after it has passed the ring, or exactly three seconds after it has begun to fall.

By performing this experiment we see that the same moving force, when applied to the same mass, will generate in two seconds of time a velocity twice as great as that which is generated in one second, or *the velocity generated by a constant force is proportional to the time during which the force has acted.*

44. *Experiment* G.—We see very easily from the above experiment that the spaces passed over by a body under the action of a constant force vary as the squares of the time.

For in Experiment A we placed the stage 0·2335 metres below the starting-point, and the box was found to have arrived there in exactly one second.

Again, in Experiment F, we placed the stage four times as far from the starting-point, and we found that the box reached it in exactly two seconds; and if we place the stage nine times as far from the starting-point (provided the apparatus be sufficiently high), we shall find that the box will reach it in exactly three seconds, thus proving that *under the influence of a constant force the spaces passed over vary as the squares of the times.*

45. *Experiment* H.—We may also make use of Atwood's Machine to illustrate the laws of uniformly retarded motion. Thus if we throw up a stone with the velocity of 9·8, gravity will, by the second law of motion, tend to impress upon it a downward velocity of 9·8 in one second, so that at the end of one second it will have no velocity at all, and will be in the act of commencing its descent. Also, it will have moved during this second with an average velocity equal to $\frac{9{\cdot}8 + 0}{2} = 4{\cdot}9$, and hence it will have ascended through a space equal to 4·9 metres.

Suppose now that the stone is projected upwards with the velocity of 19·6, or double that of the former. It will be two seconds before gravity is able to impress upon it a contrary velocity of 19·6, so as to bring it to rest. It will therefore mount upwards for two seconds, and will do so with the average velocity of $\frac{19{\cdot}6 + 0}{2} = 9{\cdot}8$, and hence the whole space described during the two seconds of ascent will be $9{\cdot}8 \times 2 = 19{\cdot}6$ metres.

Thus we see that with a double velocity of projection the stone mounts four times as high, and in like manner it might be shown that with a triple velocity it would mount up nine times as high.

In fact, we have here space described against the action of gravity just as we previously had space described by aid of the action of gravity, and the two cases are connected together by a very simple law. Thus if a stone be projected vertically upwards, it will reach to such a height that when it falls again under the influence of gravity, it will finally strike the ground with the very same velocity with which it started. During its ascent gravity has acted so as to take away its upward velocity, and during its descent it has produced just as much again in an opposite direction.

Now this law can be approximately illustrated by Atwood's Machine. Let us for instance, as in Experiment A, adhere to a total mass of 1,050 grammes, the moving force being a bar 50 grammes in weight; also place the ring at any given distance, for instance one metre below the starting-point of the heavier box, and finally let there be a ring from which at the same moment the ascending box shall take up a bar equal in weight to that dropped by the descending box. The motion will now be taking place against a moving force equal in amount to that which previously operated in its favour, and therefore, if we place a stage one metre below the ring, we shall find that the box will nearly touch that stage, and then begin to mount upwards. Thus it passed through one metre under the influence of a certain force, and by this means acquired a velocity which enabled it nearly to describe the same space of one metre against the influence of a similar force before it was finally brought to a rest, and its motion reversed.

46. It may be useful to give one or two examples in illustration of this subject.

Example I.—Neglecting the weight of the pulley in Atwood's Machine, let the one box weigh 600 grammes, and the other 400 : what will be the tension on the string during the downward motion of the heavier box?

Answer.—We have here a moving force equal to 200 × 9·8, and a mass equal to 1,000; hence the velocity acquired in one second will be $\frac{200}{1,000} \times 9{\cdot}8 = 9{\cdot}8 \times \frac{1}{5}$. The heavier mass (or 600 grammes) will therefore have attained this velocity at the end of one second, whereas, if left to itself, it would have attained through gravity a velocity equal to 9·8, or five times as much. There must, therefore, have been acting upon it a force opposed to gravity, and representing the tension of the string, which has generated in one second an upward velocity of $9{\cdot}8 \times \frac{4}{5}$. Now this is a force equal to four-fifths of that of gravity, and hence the tension on the string will be equal to four-fifths of the weight of the body; that is to say, it will be $600 \times \frac{4}{5} = 480$ grammes.

Example II.—A body is projected vertically upwards with a velocity equal to 19·6 metres per second : what will be its velocity after it has risen 14·7 metres?

Answer.—We see from Art. 45 that a body projected upwards with the velocity of 19·6 will rise 19·6 metres before it begins to descend. When it has risen 14·7 metres, it will, therefore, have sufficient velocity left to carry it up 4·9 metres more. But by Art. 45 we see also that a body having the upward velocity of 9·8 metres will rise 4·9 metres in height. Hence the body in this example, which has risen

14·7 metres in height, will then be moving upwards with the velocity of 9·8 metres, since that velocity will suffice to carry it up 4·9 metres more.

Let us now briefly recapitulate the facts connected with the action of gravity at the earth's surface.

(A.) A body, falling from rest under the influence of gravity, attains a velocity of 9·8 at the end of the first second of descent, of 9·8 × 2 or 19·6 at the end of the second second, and so on, the velocity acquired being proportional to the time.

(B.) The body will have fallen through 4·9 metres at the end of the first second, through 19·6 at the end of the second second, through 44·1 at the end of the third second, and so on, the space passed over varying as the square of the time.

(C.) If a body be projected upwards with any velocity, it will rise to such a height that when it falls again from this height it will, when it reaches the ground, have acquired, under the influence of gravity, during its descent a velocity equal but opposite to that with which it started on its upward journey.

LESSON IX.—CENTRE OF GRAVITY, ETC.

47. In a former chapter (Art. 30) it was seen that if two or more parallel pressures act upon a rod, and if a fulcrum or point of application of resistance be so chosen in the rod that the moments of the forces on the one side of the fulcrum equal those on the other side, there will be equilibrium. Now let us suppose that we have a set of downward pressures represented by heavy weights attached to such a rod, and that the system is moveable round an axis or fulcrum, such that the moments of the weights on the one side shall be equal to those on the other, there will in such a case be no tendency to rotation round the axis, but the system will balance in every position.

An obvious application of this principle is to heavy bodies of various shapes. Take for instance a uniform circular plate. Each particle of this plate is attracted to the earth by virtue of gravity, and the attractions on the various particles form a system of parallel forces, which will obviously balance round the centre of the plate.

In like manner a rectangular plate will balance round the point which constitutes the intersection of its diagonals ; in fact there is some point in every heavy substance about which it will balance. This point is called the **centre of gravity** of the Substance.

48. The following is a simple practical way of finding the position of this point. Suppose for instance (see Figs. 15 and 16) that we take a heavy plate of irregular outline, and suspend it by a string. It is evident that it will not hang as in Fig. 15, but rather as in Fig. 16.

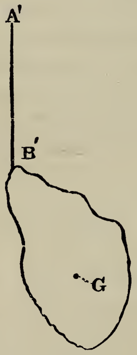

FIG 15.

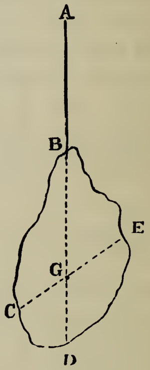

FIG. 16

The whole weight of the substance may be supposed to be concentrated in its centre of gravity, and if this point is supported there will be equilibrium.

Therefore when a heavy plate is hung by a string as above, it is clear that it will place itself in such a position that the centre of gravity will be vertically under the string, for we then have the whole weight of the body acting vertically downwards from its centre of gravity, and the tension of the string acting vertically upwards, and both forces in the same line, so that they will neutralize one another. Now draw on the heavy plate a line B D in prolongation of this string (Fig. 16). This line will evidently pass through the centre of gravity of the plate, although we do not yet know what precise point in it constitutes this centre of gravity. Let us, however, next suspend the same plate by another point, C, and mark, as before, on the plate the prolongation, C E, of the string by which it is suspended. This line must likewise contain in it the centre of gravity of the plate.

We have thus got two separate lines, B D, and C E, each containing the centre of gravity of the plate, which can therefore only be at the point G where these two lines intersect one another.

49. Let us now give a few practical illustrations of this doctrine. In the first place, if we have a heavy solid resting upon a base slanting-wise, how shall we know that it will not topple over?

We answer—It will remain stable so long as a vertical line, drawn from the centre of gravity, shall fall within the base, as it does for the lower solid in Fig. 17; for then the weight of the body, acting downwards, is neutralized by the resistance of the support. But if the solid be so high or so slanting that this line falls without the base, then it will topple over, and this will be the case for the large solid A B C D (Fig. 17).

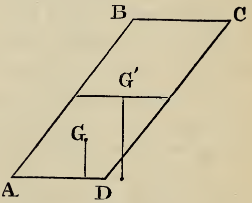

FIG. 17.

50. We have said that there will be equilibrium when the centre of gravity is supported; but this equilibrium may be of two kinds, *stable* or *unstable*.

When a body is in stable equilibrium, and a displacement takes place, it shows a tendency to recover its former position. But when a body is in unstable equilibrium, if it be displaced it shows a tendency to depart farther and farther from its original position.

Thus, for example, an egg is in stable equilibrium when resting on an extremity of its shorter axis ; and if a slight displacement be given to it, it recovers its former position. But it is in unstable equilibrium if standing on its longer axis,—that is to say, on its end ; for even if we succeed in making it stand thus, the very smallest displacement will cause it to topple over.

When we examine into the subject, we find a very simple law determining whether an equilibrium shall be stable or unstable. If a displacement tends to raise the centre of gravity, the equilibrium is stable ; and if it tends to lower it, the equilibrium is unstable.

For since we may suppose the whole weight of the body to be concentrated in its centre of gravity, raising this centre simply means raising the body itself against the action of gravity, and therefore implies an effort; inasmuch as the force of gravity will resist the raising of any substance vertically upwards ; there will, therefore, be a tendency in the body to resume its former low position.

On the other hand, if a displacement lowers the centre of gravity, it is the same thing as lowering the body, and such a displacement will be favoured and increased by the action of gravity, and not resisted as in the former case. Thus when an egg rests on its smaller axis, its centre of gravity is as low as it can possibly be, and any displacement lengthwise tends to raise this centre of gravity. Such a displacement is therefore resisted, and the egg, after several oscillations in consequence of the displacement, will ultimately recover its former position, just as a pendulum after a displacement ultimately assumes its position of verticality. But not so if the egg stands on its larger axis, for in such a position the centre of gravity is as high as it can possibly be. A displacement here tends to bring the centre of gravity, and therefore the whole mass of the egg itself, nearer to the Earth's centre, and is therefore favoured by the force of gravitation. The egg will therefore be in unstable equilibrium and the smallest touch will make it topple over.

Besides the two kinds of equilibrium to which we have alluded there is that of neutral or indifferent equilibrium, such as that of a sphere resting upon a horizontal plane.

Here, if a displacement occurs, the centre of gravity is neither raised nor lowered, but remains always at a height above the plane equal to the radius of the sphere, and in consequence the sphere will rest with indifference on any part of its surface, showing no tendency to prefer one point above another.

51. We have here an explanation of the principle of construction of those toys which, with the appearance of being top-heavy, are nevertheless so weighted beneath that the centre of gravity is raised, and not lowered, by displacement; the consequence is that when displaced they oscillate backwards and forwards until ultimately they regain the position from which they were moved. The following are a few examples in illustration of this subject :—

Example I.—A cone is placed on its apex on a flat horizontal surface : will the equilibrium be stable or unstable?

Answer.—Unstable.

Example II.—A uniformly heavy circular wooden disc has a piece of its substance taken out, and a piece of lead inserted instead. In what position will it rest on a flat horizontal surface?

Answer.—It will rest so that the lead will be below the centre of the disc and in the vertical line joining it and the point on which the disc rests. For the centre of gravity of the whole heavy disc, including the lead, will in this case be as low as possible, and any displacement will tend to raise it, and will therefore be resisted by the action of gravity.

Example III.—How will a man rising in a boat affect its stability?

Answer.—It will make it more unstable, because it will raise the centre of gravity of the system, so that an oscillation would now be more ready to swamp the boat than if its centre of gravity were low.

Example IV.—In like manner a cart loaded with hay is more liable to be overturned, owing to irregularities in the road, than one loaded with lead, because in the former case the centre of gravity of the system is high, and a comparatively small angular displacement, due to want of level in the road, may bring this centre of gravity into such a position that a line, drawn vertically downwards from it, shall pass without the wheels, in which case the system will topple over.

52. Let us now say a few words about the **balance.** This important instrument consists of a lever, having equal arms on either side of a knife-edge upon which it rests (Fig. 18).

From the extremities of these arms the scale-pans are suspended, and a pointer attached to the balance points vertically downwards when there are equal weights in the two scale-pans; but should the left-hand weight predominate, the pointer will be deflected to the right, and *vice versâ.*

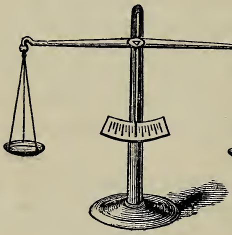

Fig. 18.

Let us begin by supposing that there is no weight in either scale-pan. The balance is so arranged that its centre of gravity is raised somewhat by displacement, so that, if displaced under these circumstances, it will come back to its original position.

If the balance be very delicate, this force tending to bring it back is a very small one, so that a very small additional weight in one scale-pan will push the pointer aside a considerable distance before it is counteracted by this force. If, for instance, the one scale-pan be a milligramme heavier than the other, the pointer may be pushed aside one division before the force of restitution is sufficient to overcome the additional pressure of the milligramme. If the additional weight be two milligrammes, the pointer will now be pushed aside two divisions, and so on. A sensitive balance enables us to ascertain with great exactness the weight of any body; for we have only to put the body to be weighed in the one scale-pan, and such a number of standard weights in the other, as to cause the pointer to point vertically downwards, when we may be sure that we have ascertained the weight of the body with very great exactness; for were the one scale-pan the smallest degree heavier than the other, it would cause an appreciable deflection of the pointer either to the one side or the other.

53. The pendulum has been already alluded to (Art. 34) as a means of measuring the force of gravity by the rapidity of its oscillations. Thus we find that the same pendulum moves somewhat slower at the equator than near the pole, and hence we conclude that the force of gravity is slightly less at the equator than at the pole. This is a result, in part at least, of the fact that, owing to the orange-like shape of the earth, a particle at the pole is really nearer the earth's centre than one at the equator. But the most frequent use of the pendulum is to regulate clocks, and the mode in which it is applied for this purpose is exhibited in Fig. 19, where B denotes the bob or heavy part of the pendulum. This pendulum in its oscillations moves backwards and forwards an escapement, C, terminated by two pallets which act upon the teeth of the escapement wheel, so that at each oscillation one tooth is allowed to escape, and thus the wheel moves

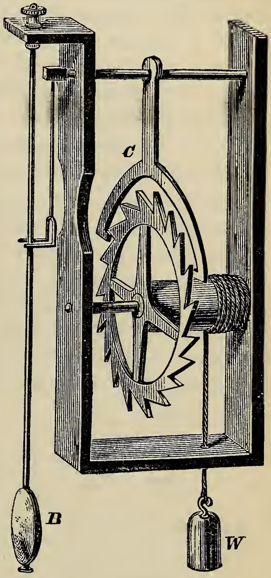

FIG. 19.

round one tooth at a time. If it were not for this connection of the escapement wheel with the pendulum, the effect of the weight, W, would be to make it move rapidly round until all the cord which is wrapped round its axis is uncoiled by the lowering of the weight. Thus the pendulum regulates the clock. The length of the *seconds* pendulum—that is to say, of the pendulum which vibrates from one of its extreme positions to the other once in a second—is very nearly one metre, and its time of oscillation is the same whether it makes small or very small swings. This property of the pendulum is called its **isochronism**, and was first discovered by Galileo, who noticed that a lamp swung by a chain in the cathedral of Pisa performed its oscillations in equal times without respect to their extent.

The time of oscillation of the pendulum will, however, be altered by altering its length. Thus, if it be four times as long, its time of oscillation will be doubled, if it be only one-fourth as long, its time of oscillation will be halved ; if it be one-ninth as long, its time will be reduced to one-third, and so on, *the time of oscillation varying as the square root of the length.*

LESSON X.—FORCES EXHIBITED IN SOLIDS.

54. In the present chapter we propose to discuss molecular and atomic forces, as they are exhibited in the three states of matter—solid, liquid, and gaseous. Let us, in the first place, briefly describe the most prominent variety of these forces.

We have in the first place cohesion, adhesion, and chemical affinity, which are attractive forces ; and in the next place we have those forces which resist any change of shape or volume in solids, and any change of volume in liquids and gases.

Cohesion denotes the attraction which the various molecules of the same substance have for one another, and in virtue of which the various particles of solids and liquids keep together.

We may use the word **adhesion** to denote the attraction exerted between particles of two different bodies when placed in contact with one another. On the other hand, when particles of different bodies have such an attraction for each other as to rush together and form a substance of a different chemical nature, then we have the operation of **chemical affinity.**

Thus we should characterize that force which holds together the various particles of a piece of glass as the force of cohesion ; but that force which causes a film of water to cling to a surface of glass we should denominate adhesion, while again that force which causes sulphuric acid and lime when brought together to unite in order to form sulphate of lime, we should term chemical affinity. Forces tending to alter the shape of a solid body may be applied to it in several different ways, each of which will be resisted by the body itself. Thus a thick metal wire, having one end fixed and a weight suspended from the other, will resist a force tending to twist the weight round ; this is called resistance to torsion. Or, again, a thick rod of metal will resist any force tending to pull its particles apart ; this is termed resistance to linear extension. And it will also resist any force tending to crush its particles together, and this is called resistance to linear compression.

Besides resisting deformation in these different ways, a solid bar, as for instance a bar of steel, will resist a force tending to bend it, thus exhibiting resistance to flexure.

Finally, it will resist any force tending to make its whole volume less, which we may term resistance to cubical compression.

We may exemplify this by a thick cylinder of india-rubber. A comparatively slight force will extend this cylinder or compress it in the direction of its length ; but when the cylinder is extended it becomes attenuated as regards its thickness, and when it is compressed it bulges out. We cannot therefore well deduce the cubical compressibility of india-rubber from such experiments, or assert how far the substance will yield to a force which tends to compress it in one direction without permitting it to bulge out in another.

We have therefore—

(1) Resistance to linear extension :
(2) Resistance to linear compression ;
(3) Resistance to cubical compression ;
(4) Resistance to torsion ;
(5) Resistance to flexure ;

all denoting various forces exhibited by solids, and the only one of these that is also exhibited by liquids and gases is the third, or resistance to cubical compression.

Before leaving this subject a few words must be said about **friction.** If we put a heavy weight on the table it will require a very strong force to move it along. But if the table were of marble and not of wood, then a much less force would make the weight slide along, while if the weight were on a sheet of ice it would move with a still smaller force. This resistance which the heavy weight offers to our efforts to push it along is called friction.

As long as the surface of the heavy weight in contact with the table remains the same, the force requisite to move it will be proportional to the weight itself, and this force or pressure required to move the weight divided by the weight itself expresses what is called the *coefficient of friction* for these particular touching surfaces. It follows as a corollary from what has now been said, that while the pressure remains the same, the friction has no relation to the magnitude of the surface. For if we suppose a plate of iron equal to four square inches and weighing four pounds, to rest upon a surface of marble or stone, and if we imagine it cut up into four squares of one inch each, then, evidently, the friction of one of these squares will be one-fourth of the friction of the whole ; but if now we take up three squares and pile them on the fourth, it is evident that the pressure on the fourth square will now be increased fourfold, and hence the friction will be increased fourfold also. It will therefore be the same as the friction of the original surface before it was cut up ;

or, in other words, the friction of a fourfold weight resting on a base of one square inch is the same as if it rested upon a base of four square inches.

What we have now said only holds approximately, for in some cases the friction is not strictly independent of the surface. Rennie has made some valuable experiments on this subject, and has derived the following laws :—

(1) In fibrous substances such as cloth, friction is increased by surface and time, and diminished by pressure and velocity. (2) In harder substances, such as woods, metals, and stones, the friction varies directly as the pressure without regard to surface, time, or velocity. (3) Friction is greatest with soft, and least with hard substances.

The application of grease in general diminishes the friction. In engines, for instance, the friction is very much diminished by means of grease.

55. With these preliminary remarks let us now consider the properties and forces of solids.

Crystallized Structure.—When a solid is formed slowly and without agitation, its particles usually arrange themselves so as to produce bodies of a definite and regular form, which are called crystals. Thus when small particles of water slowly freeze in a calm atmosphere, they form crystals of snow of beautiful symmetry and shape.

The crystallization of salts from solutions is of frequent occurrence, and in Nature we have crystals of great value, which cannot as yet be formed artificially, as for instance the diamond and the emerald.

Very often in crystals the various forces and properties are different in different directions. Thus if we prepare a long-shaped block or rod out of a crystal of Iceland spar, it will behave itself differently in many respects, according as its length is in the direction of the axis of crystallization, or in a direction at right angles to it.

56. Fibrous and Laminated Structure.—Some solids, especially those which compose organized bodies, have a fibrous structure. This is exemplified in wood, and in many vegetable products, such as flax. It is very difficult to break such a fibre as regards its length, but it is very easy to separate two contiguous fibres from each other. Wood, for instance, very easily splits along the fibre, but not at all easily across it. Again, certain artificial substances, by the processes to which they have been subjected, acquire a fibrous structure ; thus wrought-iron has acquired such by the process used in making it. Mica, oyster-shells, &c., are examples of bodies possessing a laminated structure, and the same remark that applied to fibrous bodies applies also to these ; mica, for instance, can be split very easily in one direction, but not in another.

57. Solids without Structure.—Many solids exhibit no apparent trace of structure ; glass is a familiar instance of a body of this kind, and sealing-wax is another. In many cases a solid, when first produced, exhibits no appearance of crystallization ; but afterwards, through time and vibration, the particles assume a definite structure. Thus after a cannon has been fired a great number of times its texture changes, and it becomes liable to burst. It would seem that the crystallized structure is the most natural one, and that the particles of a body have a natural tendency to assume this condition whenever circumstances permit of their doing so.

58. Cohesion in Solids.—Cohesion, as we have said, is the general name for that force in virtue of which the various particles or molecules of a body are kept together, and without which everything would fall into a state of powder. It is therefore a force tending to prevent disintegration of the body. When the force tending to rupture a body is one directly tending to pull its particles asunder, the resistance which it offers to this is termed its **tenacity**. When the tenacity of bodies is to be experimented upon, they are generally drawn out into prismatic wires, the cross-section of which is capable of being accurately measured, and being supported at one end, a weight is applied to the other. In such cases it is found that *the breaking weight is proportional to the cross-section*. Thus suppose the cross-section is one square millimetre, and the breaking weight 10 kilogrammes, if the cross-section be 2 millimetres the breaking weight will be 20 kilogrammes, and so on. The tenacity may therefore be measured by the weight necessary to break a prism of given section, say one square millimetre.

Time is, however, an element in all such experiments; thus a substance may endure for a short time a weight which if applied during a long time would be sufficient to break it.

M. Wertheim has made many experiments on the tenacity of various metals drawn into threads one millimetre in diameter, and he finds the following results :—

	Slow rupture.	Sudden rupture.
Lead drawn	2^{k}·07	2^{k}·36
„ annealed	1·80	2·04
Tin drawn	2·45	2·94
„ annealed	1·70	3·57
Gold drawn	27·00	28·40
„ annealed	10·08	11·10
Silver drawn	29·00	29·60
„ annealed	16·02	16·50
Zinc drawn	12·80	15·77
„ annealed	—	14·40
Copper drawn	40·30	41·00
„ annealed	30·54	31·55
Platinum drawn	34·10	35·00
„ annealed	23·50	27·70
Iron drawn	61·10	65·10
„ annealed	46·88	50·25
Steel thread drawn	70·00	99·10
„ annealed	40·00	53·90

From this table it will be seen that in all cases the breaking force for sudden rupture is greater than for slow rupture, thus confirming the remark made about the effect of time.

Wood, as may be imagined, requires a much larger force to break it in the direction of the fibre than in any other, and the results of some experiments of Musschenbroeck give the following breaking weights in kilogrammes for prisms of different kinds of wood, of which the section is one square millimetre :—

	Kilogrammes.		Kilogrammes.
Oak	6 to 8	Beech	8
Aspen	6 to 7	Box	14
Fir	8 to 9	Pear-tree	6
Ash	12	Mahogany	5
Elm	10·40		

But all bodies do not suddenly give way under a force tending to pull their particles asunder. When the force is long continued, the body often gradually changes shape, becoming always weaker and weaker until it yields. Sometimes this change of shape is very perceptible, and the body, by means of the applied weight, is drawn out into a thread.

59. **Ductility** denotes that property of bodies, in virtue of which they permanently change their form under the application of stretching force. Thus a piece of wax, at a tolerably high temperature, is very ductile, and may easily be drawn out into a thread; and at a high temperature a piece of glass is the same. On the other hand, it requires a very great force to pull a piece of iron or steel into a long thin wire.

60. **Malleability** is a modification of ductility. Some bodies do not stand being drawn out into very fine wires, but they may be hammered into very thin plates. Gold is the most malleable metal, and it has been beaten into leaves the thickness of which is only ·00008 of a millimetre.

61. **Brittleness.**—Disintegration may be accomplished in many ways; for instance by a sudden blow, and some bodies are particularly liable to be fractured by this means, in which case they are said to be brittle. Glass is an example of a body of this kind. Thus, in some respects, a sheet of glass, although stronger than a sheet of paper or cardboard, in resisting a pressure evenly applied, will yet be fractured by a blow which will not affect the paper. Disintegration may also take place by scratching the surface of a body.

62. Hardness, in the language of mineralogists, is that property in virtue of which a body resists the action of another tending to scratch its surface.

Therefore, if we have three bodies, A and B and C, of which A can scratch B, and B can scratch C, we say that A is harder than B, and B harder than C.

The diamond is the hardest of all known substances, and when a gem of this nature is to be cut, particles of diamond dust must be used in the operation.

Hardness is not capable of exact numerical estimation, but a scale has been adopted, by means of which the relative hardness of any substance may be easily found.

The following is the scale generally used :—

1. Talc.
2. Rock Salt.
3. Calc Spar.
4. Fluor Spar.
5. Apatite.
6. Felspar.
7. Quartz.
8. Topaz.
9. Corundum.
10. Diamond.

Thus suppose a body scratches calc spar, but not fluor spar, its hardness will be between 3 and 4, and so on.

63. Temper.—The hardness of a body may be made to vary according to the treatment which it receives. Thus if a piece of steel be heated to a great heat, and suddenly cooled, it becomes very hard ; this is called **tempering** the steel. Generally the particles of bodies which are suddenly cooled are in a state of constraint, and the body is very apt to break ; for instance, glass which is suddenly cooled is much more apt to break than that which is cooled slowly or **annealed,** as this operation of slow cooling is called ; and if the surface of a vessel of unannealed glass be scratched the whole vessel will probably fall to pieces. Prince Rupert's drops are small drops of glass, cooled by being dropped when in a melted state into water, and in these the interior is in such constraint, that when broken the whole structure goes to pieces with an explosion. There are also philosophical toys, consisting of thick glass vessels which will stand a strong blow, but go to pieces when a morsel of flint is allowed to scratch their surface.

64. Elasticity.—We have hitherto concerned ourselves with forces tending to disintegrate solids. All forces, however, do not produce this result; but provided the force applied to a solid be not too great, the solid, when the force is withdrawn, will exactly recover its previous figure. There is, in fact, a limit within which a solid may be temporarily acted upon with the certainty of its recovering its figure when the force is withdrawn, and this limit is called the **limit of perfect elasticity**, the word elasticity denoting tendency to recovery.

When this limit is overpassed the body does not recover itself, but becomes weaker and weaker until at length it yields to the applied pressure.

Therefore, in making a solid structure, such as a bridge, it must be made so strong, that the greatest possible load which it has to bear shall deform it much within the limit of perfect elasticity.

Let us now consider shortly the forces in a solid which resist displacement of its particles, all being within the limit of perfect recovery.

65. Force resisting Linear Extension.—Suppose we use a vertical rod one millimetre square and one metre in length, and apply a weight at the lower end, the rod being fixed at the other. Suppose that we take as our unit of force the weight of one kilogramme, and that it increases the length of the rod say by $\frac{1}{100}$ of a millimetre. If the weight be two kilogrammes, this increment of length will be doubled and become $\frac{2}{100}$ of a millimetre, and so on; *the elongation or increment of length being proportional to the weight or force applied.* This is the first law.

In the next place, *the elongation is proportional to the whole length of the rod;* for if the rod be very short it will obviously require a much greater pulling asunder of the particles, and consequently a much greater force to lengthen it by a definite amount than if it is long: hence the above law; so that, had the above-mentioned rod been two metres long, its elongation under the pulling force of one kilogramme would have been $\frac{2}{100}$ of a millimetre, and so on.

Thirdly, *the elongation caused by a given weight or force is inversely proportional to the cross section.* If for instance the cross section be two square millimetres, it will require a double force, or two kilogrammes, to produce the extension of $\frac{1}{100}$ of a millimetre; if the section be three square millimetres, it will require three kilogrammes, and so on; so that the extension produced by a single kilogramme will be only one-half when the cross section is doubled, and only one-third when it is tripled; that is to say, it will vary inversely as the cross section.

66. Force resisting Linear Compression.—It is difficult to avoid flexure in compressing a rod; but when the experiment is properly made, it is found that a rod will be as much compressed under the operation of a force, as it would be extended under the operation of the same force applied in the opposite direction.

The laws which we have given enable us to measure accurately the force with which a body resists linear extension or compression. Thus if we suspend by one end a rod *of any length*, having a square millimetre of cross section, and apply the weight of one kilogramme to its lower extremity, we shall find that it will be stretched out a *certain small proportion of its whole length*, this proportion being the same for different lengths, but differing, however, with the nature of the substance.

The following table, derived from the experiments of M. Wertheim made at temperatures between 15° and 20° C., shows this extension in fractional parts of the whole length of the rod for various metals:—

Lead	$\frac{1}{1727}$
Gold	$\frac{1}{5584}$
Silver	$\frac{1}{7140}$
Copper	$\frac{1}{10519}$
Platinum	$\frac{1}{15518}$
Iron	$\frac{1}{20794}$
Steel	$\frac{1}{18420}$

Thus we see that a rod of gold of one square millimetre in section will be extended $\frac{1}{5584}$ of its whole length by the weight of one kilogramme, and so on.

67. Force resisting Torsion.—Let a thread be suspended as in Fig. 20, and a weight be attached to the other end of the thread, having a pointer which moves over a dial-plate recording angular measure. If left to itself, the pointer will settle in some particular direction. If now the pointer be twisted round, this operation will be resisted by the thread, and the force which exercises this resistance is called the *force resisting torsion.*

It has been found that *the force resisting torsion is proportional to the angle through which the pointer is twisted*, thus, if the pointer be twisted 90° from its point of rest, the force tending to bring it back will be only one-half of what it would be were it twisted 180°, or one-fourth of what it would be were it twisted 360° or a whole turn.

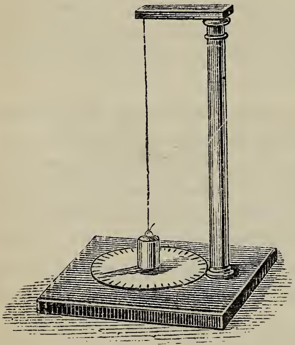

FIG. 20.

In the next place, *the force resisting torsion varies inversely as the length of the thread;* that is to say, if the thread be doubled in length it is only half as great, if it be tripled in length only one-third as great, and so on.

Finally, *it is proportional to the fourth power of the diameter of the thread.* Thus if the diameter of the thread be doubled, it will be increased $2 \times 2 \times 2 \times 2 = 16$ times; and if the diameter be tripled, it will be increased $3 \times 3 \times 3 \times 3 = 81$ times, and so on.

68. Resistance to Flexure.—The force with which a solid resists any attempt to bend it is made use of in a variety of ways. The main-spring of a watch, when coiled up, tends to uncoil itself from this cause, and the bow and the spring balance are other examples of the same. The laws of this force are best studied in the case of a beam fixed by one end into a wall, and loaded at the other extremity (Fig. 21). Rupture will be produced when this load is excessively great, but the limit will depend upon the length, the breadth, and the thickness of the beam.

In the first place, *the force necessary to produce rupture of a beam will vary inversely as the length of the beam;* so that if we double the length, a force of half the size will be sufficient, the reason being that it acts at a double leverage. Again, *the force necessary to produce rupture will be proportional to the breadth;* for if the beam be doubled in breadth, it will stand a force twice as great; and, finally, *the breaking force will vary as the square of the depth of a beam*, so that if the beam be doubled in depth it will stand four times as great a force. Therefore, in structures in which beams are heavily loaded, it is a much more advantageous use of the material to increase the depth than to increase the breadth of the beams.

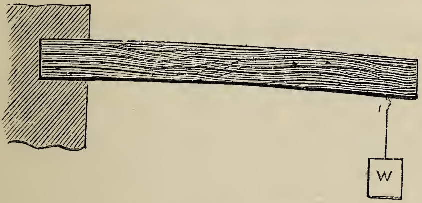

FIG 21.

What we have now said refers to forces producing rupture; but it is often desirable to study the action on a beam of forces much less than of those which rupture it. This is most conveniently done by measuring the vertical lowering of the extremity of the beam due to the applied force.

Let *a* represent this lowering of the extremity of a prismatic beam of length l, and breadth b, and depth d, for a weight w, applied at its extremity.

Then it is found, both as the result of theory and experiment, that $a \propto \frac{w\,l^3}{b\,d^3}$, or in other words, the depression of the extremity of a beam so loaded, is, in the first place, *proportional to the load.*

In the next place, *it is proportional to the cube of the length of the beam.*

Thirdly, *it is inversely proportional to the breadth of the beam.*

And lastly, *it is inversely proportional to the cube of the depth of the beam.*

LESSON XI.—FORCES EXHIBITED IN LIQUIDS.

69. Viewing adherence to shape as the essential characteristic of a solid, we come next to liquids, in which this property is almost entirely absent. In this class of bodies the particles slide along each other with very great freedom, and may easily be separated the one from the other. Cohesion is in their case very small, but yet it has not entirely disappeared, and it is only necessary to refer to drops of liquids as a proof that there is still a trace of cohesion in such bodies.

Thus a drop of water, whether pendent from a surface to which it adheres, or rolling along a surface to which it does not adhere, or a globular drop of mercury, such as we frequently see, are all instances that cohesion has by no means entirely vanished in the case of liquids.

All liquids are not equally endued with fluidity, and in many bodies an increase of heat will produce a transition from the solid to the liquid state by imperceptible degrees. We have already instanced the case of sealing-wax and glass (Art. 59) as bodies which gradually change their state. When a substance is in an imperfect state of liquidity, it is said to be **viscous**, and we need only refer to treacle or honey as examples of viscous fluids. In all such bodies time is an element which we must consider: thus, stir up the surface of a pot of honey or treacle, and it will ultimately right itself and become level; but it will take very much longer to do so than a similar surface of water or alcohol; or again, at a temperature only moderately high, support a long stick of sealing-wax horizontally by its extremities, and weight it slightly in the middle, and it will in the course of time be found to be bent into a curved shape without the appearance of fracture.

70. But while a liquid offers little or no resistance to the separation of its particles, or to their motion over one another, it offers very great resistance to a force tending to compress it into smaller volume. Thus if we fill a hollow cylinder with water, and fit into it a water-tight plunger, we shall not be able to drive this down, or to compress by its means the water in the cylinder to an appreciable extent.

So great, in fact, is the resistance to compression offered by liquids that for a long time these bodies were regarded as incompressible. It has, however, been found, by means of very delicate experiments, that this is not the case, but that for a pressure equal to one atmosphere, or 15 lbs. on every square inch of surface, mercury will become compressed about 0·000005 of its original volume, water 0·00005, and ether 0·000133. It is hardly necessary to state that when the pressure is removed the liquid recovers its original volume.

71. Equality of Pressure in all Directions.—This was a law of liquids discovered by Pascal, and it may be best seen in the case of a fluid uninfluenced by gravity. Its mode of action will be understood by reference to the annexed figure (Fig. 22), which is supposed to represent a hollow vessel containing water or any similar liquid, and having various cylindrical apertures of equal size fitted with moveable pistons. Now let a pressure, say of 10 kilogrammes, be put on the uppermost piston, A; this pressure will be transmitted by means of the particles of water, and will press out each of the various other pistons with a force in the direction of the arrow-heads, and equal to 10 kilogrammes, so that the force at A is transmitted by the fluid particles in all directions, causing on every portion of the surface, equal to that of the piston at A, a pressure perpendicular to the fluid surface, and equal to 10 kilogrammes.

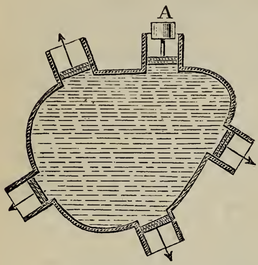

FIG. 22.

Let us now see what will happen if we vary the size of the different pistons.

In Fig. 23 let the piston A have an area equal to one square centimetre, and the piston B an area of 100 square centimetres; also let a pressure of 10 kilogrammes be applied to the piston A in a downward direction. From the above law it will follow that every square centimetre of the piston B will be pressed upwards with the force of 10 kilogrammes, because that is the pressure on one square centimetre at A, and this pressure will by Pascal's law be transmitted in all directions. Now the piston at B having the area of 100 square centimetres, there will be on the whole an upwardly acting pressure at B equal to 100 × 10, or 1,000 kilogrammes, and it will therefore lift this weight.

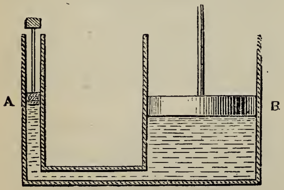

FIG. 23.

A fluid is therefore capable of forming a very powerful mechanical arrangement; for in the above machine the downward pressure of 10 kilogrammes is made to raise 1,000 kilogrammes, and, by increasing the proportion between the smaller and the larger piston, the mechanical advantage will be proportionally increased. Thus, if the area of A were one square centimetre, and that of B 500 square centimetres, and if A were pressed downwards by 10 kilogrammes as before, B would now be pressed upwards by a pressure equal to 5,000 kilogrammes, which is a very great force.

72. Bramah's Press.—Machines on this principle were introduced by Bramah, who made the force with which the large piston is moved upwards of service in pressing materials together. This contrivance is known as the hydraulic press, or Bramah's press, and is much used in many of the arts. Wool and similar materials are pressed together into small bulk by means of this machine.

73. Equilibrium of Fluids.—In what has preceded we have considered the liquid as closely shut in on all sides, and merely serving as a vehicle for transmitting pressure. Let us now take the case of a fluid in an open vessel, and find what will be the form of its surface. Suppose, for instance, that we have an open vessel (Fig. 24) containing water, and that this vessel is set on the surface of the earth.

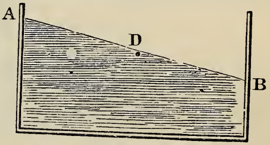

Fig. 24.

Now, as every particle of a liquid is free to move, it follows that when at rest there must be no excess of pressure urging the particle in any one direction, but every pressure must be counterbalanced by an equal and opposite pressure. Suppose now that under the action of gravity the surface of the water were to rest in an inclined position, A B, it is clear that there is a considerable weight of water above a particle, D, towards the left, and none towards the right; and this weight, forming a pressure which the particles of the water convey in all directions, will push the particle D towards B.

The particles will therefore not be at rest in such a position, and will only be so when the surface is perpendicular to the force of gravity which is acting on them; for then every heavy layer of water will act like a uniformly loaded piston on the surface below it, and will therefore do nothing but exercise a force tending to compress the particles of the water together, which will be counteracted by the resistance of the fluid.

We thus perceive that the surface of the ocean will be horizontal; that is to say, it will be perpendicular to the direction of the plumb-line at the place. For short spaces this surface may be regarded as a plane, but taken as a whole it will of course form a covering with a spherical surface surrounding the globe.

74. The **Water Level** is a useful application of this principle. Suppose (Fig. 25) that we have a tubular vessel containing water, the ends of which are bent at right angles to the middle part. When the instrument is at rest there must be equal and opposite pressures on any particle in the lower part of the tube, otherwise the particle would move either in one direction or another; hence the vertical height of the column of water above it on the left side must be equal to that of the column on the right, and hence a line drawn from the top of the water at A to the top of that at B will be strictly horizontal, as truly so as if A and B were portions of one continuous surface of a vessel of water.

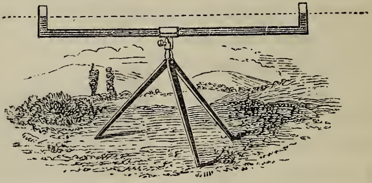

FIG. 25.

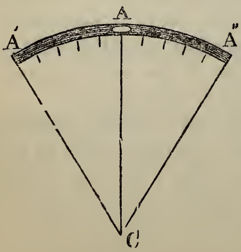

FIG. 26.

75. The air-bubble level or **spirit level** is, however, much more convenient than the water level. To understand its principle, imagine a curved hollow glass tube forming a portion of the circumference of a circle of large radius, of which C is the centre.

Let this tube be filled with spirits of wine or some mobile liquid, all except a small bubble of air. This air-bubble will always seek the highest point. In the figure (Fig. 26) this is at A; but suppose the whole arrangement turned round C, from left to right through the angle A C A′, it is clear that A′ would now be the highest point, and the bubble would be found there.

If the curved tube is graduated, we can at once read off on its scale the position of the bubble, and thus ascertain how far the instrument has been turned round the centre.

In practice this bent tube is firmly attached to a flat piece of metal, as in Fig. 27, and so adjusted that when the bottom of the instrument is horizontal the bubble shall be precisely in the middle.

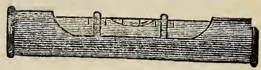

FIG. 27.

Suppose now the instrument be tilted up to the smallest degree, the bubble will no longer rest in the middle graduation, but at some other point; and noting the number of graduated divisions passed over by the centre of the bubble, and knowing also the value of each division, we shall know at once the precise angular displacement from the level position caused by the tilting of the instrument.

76. Artesian Wells.—It sometimes happens in Nature that a layer of water becomes collected between two strata of the earth's crust, which are imperviable to this fluid. In the lower part of this layer the water will exist under considerable pressure, due to the height of the highest point of the layer above the lowest. If therefore the surface of the ground be less than this height above the lowest point of the layer, and if we sink a well, the pressure of the water at the bottom of the layer will be sufficient to drive the fluid up the shaft of the well, and to cause it to flow over, and even perhaps to rise in the form of a fountain. Such wells are called Artesian Wells, the name being derived from the province of Artois, where they were first dug in modern times; but the method of procuring water by this means was known to the ancients.

77. Pressure of Liquids contained in Vessels.—It will readily be gathered from what we have said that *the pressure* *on any layer of liquid contained in an open vessel is proportional to its depth below the surface.*

The pressure is, in fact, due to the weight of the superincumbent column of the liquid.

Thus the weight of one cubic centimetre of distilled water is one gramme. If, therefore, we have a vessel of pure water, the pressure of the fluid against a square centimetre of surface immersed in the water at the distance of one centimetre below the surface will be one gramme. This surface, in fact, sustains the weight of one cubic centimetre; that is to say, of one gramme of water above it. This pressure acts by Pascal's law in all directions, upwards as well as downwards, and the upward pressure of a layer of water may be shown by means of the following simple experiment:—

Let there be a large glass tube, open at both ends, and let the one end of it be ground flat, so as to be capable of being fitted with a ground glass cover having a string attached to it. Fit the tube with this cover, thus giving it a loose bottom, and, retaining in the hand the string attached to this bottom, plunge the tube into a vessel nearly full of water. If the string be now let go, the loose bottom will not leave the tube, but will cling to it, being kept tightly fastened to it by the upward-bearing pressure of the water.

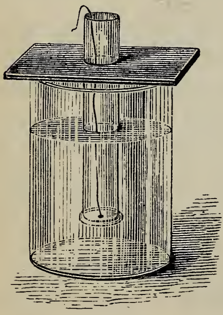

FIG. 28.

Now fill the tube itself with water, and whenever the water in the tube attains the same level as that in the vessel, the bottom will leave the tube, thereby showing that the upward-bearing pressure on the bottom of the tube is equal to the weight of a column of water extending from this bottom to the surface of water in the vessel.

78. The following examples will serve to illustrate the subject:—

Example I.—A hollow cubic decimetre, open at the top, is filled with water; what will be the pressure on the bottom and sides? *Answer.*—The depth being 10 centimetres there will be on each square centimetre of the bottom a pressure equal to that of a column of water 1 centimetre square and 10 centimetres high—that is to say, 10 grammes; and there being 100 square centimetres in 1 square decimetre, the total pressure on the bottom will be 100 × 10 = 1,000 grammes. This will, in fact, be the weight of the water which the vessel contains, and which is supported by the bottom.

Next, with regard to the sides; by the law of Pascal, since the pressure is transmitted in all directions, there will be on each small unit of area at the bottom of one of the sides a pressure represented by a column of water having this area for its base, and 10 centimetres high, while, on the other hand, the pressure on any small unit of area at the top of the side will be nothing, since it is just at the top of the water.

The mean pressure upon a side will therefore be represented by the mean of these two extreme pressures; that is to say, by a column of water equal to $\frac{0 + 10}{2}$, or 5 centimetres high, pressing upon the side.

But the area of the side is 100 centimetres. Hence 100 × 5 = 500 grammes will be the whole pressure against any side; so that were a side moveable outwards by a hinge from the bottom, it would be necessary to apply at the proper point a pressure of 500 grammes pressing inwards, in order to resist the pressure of the water tending to push the side outwards.

Example II.—A vessel contains water to the depth of a decimetre, and one of the sides of this vessel is a rectangular surface, the bottom of which is one decimetre, while the side slopes at an angle of 45°; what is the whole pressure on this side? *Answer.*—The whole surface of this side is 100 square centimetres × $\sqrt{2}$, and the mean pressure on one square centimetre of the side is as in Example I.,

$\frac{0 + 10}{2} = 5$ grammes, and this pressure, by the law of Pascal, acts perpendicularly to the side. Hence the whole pressure against the side will be $500\sqrt{2}$.

79. We have hitherto considered the pressures on the sides of a vessel containing water; but if the liquid be other than water, the pressure will of course be different. Thus if the liquid be mercury, which is 13·6 times heavier bulk for bulk than water, the pressures which we have calculated for water will have to be increased in this proportion. Or if the liquid be alcohol, which is bulk for bulk only 0·8 times as heavy as water, the pressures will be less in this proportion.

In fact, *the pressure will be proportional to the density of the liquid.*

80. Flotation.—Suppose now that in the midst of a vessel of water a portion of the water were suddenly to become rigid, retaining however its density, its volume, and all its other properties unaltered. This will not alter the conditions of equilibrium, because rigidity merely means an indisposition to certain kinds of motion; but since the system was in equilibrium there was no disposition to move before the rigidity commenced, and hence the portion of water suddenly become rigid will remain in equilibrium, and will therefore not alter its position, but will still remain suspended in the centre of the liquid. Now this isolated rigid portion will be attracted downwards by the force of gravity represented by its own weight; but since it remains at rest, this force must be counteracted by an equal but opposite force, due to the pressure of water in which it is immersed. It thus appears that the buoyancy or floating power of a fluid is sufficient to counteract the weight of a solid substance immersed in it, and of the same density as itself. If, however, a solid immersed in a fluid be of greater density than this fluid, the upward pressure will not be sufficient entirely to overcome the weight of the solid, but it will appear to lessen it by the weight of its own volume of the fluid in which it rests. If left to itself, such a solid will therefore sink to the bottom; or if supported by a thread, it will act as a heavy body, but not with the force due to its whole weight, but with a force less than this by the weight of its own bulk of the fluid in which it is weighed.

If, however, the solid be lighter than the fluid, it will not sink therein, but a certain portion of its volume will become immersed, such that the weight of a volume of the fluid equal to this immersed portion shall be equal to the whole weight of the solid. Thus, for instance, a cube of wood, of a density = 0·8 of that of water, is put into a vessel containing water; what portion of its side will be immersed? *Answer.* —Eight-tenths; for then the weight of the volume of water displaced by these eight-tenths will precisely equal the whole weight of the cube.

81. Specific Gravity.—By taking advantage of the property of liquids now mentioned, we are furnished with a means (first discovered by Archimedes) of ascertaining the specific gravity or relative density of bodies.

Suppose, for instance, we take distilled water at the temperature when it has its maximum density (4° C.) as our standard, and call its density unity. Suppose, also, that a substance weighs in vacuo 120 grammes, and when immersed under water at 4° C. only 89 grammes. It is thus when in water apparently lighter by 31 grammes, and this, according to the foregoing principles, will be the weight of its own bulk of water. Now, obviously the density of the substance will bear the same proportion to the density of water as the weight of the substance bears to the weight of its bulk of water; that is to say, density of substance : density of water or unity : : 120 : 31, and therefore the density will be $\frac{120}{31}$ = 3·87. Hence we have the following simple rule:—*Divide the whole weight of a solid body by its loss of weight when weighed in water at 4° C., and the quotient will represent the specific gravity or comparative density of the body at this temperature.*

This method, however, will only give us the specific gravity of solids; but we can obtain that of liquids by means of similar principles.

Suppose, for instance, we wish to ascertain the specific gravity of a liquid. Let us weigh a solid body first in water, and let its loss of weight be 31 grammes. Let us now weigh the solid in the liquid, and let its loss of weight be 28 grammes. This latter loss will represent the weight of a quantity of the liquid equal in volume to the solid, while 31 will represent the weight of the same volume of water. Hence—

Density of the liquid : density of water : : 28 : 31, and hence the specific gravity of the liquid will be $\frac{28}{31} = 0{\cdot}903$.

In the following table we have the specific gravities of a few of the most important solids and liquids, the comparison being made at the zero of centigrade, and the standard of density being that of water at its point of maximum density, or 4° C. :—

Platinum (rolled)	22·069
Gold (forged)	19·362
Lead (cast)	11·352
Silver (cast)	10·474
Copper (cast)	8·788
Brass	8·383
Iron (bar)	7·788
Iron (cast)	7·207
Tin (cast)	7·291
Zinc (cast)	6·861
Diamond (maximum)	3·531
Flint glass	3·329
Ivory	1·917
Melting ice	0·918
Beech	0·852
Yellow pine	0·657
Cork	0·240
Mercury	13·598
Sulphuric acid	1·841
Hydrochloric acid	1·240
Nitric acid	1·217

Sea water	1·026
Absolute alcohol	0·803
Ether	0·723

82. Capillary Phenomena.—When open tubes, having a very small bore, are placed in vessels containing liquids, we have certain phenomena which appear at first sight to contradict the laws which we have just stated. If, for instance, a glass tube of this kind be placed in a vessel containing water, the level of the water in the tube will be above that of its general surface; also the surface of the water in the tube will be concave, the whole appearance presented being that of Fig. 29.

But if the same tube be placed in a vessel of mercury, the level of the mercury in the tube, instead of being above that of the general surface, will be below it, as in Fig. 30, while the surface will be convex, and not concave.

The narrower the bore of the tube the more pronounced will these phenomena be; and if a substance full of small pores, such as a lump of sugar, be placed with its lower extremity in water, the ascent of the water into the sugar will soon moisten the whole lump.

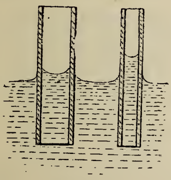

FIG. 29.

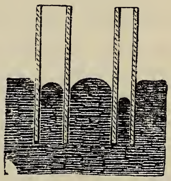

FIG. 30.

But it is only when the fluid is capable of moistening the sides of the tube that it will ascend, so that *we have two kinds of capillary action, in one of which the liquid ascends, wets the tube, and has a concave surface, and in the other of which it descends, does not wet the tube, and has a convex surface.*

Also *the capillary ascent or depression is inversely proportional to the diameter of the tube*, so that with a narrow tube we have a considerable difference of level owing to capillarity. The ascent of oil in lamp-wicks, the diffusion of moisture throughout the earth, the action of blotting-paper, sponges, and porous substances generally, afford illustrations of the laws of capillarity.

83. Endosmose and Exosmose.—Certain phenomena which much resemble capillarity are produced when two different liquids are separated by a membranous partition. In this case, generally, there is a carriage of both liquids across the membrane, but of the one liquid more than the other, so that there is an increase of substance on the one side, and a diminution of substance on the other. The current which sets so as to increase the volume is called endosmose, and when it sets in the opposite direction it is called exosmose.

Thus, for instance, if a strong syrup be placed in a membranous bag, and the whole immersed in water, it will be found that the amount of substance in the bag has increased, owing to some of the water having entered ; and at the same time it will be found that some of the syrup has mixed with the water outside.

LESSON XII.—FORCES EXHIBITED IN GASES.

84. We have seen that in liquids the force of cohesion has not entirely vanished, but in gases there is no trace of such a force, instead of which we have repulsion exerted between the various particles, so that a gas, however small in quantity, will always fill the vessel in which it is held. Nevertheless, a gas, like all other substances, possesses mass and weight. When we proceed to describe the Barometer, it will be shown that our atmosphere has weight, but in the meantime an experiment may be described illustrating the weight of a gas.

Let a large glass flask be accurately fitted with a stop-cock, and also at the extremity of the cock with a screw, by means of which it can be attached to the receiver of an air-pump. First of all let the flask, having its stop-cock open, and being of course filled with air, be weighed, attached to the scale-pan of a balance. Let it then be carried to a pump, and let the air which it contains be withdrawn ; let the stop-cock be now shut, and the flask again weighed in the balance after it has been thus deprived of air. Its weight will now be found to be sensibly less. Let it next be filled with hydrogen gas and again weighed, its weight will now be found greater than when it was empty, but less than when filled with air. Finally, let it be filled with carbonic acid gas, and it will be found to weigh heavier than when filled with air. We thus perceive that gases have weight, and that some gases weigh more than others, hydrogen being lighter than air, and carbonic acid heavier.

85. Just as solids are converted into liquids by the application of heat, so liquids are converted into gases by increasing the heat. Thus at 0° C. ice is converted into water, and at 100° C. water is converted into steam, which is a gas.

We must not, however, imagine from this example that gases are visible. The visible cloud arising from a kettle or a railway engine is not true steam, it is rather small particles of water, into which the steam has condensed through contact with the cold air.

Often, however, near the spout of a kettle or the funnel of a locomotive, the matter, which we know to be issuing out, is nevertheless invisible. Then it is true steam.

Elastic fluids have been divided for convenience' sake into **gases** and **vapours.**

A gas denotes a substance which at ordinary temperatures remains gaseous ; and a vapour denotes a substance in the gaseous form which at ordinary temperatures is solid or liquid. Thus, for instance, steam is a vapour, because the substance, water, from which it issues ordinarily appears as a liquid, and can only be driven off into vapour through the application of heat.

Carbonic acid, on the other hand, is a gas, and can only be brought into the liquid or solid state through intense pressure or intense cold. Most gases have been forced into liquids through the joint effect of these agents ; nevertheless there are six substances which we have not yet been able to condense : these are oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas.

On the other hand, there are some substances which can only be driven into the state of vapour through the most intense application of heat; and these are called refractory substances. Carbon is a body of this nature.

86. The Atmosphere.—The gaseous body with which we are best acquainted is our own atmosphere. It is chiefly composed of the two elementary gases, oxygen and nitrogen, mixed together in the proportion of 23 parts by weight of oxygen, and 77 parts by weight of nitrogen; there is likewise a little carbonic acid gas and a trace of ammonia in the atmosphere. Besides this it contains a variable proportion of aqueous vapour, which sometimes exists in it in a strictly gaseous invisible form, and is sometimes deposited in the form of a cloud.

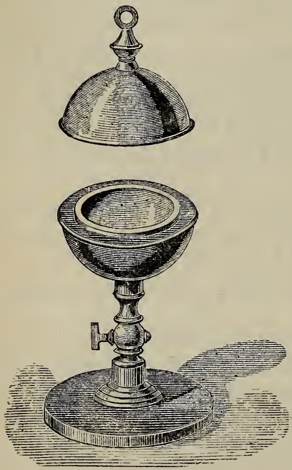

Fig. 31.

When animals breathe, or when combustion takes place, the oxygen of the air is thereby converted into carbonic acid gas. If this process were to go on without being remedied, the air would, in the course of ages, gradually deteriorate, losing its oxygen, until it became unfit for the respiration of animals. But a check is put upon this by plants, in which the reverse process takes place; that is to say, instead of inhaling oxygen and giving out carbonic acid, they inhale carbonic acid and give out oxygen; and thus, by the joint action of the animal and vegetable kingdom, a balance is kept up, and the condition of the atmosphere remains unchanged.

87. Its Weight.—The atmosphere possesses weight, and hence presses upon the substances at the earth's surface; but being a fluid, this pressure is by the law of Pascal transmitted in all directions. Thus a piece of paper is not forcibly held to the ground by the weight of the atmosphere above it, but there is as much upward pressure upon its under surface as there is downward pressure upon its upper surface ; and so we fail to perceive the traces of any pressure.

One of the most interesting experiments in illustration of this equality in all directions of atmospheric pressure, is that of the Magdeburg hemispheres, so called from Otto von Guericke, burgomaster of Magdeburg, who first invented them. They consist of two hollow brass cups (Fig. 31), capable of being fitted very accurately together. The lower of these two cups has a stop-cock attached to it, and is likewise capable of being screwed on to the receiver of an air-pump, by which, when the two cups are joined together, the whole may be deprived of air. If this be done and the stop-cock be shut, the hemispheres may then be detached from the pump, and it will be found impossible without a very great effort to force them asunder. As soon, however, as the stop-cock is opened so as to admit the air, they will come asunder with the greatest ease. The reason of their being forced together when exhausted, arises from the fact that in this state the air presses them together from without, while there is no air within to counteract this pressure ; as soon, however, as air is introduced into the interior, this pressure is counteracted, and they may be separated with ease.

88. The Barometer.—Torricelli, a pupil of Galileo, was the first to devise an instrument by which the pressure of the air can be accurately measured.

It had long been remarked that in the ordinary lifting pump when the piston was drawn up the water followed it, and the reason alleged was only a quaint way of expressing the fact without assigning any cause. It was said that Nature abhorred a vacuum ; but it was afterwards found that this abhorrence only extended to the height of about thirty feet, and that if the piston was raised higher than this, the water would not follow it. Torricelli rightly conceived that this was an effect of the pressure of the air, and that if we make use of a heavier fluid than water, this will refuse to mount long before the water will. In fact the fluid, he argued, will rise in a tube which is a vacuum to such a height, that the downward pressure of the column will exactly balance the upward pressure of the air.

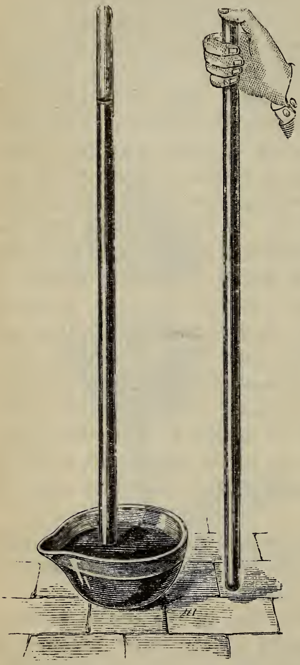

Fig. 32.

With this purpose he procured a glass tube more than thirty inches long, shut at the one end and open at the other, and having filled it with mercury (Fig. 32), which is more than thirteen times heavier than water, he inverted the tube into a basin of the same fluid, and he found that the mercury remained suspended in the tube to the height of about 760 millimetres above the level of that in the basin, while the space above the top of the mercury in the tube was entirely empty.

He therefore concluded that the pressure of the atmosphere is such as to sustain a column of mercury 760 millimetres in height, so that the whole weight of our atmosphere would be equal to that of an ocean of mercury surrounding the globe, and about 760 millimetres high.

This inverted tube is called a Barometer, which means an instrument for ascertaining the weight of the air ; and the vacuum above the column of mercury in a barometer tube is called the **Torricellian vacuum**; it is one of the most perfect that can be produced.

89. Pascal's Experiments.—The truth of Torricelli's discovery was verified in a different manner by Pascal. Arguing that the atmosphere is an ocean, he imagined that the pressure of this ocean will vary with its depth; that is to say, if we ascend a mountain, and leave a quantity of the heavy matter of the air below us, the pressure will be proportionally diminished.

Accordingly, on ascending an elevated mountain termed the Puy de Dôme, he found that at the top the mercurial column read nearly three inches lower than at the bottom, which he correctly attributed to the weight of air left beneath. Of late years the height of the mercurial column in a barometer has been very extensively used in ascertaining the height of mountains, which can thus be determined, although not with the same accuracy as by trigonometrical measurement.

The behaviour of the barometer is probably, to some extent, an index of the weather that is about to follow; but in using the indications of the barometer for this purpose, it is desirable to compare together the records at different places, and not trust too much to those at any one place. Generally speaking, when there is very rapid movement in the barometer much atmospheric disturbance may be expected, and there will be a conveyance of air from those districts where the barometer is high, that is to say, where there is a surplus of air, to those districts where it is low, that is to say, where the air is deficient in quantity.

90. Boyle's Law.—We have said that we do not feel the effects of the atmospheric pressure because it is exerted in all directions. Thus, if we have a flask full of air provided with a stop-cock, and shut the stop-cock in the open air, the glass of the flask will not be subject to any pressure on account of the air. There will, no doubt, be a very strong pressure of the atmosphere upon its outer surface; but the air which it contains, and which has been shut off when in a state of equilibrium with the outer air, will exert just as much pressure from within upon the flask, but in an opposite direction; there will thus be no tendency either to force the sides of the flask in, or to force them asunder. If, however, by any method we abstract a portion of the air within the flask, this state of things will be altered. The pressure of the particles within will now no longer be able to equal that of the atmosphere without, and the tendency will be to press together the sides of the flask. Now, it is found that if we take away half the mass of the air within the flask, the pressure on one square unit of surface will only be one-half of what it was; or if we take away three-quarters of the whole mass, so as only to leave one-quarter, the pressure will be only one-quarter of what it was, and so on. That is to say, *the pressure of a quantity of air shut up in a flask in this manner will be proportional to its mass.*

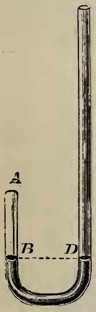

FIG. 33.

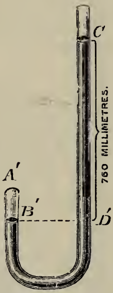

FIG. 34.

This law was discovered by Boyle, by whom it was put in a slightly different form. The truth of the law will be seen from the following simple experiment:— Let us take a tube, shaped as in Fig. 33, shut at one end, and having a uniform bore throughout, and suppose that it contains, separated from the atmosphere by a little mercury, a quantity of air, filling the tube A B. This air, let us suppose, exists at the ordinary atmospheric pressure, equal to that of 760 millimetres of mercury, and is the same in all respects as the outer air.

Let us now, as in Fig. 34, pour a quantity of mercury into the long leg of the tube until the level of the mercury in this leg is 760 millimetres above that of the mercury in the shut part. This difference of level will cause a pressure tending to compress the air in A′B′ equal to that of a column of 760 millimetres of mercury.

Besides this we have the pressure of the outer air, conveyed through the mercury, tending also to press together the air in A′B′. Hence altogether we have a pressure equal to 1,520 millimetres of mercury, or that of two atmospheres, tending to press together this air, whereas in the first figure there was only the pressure of one atmosphere. It will be found that under this double pressure the air will only occupy half the volume; that is to say, A′B′ will be one-half of A B: had the pressure been tripled, the volume would have been reduced to one-third; in fact, *the volume varies inversely as the pressure*, and this is the law of Boyle.

It will easily be seen that this is only another form of the previously stated law that the pressure of air varies as its density; for since there is the same quantity of air in A′B′ as in A B, and since the volume A′B′ is only one-half of A B, it follows that the density of the air in A′B′ is double of what it is in A B; but the pressure is also double, and hence the pressure is proportional to the density.

In what we have stated it is supposed that the temperature has remained the same throughout. It will be afterwards shown how the pressure of a gas varies with its temperature.

91. It thus appears that when a quantity of air is inclosed in a vessel, it presses against the sides of the vessel in all directions, and is in this respect very different from water, which assumes a definite surface; it further appears that this pressure on a square unit of surface is proportional to the mass of air contained in the vessel, the temperature remaining the same. This fact of gaseous pressure has induced many philosophers to imagine that the particles of a gas are continually moving about in all directions, knocking against one another, and against the sides of the vessel that contains them, and that it is the accumulated effect of these blows that constitutes gaseous pressure. This theory would appear to afford a good explanation of the fact that gaseous pressure is proportional to the density of the gas. In order to give our ideas a tangible form, let us suppose that a hollow cubic metre is filled with gas, the particles of which are exactly one millimetre apart. There will therefore be 1,000

× 1,000 × 1,000, or one thousand million particles in the vessel.

Suppose, now, that air is abstracted until the distance between two contiguous particles is increased to 2 millimetres. There will, therefore, be 500 × 500 × 500, or one hundred and twenty-five million particles in the vessel, so that the mass will be only one-eighth of what it was.

Now, in the first case there would be 1,000 × 1,000, or one million particles in immediate contact with one of the sides of the vessel, so that we should have a million little balls knocking against that side ; and when these had delivered their blows, the layer immediately behind them would follow, let us say (to fix our thoughts) at the interval of half a second, and there would thus be two million blows, delivered in one second.

But, in the second instance, there are only 500 × 500, or 250,000 particles in contact with one of the sides, so that there will only be this number of blows delivered instead of the million in the previous case. But when these blows have been delivered, we shall have to wait twice as long as in the previous instance, or one second instead of half a second, for the next broadside, for the second row of particles have now two millimetres instead of one to travel in order to come up to the side of the vessel, and deliver their blows, so that we shall have only 250,000 blows delivered in one second instead of 2,000,000, which was the number in the first case. We shall, therefore, have only one-eighth of the number of blows delivered in the same time, and hence the pressure will be reduced eight times, which is also the proportion in which the density was reduced.

92. Buoyancy of Air.—Gases as well as liquids possess buoyancy ; and just as a body immersed in water is rendered lighter by the weight of its bulk of water, so a body immersed in air is rendered lighter by the weight of its bulk of air.

A very good experiment in illustration of this is to attach to one arm of a balance a large hollow globe, and to counterpoise it by a small and heavy solid attached to the other arm (Fig. 35), so that in atmospheric air the weight of the two arms is exactly the same. If the balance be now placed under the receiver of an air-pump, and then exhausted, the two arms will no longer be in equilibrium, but the weight of the globe will preponderate. The reason of this is that in vacuo we obtain the true weight, so that the globe is really heavier than the solid used to counterpoise it; but as the volume of the globe is much larger than that of the counterpoise, the former will apparently lose more in weight through the buoyancy of the air than the latter, and hence in air they may appear to be of equal weight, although the globe is in reality the heavier.

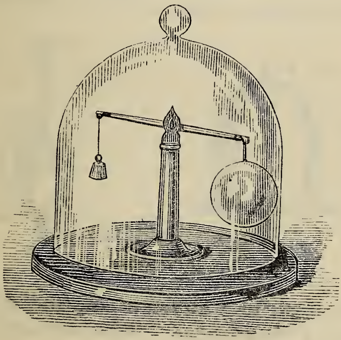

FIG. 35.

When a large globe is filled with some gas, such as hydrogen or coal-gas, that is lighter bulk for bulk than air, it will on account of this buoyancy strive to rise in the atmosphere, just as a piece of cork will strive to rise in water. A **balloon** rises from this cause; it is filled with hydrogen, or coal-gas, and the united weight of this gas, of the balloon itself, and of that which its car contains, must always be less than the weight of the same bulk of air in order that the balloon may rise.

We shall now describe the construction of several instruments which depend for their action on the pressure of the air.

93. Air-pump.—The intention of this instrument is to deprive a vessel, as far as possible, of the air which it contains, and this is done in the following way :—

Let V (Fig. 36) denote a vessel of glass, the bottom of which fits accurately on a well-ground plate of metal or other substance. Through the centre of this plate there is an opening communicating by means of a bent tube with the cylinder C, and where the pipe joins the cylinder there is a small valve, v, capable of opening upwards, but not downwards. There is also an accurately-fitting piston which moves in this cylinder, and in this piston there is another valve, v', which opens upwards, and not downwards.

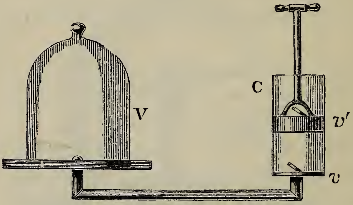

FIG. 36.

In the first place, let V be full of air, and let the piston be at the bottom of the cylinder. When we raise this piston a vacuum is immediately produced, and this cannot be filled from the outer air, since the valve in the piston only opens upwards ; it can, however, be filled from the air in the receiver V by means of the valve v, which opens upwards into the vacuum. Thus when the piston is at the top of its stroke, the air which at first filled the vessel V, will now fill both V and the cylinder. Next, let us push the piston down again, and the first effect of this will be to shut the lower valve v, and to open the upper valve v', through which the air in the cylinder will escape into the atmosphere.

Thus the effect of the double stroke has been in the first place to bring the air of the vessel V to fill both V and the cylinder; and, secondly, to drive into the outer air that portion of it which filled the cylinder.

Let us suppose that the capacity of the vessel V is four times as great as that of the cylinder, and that we have a mass of air in the vessel V equal to 100 to begin with.

Then (1) when the piston is raised to the top of the cylinder there will be 80 parts of this air in V, and 20 in the cylinder.

(2) When the piston reaches the bottom of the cylinder the 20 parts in the cylinder will have been driven out into the atmosphere, leaving 80 parts in V.

If this operation be repeated, we shall have, after the next upward stroke of the piston, 64 parts of air in V and 16 in the cylinder, and after the corresponding downward stroke there will be only these 64 parts of air left in V.

Thus, after the first double stroke, the air in V was diminished in the proportion of $\frac{4}{5}$ths, so that we had $100 \times \frac{4}{5} = 80$ parts of air left; and, in like manner, after the second double stroke, these 80 parts were diminished in the same proportion, and we had $80 \times \frac{4}{5} = 64$ parts left.

The law of diminution is very obvious; for instance, at the end of the third stroke the air left will be $100 \times \frac{4}{5} \times \frac{4}{5} \times \frac{4}{5} = 100 \left(\frac{4}{5}\right)^3$; at the end of the tenth stroke, $100 \left(\frac{4}{5}\right)^{10}$, and so on. We shall never by this means succeed in depriving the chamber completely of the air which it contains, and the limit will be reached when the pressure of the residual air is not sufficient to lift up the lower valve when the piston is raised in the cylinder.

94. Lifting-pump.—In the lifting-pump we have, besides the cylinder or barrel, a tube which extends downwards into a reservoir of water, part of which we wish to obtain. At the point where the tube enters the cylinder we have a valve, v, opening upwards, and in the piston we have another valve, v', also opening upwards. If, to begin with, we have air in the tube below the valve v, the first action of the lifting-pump is similar to that of the air-pump, and part of this air is abstracted, so that the pressure of air left in the tube is less than that of the atmosphere. Now, as the atmosphere presses upon the surface of the reservoir of water into which the tube is plunged, and as this pressure is no longer counteracted by an equal pressure from within, owing to the abstraction of a portion of the air, the water will be forced into the tube, in which it will rise, and continue doing so as the operation goes on. There will, however, be a limit to the height to which it will mount; for if the tube is more than about 30 feet long, we shall not be able to get it into the barrel or cylinder, and the pump will not work, the reason being that the pressure of the atmosphere is just about equal to that of a column of water 30 feet in height.

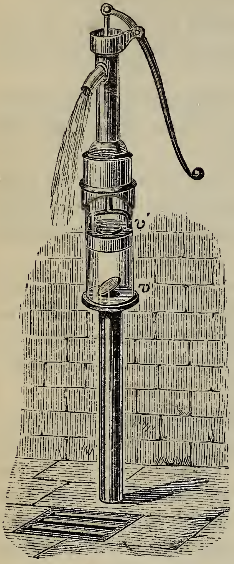

Fig. 37.

When the air has been pumped out, and the barrel is full of water, as we force the piston down, the upper valve, v', will open, while the under valve, v, will shut, and a quantity of water will thus be carried above the piston, where it may be discharged through a spout, or otherwise disposed of. Then, again, when the piston is drawn up water will follow it, rushing into the barrel through the lower valve; and as it again descends the lower valve will shut, and the upper one open, and so on.

95. Siphon.—The siphon is an apparatus for conveying liquid out of a vessel to any lower level, without the intervention of an ordinary tap. In its essential form it consists of a bent tube, open at both ends. For practical convenience, and to increase the velocity of efflux, one leg is frequently made longer than the other, but this is not necessary for its action.

The tube being filled with some of the liquid, one end is submerged beneath the surface of the liquid in the vessel;

the other end, let us suppose, is closed for the present by the thumb. Referring to Fig. 38 we see that if the vertical height of D A above the liquid surface is not more than the barometric height for the liquid, the liquid will remain within the tube.

Now by the laws of pressure in any continuous body of fluid at rest, the pressure is the same at all points in the same horizontal layer, and the pressure increases with the depth below the free surface. Hence the pressure at E is that of the atmosphere, and if the end B be lower than E, the pressure at B is greater than that of the atmosphere. Thus, when the end B is opened, the portion B E of the liquid will flow out, and the vacuum thereby created will draw down other portions of the liquid to occupy successively the same position and then flow out. The action will clearly cease as soon as the surface of the liquid in the vessel has sunk to the level of either end of the tube.

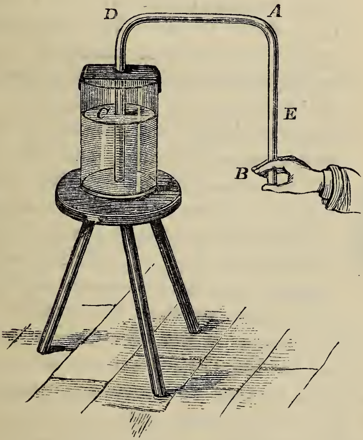

FIG. 38.

96. Diffusion of Gas.—This short sketch of the properties of gas would be incomplete without reference to diffusion—a law which was first put in a clear shape by the late Mr. Graham. His experiment consists in having a tube (Fig. 39) filled with a gas—let us say, with hydrogen—the lower extremity being immersed in a liquid, while the upper extremity is closed with a porous partition that will allow the particles of gas to move through it. Under these circumstances he found that in the course of time the liquid rose in the tube, showing that there was a diminution in the volume of gas. He also found that the quality had become changed, so that, while hydrogen had escaped through the pores of the partition, air had entered. If this process be carried on sufficiently long, all the hydrogen will escape, and the tube will be full of atmospheric air; but the volume of air will not be equal to the original volume of hydrogen, and in general, if the gas within the tube be lighter, bulk for bulk, than that outside, a greater volume will go out than will enter, so that the interior volume will diminish; but if the gas in the tube have a greater specific gravity than that outside, the reverse will take place.

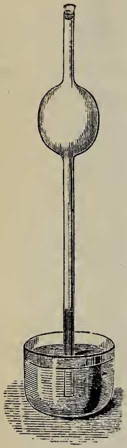

FIG. 39.

97. Absorption of Gas by Solids and Liquids.—In conclusion, we shall very briefly allude to the absorption of gases by solids and liquids. Thus charcoal has the power of absorbing or retaining in its pores a considerable quantity of various kinds of gas. Many liquids likewise have the power of absorbing or retaining gas, sometimes to a very great extent: thus water absorbs carbonic acid gas, and when strongly impregnated with this gas forms the beverage known as soda-water. It also absorbs ammoniacal gas, as well as hydrochloric acid gas, and becomes, in the one case, liquid ammonia, and in the other, liquid hydrochloric acid.

CHAPTER III.

ENERGY.

LESSON XIII.—DEFINITION OF ENERGY.

98. IT is only of late years that the laws of motion have been fully comprehended. It has, no doubt, been known since the time of Newton that there can be no action without reaction; or if we define momentum to mean the product of the mass of a moving body into its velocity of motion, then whenever this is generated in one direction an equal amount is simultaneously generated in the opposite direction, and whenever it is destroyed in one direction an equal amount is simultaneously destroyed in the opposite direction. Thus the recoil of a gun is the appropriate reaction to the forward motion of the bullet, and the ascent of a rocket to the downrush of heated gas from its orifice; and in other cases where the action of the principle is not so apparent, its truth has notwithstanding been universally admitted.

It has, for instance, been perfectly well understood for the last 200 years that if a rock be detached from the top of a precipice 144 feet high it will reach the earth with the velocity of 96 feet in a second, while the earth will in return move up to meet it, if not with the same velocity, yet with the same momentum. But, inasmuch as the mass of the earth is very great compared with that of the rock, so the velocity of the former must be very small compared with that of the latter, in order that the momentum or product of mass into velocity may be the same for both. In fact, in this case, the velocity of the earth is quite insensible, and may be disregarded.

The old conception of the laws of motion was thus sufficient to represent what takes place when the rock is in the act of traversing the air to meet the earth ; but, on the other hand, the true physical concomitants of the crash which takes place when the two bodies have come together were entirely ignored. They met, their momentum was cancelled, and that was enough for the old hypothesis.

So, when a hammer descends upon an anvil, it was considered enough to believe that the blow was stopped by the anvil ; or when a break was applied to a carriage-wheel, it was enough to imagine that the momentum of the carriage was stopped by friction. Let us now consider some of those influences which helped to prepare men's minds for the reception of a truer hypothesis.

99. Work.—We live in a world of work, of work from which we cannot possibly escape ; and those of us who do not require to work in order to eat, must yet in some sense perform work in order to live. Gradually, and by very slow steps, the true nature of work came to be understood. It was seen, for instance, that it involved a much less expenditure of **energy**[1] for a man to carry a pound weight along a level road than to carry it an equal distance up to the top of a mountain.

It is not improbable that considerations of this kind may have led the way to a numerical estimate of work.

Thus if a kilogramme be raised one metre high against the force of gravity, we may call it one **unit of work,** in which case two kilogrammes raised one metre high, or one kilogramme raised two metres high, will represent two units, and so on. We have, therefore, only to multiply the number of kilogrammes by the vertical height in metres to which they are raised, and the product will represent the work done against gravity.

The force of gravity being very nearly constant, and always in action, is a very convenient force to measure work by, and it is generally made use of for this purpose; we shall therefore take as the unit of work the **kilogrammetre**, or the work represented by one kilogramme raised one metre high, against the force of gravity at the earth's surface.

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[1] Energy means simply the power of doing work.

100. Relation between Energy and Momentum.—Having thus defined work, the next point is to connect it with momentum. Now, we have already seen (Art. 45) that a body shot upwards with the velocity of 9·8 metres per second will rise 4·9 metres in height before it stops, so that if a kilogramme be shot upwards with this velocity, it will ascend this height against the force of gravity.

Hence a man who projects a kilogramme vertically upwards with the velocity of 9·8 metres per second will thereby have imparted to the moving kilogramme an amount of energy which will enable it to raise itself 4·9 metres in height, and thus to perform 4·9 units of work. Again, it has been shown (Art. 45) that if the kilogramme be projected upwards with twice this velocity, or that of 19·6 metres per second, it will now rise four times as high; for it will rise 19·6 metres in height before it stops, instead of only 4·9 metres as before. We thus see that *the work which can be accomplished by a moving body* is increased four times by doubling the velocity; in other words, it *is proportional to the square of the velocity*. Again, if the body projected upwards have the mass of two kilogrammes, it will do double the work of a single kilogramme projected upwards with the same velocity, so that *the work which a moving body is capable of doing is proportional to its mass.*

A little reflection will convince us that the work capable of being done by a body whose mass (in kilogrammes or thousands of grammes) is m, and whose velocity is v, will be represented by the expression $\frac{mv^2}{19{\cdot}6}$. . . . . (A).

Thus if $m = 1$, that is to say if the mass be one kilogramme, and if $v = 9{\cdot}8$, that is to say, if it be projected upwards with the velocity of 9·8 metres a second, we shall have by the above expression

$$\text{Capacity for doing work, or energy,} = \frac{(9{\cdot}8)^2}{19{\cdot}6} = 4{\cdot}9\,;$$

while if the mass remain the same as before, and if the velocity be 19·6, we shall have

$$\text{Capacity for work, or energy,} = \frac{(19{\cdot}6)^2}{19{\cdot}6} = 19{\cdot}6.$$

Now, these numbers, as we have already seen, represent the heights attained, and hence the work done in these two instances, so that the expression (A) appears to be correct.

The following example will illustrate the connection between energy and momentum :—

Question.—What is the energy of a body weighing 64 grammes projected upwards with the velocity of 60 metres in one second? *Answer.*—Its energy or capacity for doing work is $\frac{64}{1{,}000} \times \frac{(60)^2}{19{\cdot}6} = 11{\cdot}76$, the first of the two factors being applied with the purpose of reducing the mass into kilogrammes.

The statement that the energy of a moving body is proportional to the square of its velocity may be seen to hold from other points of view. Thus, for instance, if a number of similar oak planks be placed the one behind the other, and if a rifle-ball with a certain velocity pierce through three of them, a rifle-ball with double the velocity will pass not only through six, or twice three, but through twelve, or four times three planks ; thus the resistance of the planks supplies the place of the force of gravity in the previous examples just given.

Again, let us suppose that on a line of rail a collision is about to take place between two similar trains, each moving towards the other at the rate of thirty miles an hour. If we denote by unity the energy of each train, the whole amount of energy spent upon the crash will obviously be two units.

Suppose now that the one train is at rest while the other is rushing towards it with the double velocity of sixty miles an hour. It is very easy to see that the energy of the latter must be four units. For in the first place, the rate of approach in this case being the same as that in the previous case above mentioned, the same amount of energy will be spent on the collision, which will thus represent two units of energy. But besides this, the double train will by the laws of momentum be in motion after the collision at the rate of thirty miles an hour—this representing other two units of energy. On the whole, therefore, the energy represented by the single train moving at sixty miles an hour will be four units.

101. Energy is of Two Types.—If we define "energy" to mean the power of doing work, it thus appears that a stone shot upwards with great velocity may be said to have in it a great deal of actual energy, because it has the power of overcoming up to a great height the obstacle interposed by gravity to its ascent, just as a man of great energy has the power of overcoming obstacles. But this stone as it continues to mount upwards will do so with a gradually decreasing velocity, until at the summit of its flight all the actual energy with which it started will have been spent in raising it against the force of gravity to this elevated position. It is now moving with no velocity—just, in fact, beginning to turn—and we may suppose it to be caught and lodged upon the top of a house. Here then, it remains at rest, without the slightest tendency to motion of any kind, and we are led to ask, What has become of the energy with which it began its flight? Has this energy disappeared from the universe without leaving behind it any equivalent? Is it lost for ever, and utterly wasted?

When the stone began to mount it contained, in virtue of its velocity, an amount of energy which, by means of appropriate contrivances, might have been spent in grinding corn, or pumping water, or turning a wheel, or in a variety of useful ways, instead of which we have allowed the stone to mount against the force of gravity as far as it will rise. Have we now lost for ever the opportunity of utilizing this energy of the stone? Far from it—doubtless the stone is at rest on the top of the house, and hence possesses no *energy of motion*; but it nevertheless possesses *energy* of another kind, *in virtue of its position*; for we can at any time cause it to drop down upon a pile, and thus drive it into the ground, or make use of its downward momentum to grind corn, or to turn a wheel, or in a variety of useful ways.

It thus appears that when a stone which has been projected upwards has been caught at the summit of its flight, and lodged on the top of a house, the energy of actual motion with which it started has been changed into another form of energy, which we denominate Energy of Position or Potential Energy, and that by allowing the stone again to fall we may change this energy of position once more into actual energy, so that the stone will reach the ground with a velocity, and hence with an energy, equal precisely to that with which it was originally projected upwards.

There are, therefore, two kinds of energy which are being continually changed into one another, and these are the energy of actual motion and the energy of position. As an example of the first kind we have a stone projected vertically upwards, or indeed projected with velocity in any direction; for it is the velocity which is material, and not the direction of motion, since by means of appropriate contrivances we may utilize a horizontal velocity as much as a vertical one.

Again, as an instance of the second kind of energy we have a stone on the top of a house, or any substance, such as a head of water, occupying a position of advantage with respect to gravity, or any other force.

LESSON XIV.—VARIETIES OF ENERGY.

102. In the preceding lesson we defined energy to mean the power of doing work, and we found that this working power might be exhibited in two ways; being in the first place possessed by a body in actual motion, and in the second place by a body which occupies a position of advantage with respect to any force. These two types (called also **kinetic** energy and **potential** energy) were illustrated with reference to the force of gravity. Thus a stone has a tendency to fall towards the centre of the earth; and if it be removed as far as possible from this centre, it may be said to occupy a position of advantage with respect to the force of gravity, as for instance if it be lodged on the top of a house. Again, when the stone is in the act of falling from the top of the house, this energy of position, or potential energy, undergoes reconversion into that of actual motion, or kinetic energy, until at length when the stone once more reaches the ground it does so with the very same velocity with which it was originally projected upwards.

103. Varieties of Energy.—But there are other forces besides gravity, and one of the most active of these is chemical affinity. Thus for instance, an atom of oxygen has a very strong attraction for one of carbon, and we may compare these two atoms to the earth and the stone in the illustration given above, the difference being that both atoms are very small, much of the same mass, and attract one another at insensible distances only. Nevertheless, within certain limits this attraction is intensely powerful, so that when an atom of carbon and one of oxygen have been separated from each other, we have a species of energy of position just as truly as when a stone has been separated from the earth. Thus, by having a large quantity of oxygen and a large quantity of carbon in separate states, we are in possession of a large store of energy of position. Now we have seen that when we allowed the stone and the earth to rush together, this energy of position was transformed into that of actual motion, which we might use in a variety of useful ways, and we should therefore expect something similar to happen when the separated carbon and oxygen are allowed to rush together. This takes place when we burn coal in our fires, and the primary result, as far as energy is concerned, is the production of a large amount of heat. We are therefore led to conjecture that heat may denote a motion of particles on the small scale, just as the rushing together of the stone and the earth denote a motion on the large. We are further led to think that when we use this heat wherewith to work our engine, we are utilizing a species of energy of motion just as truly as when we employ the energy of motion of water to drive our mills, or the energy of motion of a weight to drive a pile into the ground.

It thus appears that we may have invisible molecular energy as well as visible mechanical energy, and before proceeding further it may be desirable to give a short account of the various forms of energy both visible and invisible.

For this purpose it will be convenient to divide the various energies as we know them into three groups. The first of these will be **the group of Visible Energies**; and this will embrace two varieties, namely (1) *the energy of visible motion*, and (2) *the potential energy of visible arrangements.*

The second of these will be **the group of Heat Energies,** consisting (3) of *the kinetic energy of absorbed heat*, (4) of *molecular separation*, and (5) of *radiant light and heat.*

The third of these will be **the group of Electrical and Chemical Energies,** consisting (6) of *electrical separation*, (7) of *electricity in motion*, and (8) of *chemical separation.*

104. Visible Energy of both kinds (1 and 2).—In the first place we have visible energy on the large scale, embracing that which is due to position, and that which is due to actual motion. Of that which is due to position we have a head of water, a stone at the top of a cliff, a cross-bow bent (which is in a position of advantage with regard to the elastic force of the bow), a clock wound up, and so on. Then with regard to actual energy, we have that of a cannon-ball, or of a meteor, or of a gale of wind, or of a flowing river.

105. Heat Group: Absorbed Heat and Molecular Separation (3 and 4).—To come now to invisible energy, we have that well-known form of it which we call heat. When a body is greatly heated we have reason to believe that its particles are in a state of intense motion among themselves, although the body as a whole is at rest. But what in this case is latent heat, for (as we shall afterwards see) it requires a large amount of heat to convert boiling water into steam, notwithstanding which the steam is no hotter than the boiling water? In such a case we have reason to believe that much of the species of energy which we call heat has passed from the form of energy of motion into energy of position, and spent itself in forcing the particles of water to a great distance from each other, just as a stone, thrown upwards and lodged on the roof of a house, has spent its actual energy in separating itself from the earth, and has thus acquired an equivalent in energy of position. We may perhaps compare the heating of a particle to a man whirling round his head a heavy weight attached to his hand by a thick india-rubber string. Part of his energy will be spent in imparting velocity to the weight, and part also in stretching the string. The first is a kinetic, the latter a potential form of energy. Thus we have in the molecular world, as well as in the mechanical world, energy of motion and energy of position. We may carry the analogy yet a step further. In the visible mechanical world, whenever a body is in rapid motion, part of its energy is carried off by the air in the shape of sound and other motions of the air; thus when a string vibrates or a bell is struck the sound which reaches us represents so much of the energy of the moving particles which has been carried off by the air.

106. Radiant Light and Heat (5).—Now there is a medium pervading all space, which we call the ethereal medium, and which carries off part of the energy of motion of molecules, just as the air does in the case of large moving bodies. Thus the molecular energy which we have now described as heat is given out by a hot body to this medium which surrounds it, and by it is transmitted in a series of waves, moving at the enormous rate of 186,000 miles in one second of time. This undulatory energy is known as radiant light and heat.

107. Electrical and Chemical Group: Electrical Separation (6).—Besides the kinds of invisible energy now mentioned, we have those very important forms of it connected with electricity and chemical affinity. Thus when two bodies charged with opposite electricities are apart from one another, we have a species of energy due to the position of advantage occupied by these bodies as respects electrical force, and the bodies will have a tendency to rush together, just as a stone at the top of a cliff has a tendency to rush to the earth. Now, if they be allowed to meet one another, their energy of position will be converted into that of visible motion, just as when the stone is allowed to drop from the cliff its energy of position is converted into that of visible motion.

108. Electricity in Motion (7).—We come next to the energy represented by electricity in motion. Whenever an electric circuit has been completed, there is a power or energy pervading it which we term the electric current; and if part of the circuit be formed of a metallic wire, we can by its means convey this power into any place we choose, and as it were *lay on* so much energy which, properly applied, may be instrumental in doing useful work. Thus, while in ordinary cases the work is done by the side of the engine, in the case of an electric current we may have the battery or source of energy by our side, and by means of conducting wires perform our work fifty miles away.

109. Chemical Separation (8).—Finally, that description of energy represented by chemical separation has been already referred to and illustrated in the case of carbon, which we supposed separated from oxygen, for which it has an intense attraction.

Let us now briefly recapitulate what has been said regarding the various forms of energy.

We have, *in the first place*, visible mechanical energy, both actual and potential; in the *next* place, we have the heat group of energies—sensible heat probably representing an energy of motion, and latent heat denoting rather an energy of position: while belonging to this group we have also radiant light and heat; *thirdly* we have the electrical and chemical group of energies, embracing that form of energy of position which is represented by the separation of differently electrified bodies, embracing also electricity in motion; and *lastly*, belonging to this group, we have that form of energy of position represented by the separation of bodies having a strong chemical affinity for each other.

The remainder of this work will be chiefly devoted to a description of these various forms of energy, and of the laws according to which they are transmuted into one another. In the meantime let us describe the great principle which governs all such transmutations.

LESSON XV.—CONSERVATION OF ENERGY.

110. The most important principle connected with this subject is that known as the conservation of energy.

The production of a "Perpetual Motion" has long been one of the dreams of enthusiasts.

Their great ideal of mechanical triumphs was a machine that, without requiring to have any labour bestowed upon it, or to be fed with fuel of any kind, should continue to perform work for ever; a clock which could wind itself up, or an engine that could go on without coals, would be a machine of this description. In their endeavours to attain their object the advocates of a "perpetual motion" must often have started questions which the natural philosopher is not always able to reply to. We do not know all the properties of matter, and we are not always able to predict what will happen under every conceivable combination of natural forces. At last, in an inspired moment, the philosopher conceived the idea of replying to all the questions of the enthusiast by denying the possibility of perpetual motion, and by asserting that it is just as impossible either to create or destroy energy as it is to create or destroy matter. Now, it is clear that the only way of establishing the truth of a principle of this kind is by trying it in a number of cases; and if it succeeds in explaining the peculiarities of each case, we have strong grounds for believing in its truth: it is a tree that must be tested by its fruit. The principle of the conservation of energy has stood the test, and not only so, but it has also greatly assisted us in finding out new facts and laws of matter, so that we have much reason for believing in its truth.

111. Motion of a Stone.—Let us first of all apply it to the case of a stone projected vertically upwards, and to simplify matters, let us suppose that the stone weighs exactly one kilogramme, and that its velocity of projection is that of 19·6 metres in one second, which, as we have seen, represents 19·6 units of work. Let us consider the state of things at the precise moment when the stone is 14·7 metres high; it will then have an actual velocity of 9·8 metres per second (Art. 46), which, as we have seen, will represent 4·9 units of work.

But it started from the ground with 19·6 units of work in it; what, therefore, has become of the difference, or 14·7 units?

Evidently it has disappeared as actual energy; but the stone, being 14·7 metres high, has acquired in its place an energy of position represented by 14·7 units, so that at this precise moment of its flight its actual energy (4·9), *plus* its energy of position (14·7), are together equal to the whole energy (19·6) with which it started.

Thus, as the stone mounts up, there is no annihilation of energy, but merely the transformation of it from actual energy to that implied by position; nor have we any creation of energy when the stone is again on its downward flight, but merely the retransformation of the energy of position into the original form of actual energy.

112. Energy is not destroyed by Impact.—We have thus gauged the energy of the stone throughout its upward and downward flight, and have found this to be strictly constant. We have not yet, however, done with the case of the stone; in fact, the most difficult part of the whole problem yet remains to be solved; for what becomes of the energy of the stone after it has struck the earth? This question may be varied in a great number of ways. We may, for instance, ask what becomes of the energy of a railway train when it is suddenly stopped? what becomes of the energy of the hammer after it has struck the anvil? of the cannon-ball after it has struck the target? and so on.

In all these varieties of the question we perceive that either *percussion* or *friction* is at work. It is friction that stops a railway train, and it is percussion that stops the motion of a falling stone or a falling hammer, so that our question is, in reality, What becomes of the energy of visible motion when it has been stopped by percussion or friction?

113. It is then converted into Heat.—Rumford and Davy were the pioneers in replying to this important question. Rumford found, during the process of boring cannon at Munich, that the heat generated was sometimes so great as to cause water to boil, and he supposed that ordinary mechanical work became changed into heat through the friction produced in the process of boring. Davy, again, melted two pieces of ice by causing them to rub against each other, and he likewise concluded that the work spent on this process had been converted into heat. We see now why, by hammering a coin on an anvil, we can heat it very greatly, or why on a dark night the sparks are seen to fly out from the break-wheel which stops the motion of the railway train, or why by rubbing a metal button violently backwards and forwards against a piece of wood we can render it so hot as to scorch the hand, for in all these cases it is the energy of visible motion which has been converted into heat. There has, in fine, been an *annihilation* of visible energy, simultaneously with the *creation* of so much heat.

114. Mechanical Equivalent of Heat.—Grove in this country, and Mayer on the Continent, were the first to point out the probability of a connection between the various forms of energy; but it was reserved for Joule, Colding, Thomson, and others, to establish these relations on a scientific basis.

The researches of Joule led him to the exact numerical relation subsisting between that species of energy which we call visible motion and that which we call heat.

The result of his numerous and laborious experiments was that, if a kilogramme of water be dropped from a height of 424 metres under the influence of gravity, and if the velocity which it attains be suddenly stopped by the earth and converted into heat, this heat will be sufficient to raise the whole mass 1° centigrade in temperature. From this he concluded that when a kilogramme of water is raised 1° centigrade in temperature, an amount of molecular energy enters into the water which is equivalent to 424 kilogrammetres; that is to say, to 424 units of work. By this means an exact relation is established between heat and work.

115. Galvanic Circuit.—No better example of the connection between the various kinds of energy can be given than what takes place in a galvanic battery. Let us suppose that zinc is the metal used. Here the source of energy is in reality the burning of the zinc, or, at least, its chemical combination with oxygen in order to form a salt of zinc. The source of energy is, in fact, much the same as when coal is burned in the fire. Now, as we have said, the zinc combines with the oxygen, and a salt of zinc is produced ; but the actual energy called forth by the union does not at first exhibit itself in the form of heat, but rather in that of an electric current.

No doubt a large portion of the energy of the electric current is ultimately spent in heat, but we may, if we choose, spend part in promoting chemical decomposition ; we may, for instance, decompose water. In this case part of the energy of the battery, derived, as has been stated, from the burning of the zinc, is spent in heat, and part in decomposing the water, and hence we shall have less heat than if there were no water to be decomposed. But if when we have decomposed the water we mix together the two gases—hydrogen and oxygen—which are the result of the decomposition, and explode them, we shall recover the precise deficiency of heat. Without the decomposition, the burning in the battery of a certain weight of zinc will give us, let us say, heat equal to 100, while with the decomposition we shall only obtain 80 ; twenty units of energy have therefore become spent in the decomposition ; but if we explode the mixed gas we shall get back heat equal to 20, and thus make the whole result of the burning of the zinc 100 units of energy as before. But it is unnecessary to enter at present into great detail regarding the various changes of energy from one form into another ; suffice it to say, that amid all these changes of form the element of quantity remains the same, so that if we adopt the notation of algebra, and denote by s, t, u, v, w, x, y, z the quantity of energy of the eight varieties already mentioned as present in the universe, these letters representing variable quantities, then we shall always have $s + t + u + v + w + x + y + z =$ a constant quantity; that is to say, while u may change into v or into w, and, in fact, while the various forms of energy may change into each other, according to the laws which regulate such changes, nevertheless the sum of all the energies present in the universe will always remain constant in amount; and this is the doctrine known as the *Conservation of Energy.*

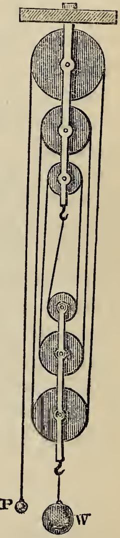

FIG. 40.

116. Function of a Machine.—To realize the truth of this doctrine, let us take one of the ordinary mechanical combinations, such as a system of pulleys (Fig. 40), and see what we gain by its employment.

In this system there are two blocks, the lower one moveable and the upper one fixed; while the same string goes round all the pulleys.

The power P is applied to the extremity of this string, so that the tension of all parts of this string is equal to the weight of P.

Now W is supported by six strings: hence we see that W must be 6 times as great as P in order that there may be equilibrium.

Suppose, now, that P is equal to 1, and W to 6 kilogrammes, and that P is pulled down 6 metres, we have thus spent a quantity of energy upon the machine represented by $1 \times 6 = 6$. Our gain is that we have caused the weight W of 6 kilogrammes to rise, but in order that the law of the conservation of energy should hold good, W ought not to rise higher than one metre; for if it did, we should get back more energy than we had spent upon the machine. Now it will readily be seen that the rise of W will be 6 times less than that of P, because W is supported by six strings, while P is only supported by one; therefore by lowering P through six metres, each of these strings of W, and hence W itself, will be raised one metre; and hence the gain of energy by the raising of W into a position of advantage will be 6 × 1 = 6, or precisely what was spent in lowering P.

We thus see that in such a machine *what we gain in force we lose in space.*

The same law will hold for the hydraulic press (Fig. 23). In this case, if the area of the two pistons be as 1 : 100 and a weight of 10 kilogrammes be applied to the small piston, it will raise 1000 kilogrammes if put on the large one; but since the *volume* of water remains constant, the rise of the larger piston will only be $\frac{1}{100}$th of the fall of the smaller.

Now if the smaller piston falls one metre with a weight of 10 kilogrammes, we have spent 10 units of energy on our machine.

But the large piston containing 1000 kilogrammes will have risen $\frac{1}{100}$th of a metre, so that we shall have recovered by its rise, an amount of energy equal to $1000 \times \frac{1}{100} = 10$; that is to say, neither more nor less than the 10 units of energy which we spent.

Here, too, therefore, and indeed in all machines, we do not create energy, but simply transform it into a kind more convenient for us, and the law holds universally that what we gain in force we lose in space, so that the power multiplied by its space of descent is always equal to the weight multiplied by its space of ascent

CHAPTER IV.

VISIBLE ENERGY AND ITS TRANSMUTATIONS.

LESSON XVI.—VARIETIES OF VISIBLE ENERGY.

117.—BY visible energy we mean the energy of visible motions and arrangements. Thus, for instance, a cannon-ball during its flight, or a flowing river, form examples of visible motion, and a stone on the top of a cliff is an example of a visible advantageous arrangement as far as energy is concerned. To begin with the first description of energy, or that due to visible motion; there are many varieties of this. Thus we have, first of all, the energy of a body in actual visible linear velocity, such as a railway train, a cannon-ball, a gale of wind, a stream of water, a meteor.

But there is also the energy due to rotatory motion; as for instance, that of a top in rapid rotation, or that of the earth in its rotation round its axis.

In the next place, there is the energy of oscillating and vibratory motion; in the former category we may place the motion of a pendulum, while the string of a musical instrument is a very good illustration of the latter. The whole phenomena of sound are to be included under this last head, for although the vibrations of sounding bodies are sometimes so rapid as to be invisible, yet they result from an arrangement and motion of particles on the large scale, and not from strictly molecular motions and arrangements, as is the case with that species of vibration which forms light.

Lastly, we have the potential energy of a body occupying a position of visible advantage with respect to some force. If the force be that of gravity, we have the energy of a stone at the top of a cliff, of a head of water, of a clock wound up, and so on ; or again, if the force be that due to elasticity, we have the energy of position of a cross-bow bent, or of a spring stretched, with many other similar instances.

Now, under certain conditions, these various forms of visible energy are transmuted into one another, while under other conditions they are transmuted into the various forms of molecular energy : but as these last will form the subject of future chapters, we shall at present mainly confine ourselves to a description of the various forms of visible energy and their transmutations into one another.

118. Linear Velocity.—Let us begin with the energy of a *rifle-ball.* In its rapid flight through the air the ball imparts some of its motion to the particles of air with which it comes in contact ; but neglecting this in the meantime, let us suppose that it ultimately strikes a heavy mass of wood hung by a string, and so forming a pendulum, in the centre of which it lodges.

Let us suppose that the weight of the ball is 20 grammes and its velocity 200 metres per second, and that the weight of the heavy block of wood in which it lodges is 20 kilogrammes. Before impact the momentum of the ball was $20 \times 200 = 4{,}000$, representing a mass equal to 20 moving with a velocity equal to 200. After the impact, we have, of course, the same momentum of 4,000, but it will now represent a mass equal to 20,020, moving with a velocity equal to 0·2 nearly.

Now, according to the method of estimating energy (Art. 100), that of the ball before impact will be $\frac{20}{1000} \times \frac{(200)^2}{19{\cdot}6} = 40{\cdot}8$ nearly, whereas after impact the energy of the united mass (ball *plus* pendulum) will be $\frac{20{,}020}{1000} \times \frac{(0{\cdot}2)^2}{19{\cdot}6} = 0{\cdot}0408$. We thus see that although in conformity with the third law of motion the momentum is preserved, yet the energy after impact is a thousand times less than the energy before, so that most of this energy has disappeared from the category of visible motion. Into what form, therefore, has it been transmuted? We answer, the ball has *worked* its way into the heart of the log of wood. In doing so, its energy has been spent in accomplishing the disintegration of the log of wood; it has, in fact, been spent against a species of friction or resistance opposing its passage, and it will be found that the production of heat has been the result. So that in this case the result of the transference of a quantity of momentum from a small to a large mass has been the conversion of visible energy into heat.

119. Resistance of Air.—So in like manner the momentum originally communicated to the air by the passage of the ball gradually becomes distributed over larger and larger masses of air, and in this process the forward momentum in the direction of motion of the ball is strictly preserved, but the energy represented by this momentum becomes less according as the moving mass of air becomes greater. As we know there is no loss of energy, we conclude that it has passed into heat; and could we only perform the experiment, we should find that when the disturbance produced in the air by the ball had become so spent as to be insensible, there would be a certain increase of temperature, representing the energy derived from the ball.

We are thus prepared to recognise an extension of the first law of motion; for, in the first place, when the moving body is not acted upon by any external force it will continue moving for ever with a uniform velocity, neither losing momentum nor energy; while, again, if it be acted upon by some external force, such as the resistance of the air, it loses both momentum and energy; and while the momentum which it loses is being communicated to larger and larger masses of air, and is thus preserved, the energy lost by it ultimately takes the shape of heat, and is thus preserved likewise.

120. Impact of Inelastic Bodies.—Let us now vary the case by considering two inelastic solids striking against each other. Let the one weigh 20 grammes, and have a velocity equal to 20, and let the other weigh 10 grammes, and have a velocity of 16 in an opposite direction; we have thus a momentum equal to 400 in one direction, and one equal to 160 in the opposite, giving an excess in the direction of motion of the larger solid equal to 400 - 160, or 240. Now this residual momentum must be preserved after impact, by the third law of motion, and hence the united mass, or 30 (for the balls, being inelastic, will move together), will, after impact, move with a velocity equal to 8. But the united energy before impact was $\frac{20}{1000} \times \frac{(20)^2}{19{\cdot}6} + \frac{10}{1000} \times \frac{(16)^2}{19{\cdot}6} = 0{\cdot}539$, and that after impact is only $\frac{30}{1000} \times \frac{(8)^2}{19{\cdot}6} = 0{\cdot}098$.

What, therefore, has become of the remainder of the energy? We reply, as before, it has been transmuted into heat. It would thus appear that the collision of inelastic balls results in a transfer of visible motion into heat.

121. Impact of Elastic Bodies.—But the case is altered if the balls be perfectly elastic,[1] for there is then no transmutation into heat, but the *energy* of visible motion is preserved, as well as the *momentum*, and is the same both before and after impact.[2]

For example, let two perfectly elastic balls weighing respectively 4 and 3 kilogrammes, moving in the same direction, with velocities 5 and 4, impinge against each other, then we know, by the laws of elastic bodies, that after impact the velocity of the first or largest ball will be $\frac{29}{7}$, and that of the second or smallest ball $\frac{36}{7}$. Now, in the first place, the momentum before impact was $4{,}000 \times 5 + 3{,}000 \times 4 = 32{,}000$, while after impact it is $4{,}000 \times \frac{29}{7} + 3{,}000 \times \frac{36}{7} = 32{,}000$, which is the same as before. Also the energy before impact was $4 \times \frac{(5)^2}{19{\cdot}6} + 3 \times \frac{(4)^2}{19{\cdot}6} = \frac{148}{19{\cdot}6}$, while that after impact is $4 \times \frac{\left(\frac{29}{7}\right)^2}{19{\cdot}6} + 3\frac{\left(\frac{36}{7}\right)^2}{19{\cdot}6} = \frac{148}{19{\cdot}6}$, also the same as before. Wherefore both the momentum and the energy are unaltered by impact.

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[1] Two bodies are perfectly elastic when the momentum impressed during restitution is equal to that spent in producing compression.

[2] Part of the energy is probably changed into vibrations of the elastic bodies, but for our present purpose this may be neglected.

The most interesting case of impact is when one elastic ball strikes centrically another of the same size at rest, in which case the first ball entirely loses its motion, which is transferred to the second. Therefore if we have a row of such balls placed near each other, and if an impulse be communicated to the first of them in the direction of the row, it will in time be transmitted along the whole series until it reaches the last ball, which will then (being the last) start off and leave the series.

122. Energy of Rotation.—Let us next consider very shortly the case of a disc in rapid rotation. We have already (Art. 17) explained that such a motion implies great force of cohesion, for the particles at the circumference of the disc have, by the first law of motion, a tendency to move in a straight line with a uniform velocity; as, however, they move in a circle, they must be continually acted upon by some force. The tendency to rectilinear motion is in fact continually resisted by the force of cohesion, tending to prevent the separation of the particles of the disc, and the resulting motion is a compromise between the centrifugal tendency and this force. But while there is thus a continual change in the direction of motion of a particle, the velocity will remain the same, for if the velocity of the various particles composing the disc were to change, it would imply that the energy of motion of the whole disc had changed also; but this cannot be, for the disc will retain its energy unchanged by the law of the conservation of energy, unless it be acted upon by friction or resistance, in which case the energy of the disc will be gradually transferred to the bodies rubbing against it.

And generally speaking, wherever we have in nature a strictly circular orbit of particles round a central force we have a uniform velocity, and the energy of visible motion of the mass remains constant.

123. Energy of a Body moving in an Ellipse.—Not so, however, if the orbit be elliptical. Let us consider, for instance, the motion of a comet—a body which describes a very elongated elliptical orbit, having the sun in one of its foci.

Let the comet be farthest from the sun at B, and nearest him at A. Now the comet, while moving from B to A, has gradually been approaching the sun, and the same thing will happen to it as when a stone falls to the earth.

In this case we know that the energy of position of the stone is gradually changed into the energy of actual motion, and so in like manner the energy of position which the comet has at B (being there very far from the centre of gravitating force) becomes changed as it approaches the sun into the energy of actual motion, until at A it is moving with a very great velocity; it has in fact fallen towards the sun, from the distance B S to the distance A S, and its increase of energy will be that which would be acquired by a body of its own weight falling direct towards the sun from a distance B S to one A S, without any regard to the path by which it has passed from the one position to the other. The same thing takes place in the case of the earth and the other planets. Thus, when the earth is nearest the sun it is moving fastest. If we assume the greatest distance of the earth from the sun to be 92,965,000 miles, and the least distance 89,895,000 miles, the difference, or 3,070,000 miles, represents the distance through which the earth has fallen towards the sun; the energy of actual motion of the earth will therefore be greater at perihelion than at aphelion, by that due to the mass of the earth falling through 3,070,000 miles under the attraction of the sun's gravitating force.

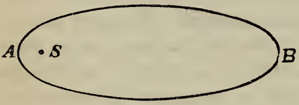


FIG. 41.

124. Energy of a Body falling down a Plane.—The laws of energy enable us to determine at once the velocity acquired by bodies falling down inclined or curved planes.

Let us, for instance, suppose that a body slides or rolls down a smooth plane, of which the friction may be neglected, and let the *vertical* height of the plane be 10 metres. The body is therefore nearer the centre of the earth by this amount at the end of its journey than at the beginning, and, without regard to the slope or curvature of the journey through which it has passed, its energy of position will at the end of this journey be less, and its actual energy greater by the precise amount which would be acquired by the body if it fell directly downwards to the earth's centre through the space of 10 metres. We thus reproduce the well-known proposition in mechanics which tells us that the velocity of a body which has slid down an inclined plane depends only upon the vertical height of the plane, without regard to its slope.

If, however, the plane be rough, or if it be made up of a series of inclined planes at definite angles to one another, this proposition will no longer hold, for in a rough plane part of the energy is lost through friction, and in the case where the slope changes suddenly, part is lost by oblique impact at the angles of the plane.

125. Visible Energy of Position.—We have already in our remarks on energy alluded sufficiently to visible energy of position. This kind of energy is exhibited by a stone at the top of a cliff, by a head of water, or by a clock wound up, if *gravity* be the force ; it is also exhibited by a cross-bow bent, or by a watch wound up, both of which occupy a position of advantage with reference to the force of *elasticity*. All these forms of potential energy are naturally transmuted into visible energy of motion. Thus the stone is hurled over the cliff, the head of water is used to drive a mill-wheel, the clock-weight drives the clock-wheels, the cross-bow discharges its bolt, the watch-spring drives the watch-wheels, and so on.

126. Energy of a Pendulum.—We come now to consider the case of oscillatory and vibratory motions. In these the energy of the body is alternately that due to actual energy and that due to position.

Let us take a pendulum, as the simplest instance of oscillatory motion. When the bob of the pendulum is at the summit of its oscillation, and about to turn, its position is similar to that of a stone which has attained the summit of its flight and is about to fall; in both cases the energy is entirely due to the position of advantage which the body has attained with regard to the force of gravity. When, again, the bob of the pendulum has reached its lowest point, all its energy of position has been converted into that of actual motion, and it is then moving with sufficient velocity to enable it to rise (were there neither friction nor resistance) to a height equal to that through which it has fallen, but on the other side of the vertical. When it has attained the summit of its swing on this side its energy, as before, has become converted into that of position, and it begins to descend once again; and so on it goes swinging alternately from left to right and from right to left, always moving fastest as it passes its lowest point, and at the summit of its swing having only energy of position. We can at once, by means of the laws of energy, determine the velocity of such a pendulum at any point of its oscillation, as will be seen from the following example.

Example.—A pendulum bob weighing one kilogramme is so swung that it is higher from the centre of the earth at the summit of its oscillation than at its lowest point by the space of one decimetre; what is its velocity at its lowest point?

Answer.—Its *energy* is precisely that which will be acquired by one kilogramme falling through the space of one decimetre under the influence of gravity; that is to say, it will be 0·1 if unity denote the energy acquired by one kilogramme falling through one metre, and hence its *velocity* (Art. 100) will be found from the expression $\frac{v^2}{19·6} = 0·1$ whence $v = 1·4$; this, therefore, is the velocity acquired by the pendulum at its lowest point.

127. Foucault's Experiment.—Let us now suppose our pendulum to consist of a heavy weight suspended from a fine thread, the only influence of the thread upon the pendulum being that due to its tension, which thus enables it to support the weight and keep it swinging. It is clear that the oscillations of such a pendulum will always continue to be performed in the same vertical plane. We may, for instance, have the means of setting round the suspension pin to which the thread of the pendulum is attached, but we shall not succeed by this means in altering the plane of the oscillation.

For not only is the actual motion performed in this plane, but the forces which vary the motion, namely, the tension of the string and the force of gravity, are all in the same vertical plane with the motion itself; it is clear therefore that the motion will continue to be in this plane. Let us now imagine that we are standing at the very north pole of the earth, and that we have there set our pendulum in motion in a plane passing through the meridian of Greenwich: the motion of the pendulum will continue, as we have seen, to be performed in the *very same plane* in which it was started, but as the earth turns round its axis the different meridians turn with it, so that in six hours' time the vertical plane passing through the meridian of Greenwich will be at right angles to its first position. These meridians may be denoted by lines drawn on the ground under the pendulum, and at the earth's pole this system of lines will turn round their central point once in 24 hours. Not so, however, the plane of the pendulum's motion, which, as we have already seen, will remain stationary, and the consequence will be that the meridian lines will move across the plane of motion of the pendulum; or, which is the same thing, the plane of motion of the pendulum will appear to travel in the opposite direction round the system of meridian lines once in 24 hours, forming, as it were, a species of clock. We have chosen the pole for the sake of simplicity, the results being less simple if the pendulum be swung at any other latitude. This ingenious experiment was devised by the late M. Foucault in order to afford an independent proof of the rotation of the earth.

128. Energy of Vibrations.—Let us now pass on to the case of vibratory motion, such as we see in the string of a musical instrument, or in a bell. This motion is very analogous to that of the pendulum. In vibratory as well as oscillatory motion the various particles of the body are alternately in a state of energy due to actual motion, and of energy due to position; the particles vibrate alternately on both sides of their position of rest, and in passing through this position they are always, like the bob of the pendulum, moving fastest, and when they have attained the limit of their excursion, and are about to return, their energy of actual motion is *nil*, because it has been wholly converted into energy of position. There is another bond of similarity between a pendulum and a vibrating body. Thus a pendulum gradually loses its motion from two causes; the first of these being the resistance of the air, and the second the friction of the support; the motion lost through the air ultimately passes into heat, while that dissipated by friction ultimately takes the same shape.

Now in a vibrating body part of the motion is carried off by the air, at first producing that audible undulation of the air which we call sound, but afterwards assuming the shape of heat; part also is converted into heat through the friction or rubbing against each other of the various parts of the vibrating body.

Thus we perceive that the energy of visible vibrations is ultimately converted into heat, but not before it produces an undulatory motion, which affects the ear as sound. We shall study this undulatory motion at greater length in what remains of this chapter; meanwhile let us recapitulate the various kinds of visible energy which we have mentioned.

129. Recapitulation.—There is, *first*, the energy due to actual rectilinear velocity, ultimately converted at our earth's surface into heat through friction and resistance.

There is, *secondly*, the energy due to rotatory motion.

There is, *thirdly*, the energy due to a body moving in an elliptical orbit, in which case there is a change from potential to actual energy, and back again in the various parts of the orbit, the body having most actual energy when nearest the centre of force, and most energy of position when farthest away from it.

There is, *fourthly*, the energy due to a stone on the top of a cliff, or to some body in a position of visible advantage with respect to some force.

Then, *fifthly*, we have the energy of oscillatory motion, such as that of a pendulum, which is alternately energy of position and actual energy, but which through friction and resistance will ultimately be dissipated, and assume the form of heat.

And *sixthly* and lastly, there is the energy of vibration of a vibratory chord or plate, which is similar to that of an oscillating pendulum, inasmuch as the energy of each particle is alternately actual and potential, and which also ultimately assumes the form of heat.

LESSON XVII.—UNDULATIONS.

130. As a preliminary to sound let us first consider in some detail the subject of vibratory motion.

In Fig. 42 let a simple pendulum, consisting of a heavy ball attached to a thread, be swung from C, and let A be the lowest point of its swing. It is very easy to find the force which urges the ball towards A at any point of its progress, such as B. Thus we have at B the weight of the ball acting vertically downwards, and which we may represent by the line B D. Now, by means of the parallelogram of forces we may decompose B D into two forces, B E and B F, of which B E is in the direction of the string, and hence only constitutes a pull upon the string without affecting the motion of the ball. The resolved portion B F is, however, precisely in the direction of motion of the ball, and is therefore wholly available in increasing the velocity of this motion ; B F will, therefore, represent the force which

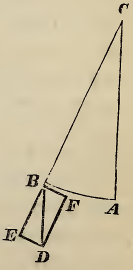


FIG. 42.

urges the pendulum towards A at any point of its path B. Now B F = B D sin B D F = B D sin E B D (since E C is parallel to D F) = B D sin C (since B D is parallel to C A); but B D represents the weight of the ball: hence the force at any point is equal to the weight of the ball multiplied by the sine of the angle which the string at that point makes with the vertical; that is to say, this force is proportional to the sine of this angle C. Now, if the pendulum only moves to short distances on either side of the vertical, the arc B A, and hence the angle C, will be very small; and since in such cases the sines of angles are as nearly as possible proportional to the arcs themselves, the force at B, which is proportional to the sine of the angle C, will become proportional simply to the arc B A; but B A is the distance of the ball from its position of rest: hence the force which urges on the ball is proportional to the distance of the ball from its point of rest, or in other words, the force is proportional to the displacement.

131. Isochronism.—We thus see that in a simple pendulum making small vibrations the force is proportional to the displacement, becoming greatest when the pendulum is farthest from its position of rest, that is to say, when its velocity is *nil*, and vanishing when it is at its lowest point, that is to say, when its velocity is greatest.

Now, this is precisely what takes place in elastic bodies, such as springs, &c.; for in all these the force of restitution, or force tending to bring the spring back to its point of rest, is proportional to the displacement: and it is furthermore found that in all such cases a series of oscillations or vibrations will be performed on either side of the point of rest, and that these vibrations will always be performed in the same time, without reference to their size (see Art. 53). Thus, if we bend a spring so as to produce a displacement equal to unity, and it vibrates backwards and forwards at the rate of one vibration in a second, and if we now distend the spring so as to produce a displacement equal to 2, the vibrations will in this case also be performed at the rate of one in a second, the only difference being that their range will now be twice as great as in the former case.

This principle of isochronism, or the faculty of performing vibrations in the same time independent of their extent, applies to all elastic bodies. Thus, if I bend an elastic rod, it will vibrate in the same time whether it be pulled lightly or strongly, whether the point bent be forced one millimetre or two millimetres from its position of rest; and if I bend another and a different rod, it also will perform each of its vibrations in the same time, independent of their extent: but the time of vibration of the second rod will not necessarily be the same as that of the first.

132. Time of Vibration.—Now, on what does this time of vibration depend? It may be desirable to enter somewhat fully into this question, because it is of much importance, while the dynamical principles upon which its solution is founded are capable of being exhibited without the aid of the higher mathematics.

Suppose that we have two bodies of equal mass vibrating in straight lines backwards and forwards, on either side of their positions of rest, to which each is attracted by a force proportional to the distance from that point. Let the magnitude of this force be the same for both bodies when their distance from their respective points of rest is the same.

Finally, to fix our conceptions, let us suppose that the range of oscillation of the second body is double that of the first.

In the next place divide each range into a great number of small divisions, the number of these being the same for each range.

Also let each division of the path of double range be double that of the corresponding division of the path of single range, so that, in fact, the large path and its various divisions will represent the small path and its various divisions magnified two times.

Let us next consider the time of passage of each body through the extreme division of its path as it starts to approach its centre of force.

Now the length of tnis division as well as the force is double in the case of the body of double range, while again we may suppose this force to remain constant for each body while it passes across one such very small division.

It follows from this that the time of passage across the extreme division will be the same for both bodies, since, while the one division is double of the other, the force is likewise doubled. This will be evident if we reflect that if gravity were double of what it is, a stone would fall through a double space in the first second.

It follows, moreover, that the velocity with which the body enters its second division is twice as great in the case of the body of double range. This will be evident if we reflect that, were the force of gravity double, the velocity generated by a body that had fallen from rest through a given time would be doubled also.

Now the size of the second division is double in the case of the body of double range, but this body enters the double space with a double initial velocity, and being at twice the distance from its position of rest is acted on throughout by a constant force the double of that acting on the other body. Hence the time of passage through the second division is the same for both bodies, and the increment of velocity of the more distant body is double that of the other.

A continuation of this process of argument will suffice to show that the time of passage of the first body through any one of its divisions is equal to the time of passage of the second body through the corresponding division, and hence the whole time of passage of the first body from its extreme to its point of rest will be equal to that of the second body between these corresponding points.

Here, therefore, we have a proof of the isochronism of such vibrations, inasmuch as under a force proportional to the distance the body of double range vibrates as fast as the body of single range.

In the next place let us take two paths precisely similar in size, and divide each into a great number of small divisions after the manner already specified. It is clear that the divisions of the one path will be precisely equal and similar to those of the other. Let, however, the mass of the body be four times as great in the second path, while notwithstanding the force remains the same as before. During the time that the first body has taken to pass through the first division, the body of fourfold mass will evidently only have passed through one-fourth of this division, and (since under a constant force the spaces described vary as the squares of the times) in double this time it will have just passed through its division.

The time of passage through the first division is therefore double in the case of the body of fourfold mass.

It is also apparent that while the mean velocity of passage through the first division is only one-half in the case of the body of fourfold mass, the velocity with which it enters the next division will also only be one-half of that for the other body. Thus in the case of the body of fourfold mass the velocity with which it enters one division, as well as its increment of velocity in that division, will always be only one half of the corresponding elements for the less massive body —in fine, the time of passage of the more massive body through any one of its divisions will be double of that for the other body, and hence the whole time of passage through all the divisions will likewise be double in the case of the body of fourfold mass. Thus by keeping the force the same and increasing the mass four times, the time of vibration is doubled. It is easily seen that in like manner, by keeping the mass the same and diminishing the force to one quarter of its former amount, the time of vibration will be likewise doubled.

In fine, the time of vibration depends upon the ratio between the mass and the force, or on $\frac{\text{mass}}{\text{force}}$ in such a manner that by increasing the mass four times we double the time. Therefore the time of vibration $\propto \sqrt{\frac{\text{mass}}{\text{force}}}$.

We may illustrate what we have now said by fastening a steel rod at one end and striking the other, when the vibrations will be very rapid ; but if the end be loaded with a lump of lead the vibrations will be very slow.

133. Wave Motion.—Hitherto we have been considering the vibration of a single body or particle only; let us now consider a line of particles propagating what is termed an undulation or wave. If we cause an ordinary corkscrew to turn round upon its axis, we perceive the progress of such a *form* or *wave* from the one end to the other of the screw: nevertheless we are perfectly certain that no individual particle of the screw has travelled from the one end to the other. To use the words of a well-known writer, an undulation or wave consists in the continued transmission of a relative state of particles, while the motion of each particle separately considered is a reciprocating motion. There are many familiar examples of this kind of motion which will at once occur to our readers. Thus, for instance, when the wind blows over a field of corn, we see a form progressing across the field, but we know that notwithstanding this the individual ears of corn do not move from their places, but only vibrate backwards and forwards. In like manner, if we throw a stone into a pool, we perceive a series of waves spreading outwards from the centre of disturbance, while at the same time a little reflection will convince us that the individual particles of water do not so move.

134. Up and Down Waves.—In order to illustrate wave motion, let us first take a case where the motion of individual particles is at right angles to the direction of transmission, as in a wave such as may be seen proceeding over the surface of a pool.

Let the upper line of figure 43 represent a series of waves of this kind, the particles 1, 11, 21, &c. being at the extremity of their upward motion, at the same time that the particles 6, 16, &c. are at the extremity of their downward motion, so that 1, 11, 21, &c. form the crests of the waves, and 6, 16, &c. their troughs, the distance between two contiguous crests, such as 1 and 11, or between two contiguous hollows, such as 6 and 16, constituting what is termed a **wave-length.**

Let us now suppose that a short time has elapsed, and that when we again view the phenomenon we find it as in the second line of Fig. 43. The particle 1 has now descended, while the particle 3 is at the summit of its upward motion, and constitutes the top of the wave. In fine, the state of things of the upper line has been pushed forward in a direction from left to right by the breadth between 1 and 3, or the wave has appeared to move over this distance.

In the next figure the undulations have moved forward yet another step, until now the particles that were at first at the extreme limit of their downward motion have attained the extreme limit of their upward motion, and the state of things has progressed a distance equal to half a wave-length since we first viewed it. This progression of a relative state of particles will continue to go on in the same direction, from left to right, until the wave whose summit was at 1, shall have travelled over a whole wave-length, and taken up the

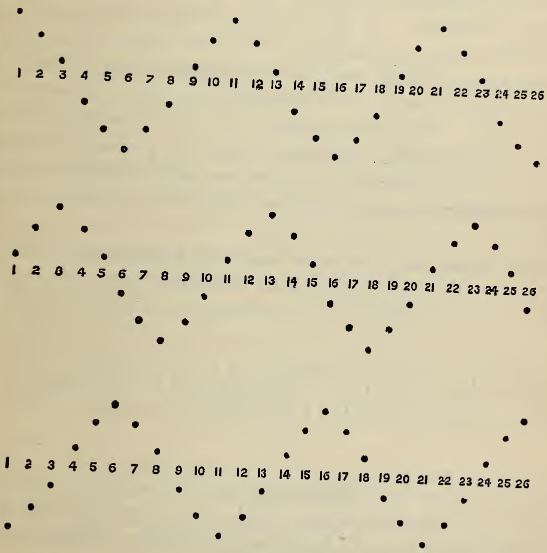


FIG. 43.

position 11; while the wave-crest at 11 will have travelled to 21, and, in fact, everything will have travelled forward one complete wave-length.

If we now pause to regard the state of things, we shall find that the particles have come into the very same position which they had at first, 1, 11, 21, &c., representing crests, and 6, 16, &c., representing hollows. During this interval, therefore, each particle must have performed a complete double vibration. It thus appears that, during the time which the wave has taken to travel over one complete wave-length, each particle has made a complete double vibration; so that if l denote the wave-length, or distance from one crest to the next, and v denote the velocity with which the wave is propagated, and t be the time of a complete double vibration of a particle, then l, or one wave-length, will have been travelled over by the wave in the time t,—that is to say, $l = v\,t$.

135. Waves of Condensation and Rarefaction.—But we may have other waves than those now described; for instance, the motion of a particle may not be in a direction perpendicular to that of transmission, but it may be in the same direction; the wave may, indeed, not be one of up-and-down motion, but of backward and forward motion,—in fact, a wave of condensation and rarefaction.

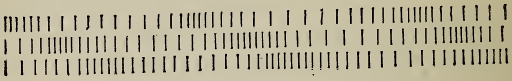


FIG. 44.

The progress of such a wave is seen in Fig. 44, where the wave-length as well as the states of progress exhibited are analogous to those already given in the case of an up-and-down wave. Here also we see that a complete vibration of a particle is performed in the time during which the undulation progresses one wave-length.

136. Having described what is meant by wave-length, we shall now explain what is meant by the **phase** of a vibrating particle. The phase of a particle at any given moment denotes its place and direction of motion at that moment, as regards its vibration. If it should happen to be at its point of rest, that would be one phase; if at the extremity of its upper range, that would denote another phase; while half-way between the two would be a third, and so on. But in order to represent the phase of a vibrating particle with perfect precision, it is necessary to have recourse to a mathematical expression.

It will be seen that in an undulation no two contiguous particles in the direction in which the wave is advancing are in the same phase; this, indeed, is the essential peculiarity of an undulation, for if all these particles were at precisely the same moment pulled in the same direction and to the same extent from their original positions of rest, the motion would be that of the whole body and not an undulation. It is, indeed, the distortion implied by the fact that different particles are differently placed at the same moment that gives rise to the forces which propagate the motion.

When a wave is advancing,—by the **front** of this wave we mean all those portions of it that are in the same phase. Thus on the sea-shore the well-known ridge of water forms the front of the advancing wave. In general we may regard the line of front as perpendicular to the direction in which the wave is advancing.

137.—The **amplitude** means the extent of vibration of any particle on either side of its position of rest. Now, we have seen (Art. 131) that the time of vibration of the particles of elastic bodies is independent of the extent or amplitude, and we thus see that we may have two sizes of waves, in both of which the wave-length and velocity of propagation shall be the same, although the extent of vibration of the individual particles is very different for the two. If the undulation be like that of Fig. 43, we can imagine the wave-length, or distance between I and II, to remain the same, while the amount of elevation and depression of the various particles is much altered: and again, if the wave be like that of Fig. 44, we can equally well imagine the wave-length to remain unchanged, while the amount of condensation and rarefaction of the various particles is lessened or increased. In fine, the wave-length does not depend upon the extent of vibration of the individual particles.

LESSON XVIII.—SOUND.

138. Definition of Acoustics.—That branch of physics which relates to sound is termed acoustics.

The word "sound" is used in common language sometimes to denote the physiological sensation caused in the organs of hearing by an aërial impulse, and sometimes it is used to denote this impulse itself. It is in this latter or physical sense that we shall here use it; so that when we speak of the velocity of propagation of a sound, we mean the rate of transmission of an aërial impulse regarded as an external existence, independent of our capacity of hearing.

139. Musical Sound and Noise.—When a sudden blow is delivered to the air, as when a cannon is discharged or an electric spark is made to pass, the impulse is propagated through the air, and, ultimately reaching our ear, affects us as a *noise*. A noise, therefore, represents a sudden and single blow, and has no perceptible wave-length. But if this blow is periodically delivered,—say, for instance, if the air is struck 100 times in a second,—then it is clear that a particle of air to which the first blow has been propagated will in one-hundredth of a second be reached by the second blow, and so on; in fact, every hundredth of a second this particle of air will be similarly affected, so that the particle may be supposed to make a complete vibration in this time. Now, let us suppose that the impulse is propagated at the rate of 340 metres in one second, then every hundredth of a second it will have advanced 3·40 metres. But we have seen (Art. 134) that in the time of a complete vibration the impulse advances one wave-length, so that in the instance now given the wave-length is obviously 3·40 metres. Thus we see that the uniform repetition of blows has given an element to the sound which was wanting in a single blow, for it has now acquired a definite wave-length.

When this succession of aërial impulses reaches the ear, they will not be perceived as separate noises, but they will be sufficiently rapid to produce a continuous and pleasing impression.

To the well-organized ear, the nature of this impression will depend upon the wave-length; and when the sound has a certain wave-length, such an ear will at once pronounce it to be of a certain **pitch**, or to have a certain value in the musical scale. We thus see that wave-length is a physical fact, independent of the ear, while musical value or pitch refers rather to the physiological impression produced on the organs of hearing by a sound of a given wave-length.

Now, if the string of a musical instrument makes a complete vibration in one-hundredth of a second, then each particle of this string will be in precisely the same *vibrating phase* 100 times every second, and hence 100 times each second will this string communicate the same sort of impulse or blow to the air, of which the wave-length will be precisely the same as if the air had been struck 100 times every second.

Besides wave-length the ear perceives **intensity**; thus the same sound will appear much less intense to one who is at a distance from the sounding body than to one who is near it. Again, the same note will have a different effect upon the ear if produced by two different instruments, so that we recognize something in a note besides pitch and intensity. This something is analogous to quality, and is called the **timbre** of the note. Indeed no note is ever entirely pure, and the timbre has been found by Helmholtz to depend upon the number and intensity of the various tones that constitute the compound note.

140. Nature of Sound Waves.—From all this we see that the *air is struck* by a sounding body, and that the nearer particles of air communicate the blow to those next them, and those again to the next layer, and so on, just as if the same particles were a series of equal elastic balls, in which case, as we have already seen (Art. 121), an impulse is propagated from ball to ball very much after the manner in which the air propagates sound. Sound thus represents a wave of condensation and rarefaction, and not an up-and-down wave, and the motion of the aërial particles is backwards and forwards in the line of propagation.

141. Sound is not propagated in Vacuo.—But if the vibrating body, such as a string or plate, be placed in vacuo, and there struck, we shall have no sound, because sound denotes the communication of part of the energy of the vibrating body to the substance or medium with which it is in contact, and hence if it be not in contact with a suitable medium it cannot communicate any of its energy. This may be shown by the following experiment. Place a small metallic bell, which is periodically struck by a hammer, in the receiver of an air-pump, and gradually exhaust the air. It will be found that as the process of exhaustion goes on the sound will become weaker and weaker, and could we succeed in surrounding the body by a complete vacuum we should not hear any sound whatever.

142. Reflexion of Sound.—When a wave of sound in its progress through the air strikes upon an obstacle, it is reflected from it, and this reflexion will take place according to the following law :—

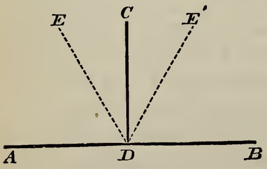


FIG. 45

Let A D B (Fig. 45) represent a plane perpendicular to the paper, and let a wave or ray of sound E D strike it at the point D; at D draw D C perpendicular to the plane A D B, then the wave will be thrown off by the surface, or in other words reflected in the direction D E′, such that the angle E D C shall be equal to C D E′, and the three lines E D, D C, and D E′, shall be in the same plane, this plane being perpendicular to that of A D B. If we call C D E the angle of incidence, and C D E′ the angle of reflexion, we have the following laws :—

(1.) *The angle of reflexion is equal to the angle of incidence.*

(2.) *The incident and the reflected ray are both in the same plane, which is also perpendicular to that of the reflecting surface.*

It will be afterwards seen that the laws of reflexion are the same both for sound and light.

143. Echoes.—When a sound strikes an obstacle, and is reflected by it so as to cause a repetition, this constitutes an echo.

Sometimes the reflected sound is much more audible than the original. For instance, the direct sound of a peal of bells may be prevented by some obstacle from impressing the ear, while their echo from a row of houses may be quite audible.

In order that a word may be distinctly repeated by an echo, a small but appreciable interval of time must elapse between the end of the word and the beginning of the echo, so that if the word is a long one the reflecting surface will require to be farther away than if it be short. For a word of one syllable the reflecting surface will require to be not less than 42 metres distant from the ear; for words of two syllables this distance will have to be doubled, and so on. Sometimes the reflected sound is again reflected; and we have accounts of curious echoes where the original sound is repeated as often as twenty times. In whispering galleries the sound is successively repeated from the smooth walls of the gallery. In that of St. Paul's, London, a whisper at one side of the dome is conveyed to the other without being heard at any intermediate point.

144. Conjugate Reflectors.—By employing two conjugate reflectors, as in Fig. 46, and by placing a sounding body at the focus of the one mirror, the sound which diverges from it will be reflected as in the figure, until it ultimately comes to a point at the focus *f* of the other mirror. Thus, if a watch be placed at the one focus, its ticking will be heard very distinctly at the other.

It is related that at the Cathedral of Girgenti, in Sicily, the slightest whisper is conveyed from the great western door to the cornice behind the high altar, through a distance of 250 feet, and that, unfortunately, the focus at the former station was chosen for the place of the confessional. The result was that a listener placed at the opposite focus often heard what was never intended for the public ear, until at length the circumstance became known and another site was chosen.

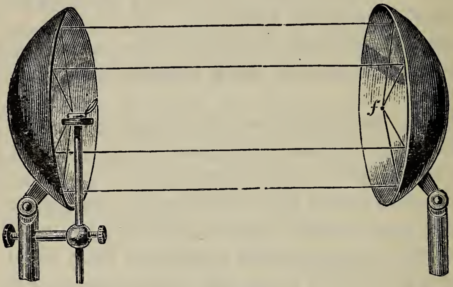


FIG. 46.

145. Refraction of Sound.—But there is, besides reflexion, another bond of similarity between the two agents sound and light, for both are capable of refraction or bending. Thus a convex lens made of glass, or in fact of any transparent substance denser than the air, has the power of concentrating light or bending it to a point; and if a lens of this kind (Fig. 47) be placed so as to receive the rays from a luminous body at S directly upon it, these rays will be brought to a focus at some point *f* after having passed through the lens, and will there be so concentrated as probably to cause ignition if a combustible substance be placed at *f*.

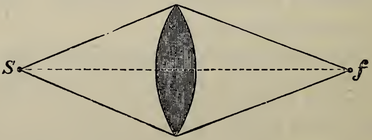


FIG. 47.

M. Sondhauss has made similar experiments with regard to sound. He made a lenticular bag, which he filled with carbonic acid gas, a substance which is denser than ordinary atmospheric air. Placing a watch at one side of the lens, he found that there was one particular point or focus at the other side where the sound of the watch could be most distinctly heard, and he argued from this that the carbonic acid gas refracted the sound just as a lens refracts light. The refraction of sound by the atmosphere will be discussed in Art. 153.

146. Velocity of Sound in Air.—Both light and sound travel through the air with certain definite velocities, but that of light is very great as compared with sound. The consequence is, that if a cannon be discharged at a great distance, and if we record the moment when we see the flash, we may be sure that this is as nearly as possible the exact moment when the cannon was fired, since the light cannot have occupied an appreciable time in reaching our eye. If we listen attentively after having seen the flash, we shall presently hear the report, and the interval between the flash and the report will denote the length of time which the sound has taken to travel from the cannon to us. By this means observers have determined the velocity of sound, which is found to be very nearly 340 metres per second under ordinary atmospheric conditions.

Sounds of various wave-lengths all travel at the same rate, and Biot found that an air played at one end of a tube 1,040 yards long was heard without alteration at the other end. It is, however, believed that a very loud sound, such as the report of a gun or thunder, is propagated somewhat more rapidly than a very weak one.

147. Velocity in other Gases.—The velocity of sound depends upon the nature of the gaseous medium. Suppose, for instance, that we have a gas which under the ordinary atmospheric pressure is only half as dense as air, and imagine that two precisely similar undulations are travelling in the two media; then, while the differences of pressure which give rise to the vibrations are the same in both, the mass to be moved is only half as large in the case of the lighter gas. The vibrations of the particles of this gas will therefore be performed in less time, while the velocity of propagation of the wave will be proportionally increased, the velocity of propagation being obviously inversely proportional to the time of vibration of a particle, which by Art. 134 is seen to be the time which the wave takes to advance through one wave-length. Dulong has made the following determination of the velocity of sound (at 0° C.) in different gases :—

Carbonic acid . .	261·6	metres	per second.
Oxygen	317·2	„	„
Air.	333·0	„	„
Carbonic oxide .	337·4	„	„
Hydrogen . . .	1269·5	„	„

Again, it is well known that the relative densities of these various gases are as follows—that of air being the unit :—

Name	Density.
Carbonic acid	1·529
Oxygen	1·106
Air	1·000
Carbonic oxide	0·968
Hydrogen	0·069

It will easily be seen that in accordance with the formula of Art. 132, the relative velocity of sound in various gases is in point of fact as well as from theory inversely proportional to the square root of the density of the gas. For the density of the gas indicates the extent to which the mass is increased, the forces concerned in the propagation meanwhile remaining the same. Now the formula tells us that the time of vibration $\propto \sqrt{\frac{\text{mass}}{\text{force}}}$, and since the velocity of propagation is obviously inversely proportional to the time of vibration, we have velocity of propagation, $\propto \sqrt{\frac{\text{force}}{\text{mass}}}$, and thus while the force remains constant the velocity will vary inversely as the square root of the mass. That it does so as a matter of fact will be seen from the following table :—

Name.	$\sqrt{\frac{1}{\text{Mass.}}}$	Relative velocity of sound; that in Air = 1.
Carbonic acid	0·809	0·790
Oxygen	0·951	0·952
Air	1·000	1·000
Carbonic oxide	1·016	1·013
Hydrogen	3·799	3·810

In this table it will be seen that the two columns, one of which denotes the calculated and the other the observed velocity of sound, are as nearly as possible alike.

148. Velocity does not vary with Density.—We have seen that the *quantity of sound* carried off by the air from a vibrating body depends upon the density of the air; nevertheless, the velocity with which sound is propagated will be the same in rare as in dense air; that is to say, provided the chemical nature of a gas remains unchanged, the velocity of sound does not vary with its density, the reason being that in the same gas, although the pressure differences which cause the vibrations are proportional to the density of the gas, yet the mass to be moved is increased in the same proportion. We have thus a double pressure difference to move a double mass, so that the two things precisely counteract each other, and the velocity remains unaltered.

149. Velocity varies with Temperature.—If, however, the gas in question be greatly raised in temperature, then we may have the same pressure difference, and hence the same moving force with a very much smaller mass, and the velocity of propagation will therefore be increased. Hence sound travels faster in warm than in cold air, and this is the reason why in the table given above the temperature at which the observations were made is recorded as well as the nature of the gas.

150. Velocity in Liquids and Solids.—By means of experiments made at the Lake of Geneva, it has been ascertained that the velocity of sound in water is nearly four times as great as in air. In solids it is still greater; thus in wood, for instance, sound travels from 10 to 16 times as fast as in air.

151. Circumstances affecting intensity of Sound.—When the air is disturbed so as to cause a sound, this disturbance spreads out on all sides of the body which causes it. Let us suppose that the source of disturbance is in mid-air, and that at a given instant the sound has spread to the distance of 100 metres on all sides of this source, the disturbance will therefore occupy the surface of a sphere, of which the radius is 100 metres. But the radius of the disturbed sphere will evidently go on increasing, until in a very short time the disturbance will occupy the surface of a sphere the radius of which is 200 metres. Now, in the latter case, the disturbed surface is four times as great as in the former, since the surfaces of spheres vary as the squares of their radii, and hence in this latter case the same amount of energy will be spread over four times the surface, the result of which will be that the amount of energy corresponding to unit of surface, that is to say the intensity, will be four times less. Thus by doubling the distance from the centre of disturbance, we diminish the intensity four times; that is to say, *the intensity varies inversely as the square of the distance.*

It is clear that in this demonstration we have supposed that the whole of the energy which occupied the surface of the sphere of 100 metres in radius will be conveyed outwards as the undulation progresses without diminution, so as afterwards to occupy the surface of a sphere of 200 metres in radius. If, however, any of the energy be absorbed in its passage outwards, this law will no longer hold good, but the intensity will in this case diminish more rapidly than the inverse square of the distance. Now probably a small part of the energy of sound is converted into heat as it passes along the air, in consequence of which the intensity will diminish somewhat more rapidly with the distance than we have indicated.

152. The intensity of Sound depends upon the density of the Air.—The experiment with the receiver showed us that the more air was rarefied the less capable was it of transmitting the sound of the bell. Thus, also, at the top of Mont Blanc, where the air is very much rarefied, the discharge of a pistol does not produce such a noise as it does at the earth's surface. In like manner sound is enfeebled in hydrogen gas, which is much less dense than air ; in fine, if the particles of a medium be small and far apart, they do not carry off the energy of a vibrating body to the same extent as if they were large and close together.

153. Atmospheric Conditions affecting the audibility of Sounds.—It is well known that sound is better heard when travelling with the wind than when travelling against it, and the reason of this has been explained by Professor Stokes. The velocity of the wind quite close to the ground is much reduced by friction, so that as we ascend vertically the wind increases in velocity. Now an undulatory disturbance such as sound partakes of course of the movement in space of the body through which it is passing, whether this be the movement of the earth in its orbit or the movement along the earth of the column of air conveying the sound.

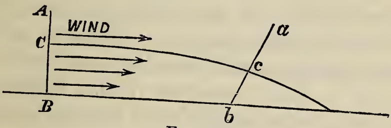


FIG. 47*a*.

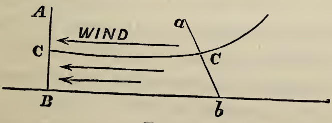


FIG. 47*b*.

In order to appreciate the influence of wind on the audibility of sound let us, first of all, imagine a sound progressing in the same direction as the wind (Fig. 47*a*), this wind being more violent at a point C above the earth than it is at a point B at the earth's surface; now let A B denote the front of the wave to begin with (Art. 136), the direction of propagation being perpendicular to this front. It is clear that the disturbance at C will be carried more quickly forward than that at B, since it is moving with a stronger wind. Hence after a time the front of the wave will be in the position *a b*, and the direction of propagation being always perpendicular to the front will tend to carry the sound down toward the earth; thus the sound will be well heard.

On the other hand, if the sound be moving against the wind, as in Fig 47*b*, the front will change in such a manner as to carry the sound upwards rather than downwards, in consequence of which it will not be well heard.

Professor Osborne Reynolds has recently extended this reasoning to layers of air of different temperature. If there be a rapid falling off in the temperature of the air as we ascend then the velocity of propagation of the impulse for the upper layers will (Art. 149) be diminished by this cause, just as in Fig. 47*b*, with this difference, that the diminution will now be irrespective of the direction in which the sound is travelling. The sound will therefore be thrown upwards, and will not be well heard. On the other hand, if the upper layers be warmer than the under layers, as they are under certain abnormal atmospheric conditions, the sound will be thrown down, and will be heard extremely well. Besides all this it may be inferred that when the air is homogeneous sound is much better heard, and the experiments of Professor Tyndall go to show that ascending and descending currents of air of different temperatures serve to scatter the sound by reflecting it from their surfaces, and are thus prejudicial to audibility.

154. The intensity of the sound of a musical string is strengthened by a sounding-box.—This is a hollow box, which along with its air vibrates in unison with the string. By an arrangement of this kind, the sound that would otherwise have passed away is caught up by the box, and given out by it as if by a second source, thereby increasing the effect.

LESSON XIX.—VIBRATIONS OF SOUNDING BODIES.

155. Vibration of Strings.—By striking or pulling a stretched string transversal vibrations are produced in it depending upon the nature, the tension, and the thickness of the string, and these vibrations give rise to a musical note (Art. 139).

If a vibratory chord (Fig. 48) have a stop B inserted (say at one-third the whole length of the chord from one of its extremities D), the point B will always remain at rest, and the tendency will be to cause the whole chord to vibrate in the manner represented in the figure. Here not only will the point B remain at rest, but the point C, which is a point as far from the other extremity A as B is from D, will also be at rest; the two opposite stages of the vibration being represented by the continuous and the dotted lines in the figure.

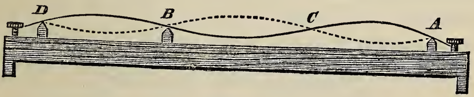


FIG. 48.

The points B and C are called **Nodal Points** or nodes, and the middle part between two nodes is called a **Loop** or **Ventral Segment.**

The ratio of D B to B A must be represented by some whole number, as 1 : 2 or 1 : 3 or 2 : 3, otherwise the vibrations will interfere with one another.

The existence of nodal lines is also rendered very evident by vibratory plates. These contain nodal lines varying in number and figure according to the form and substance of the plates, and according also to the method in which the plates are fixed. In order to render visible the existence of these lines we may sprinkle the plate with sand before it begins to vibrate.

The sand, during the vibration, is thrown off the ventral segments or vibrating parts of the plate, and accumulates around the nodes or the lines which remain at rest.

The following are the laws which connect the nature of the vibrations with the properties of the string :—

(1) *When the stretching weight is constant the time of vibration varies as the length of the string.*

(2) *The time of vibration varies as the radius of the string.*

(3) *The time likewise varies inversely as the square root of the stretching weight.*

(4) *The time finally varies as the square root of the density of the string.*

These various laws regarding strings are capable of being proved in a somewhat simple manner. Let us first take that which tells us how the time of vibration varies with the length of the string.

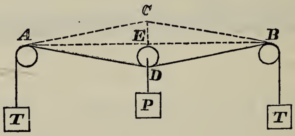


FIG. 49.

In Fig. 49 we have before us the equilibrium which takes place in the case of the single moveable pulley with inclined strings. Let us suppose that the inclination of these strings is very small, or that A D B is nearly a straight line. The tension of the string at A or B which we have called T will represent the tension of a sounding chord, while P will represent the force with which this chord is solicited back towards its position of rest A E B.

Drawing A C parallel to B D and B C to A D, the figure A C B D becomes a parallelogram of forces in which D A represents the tension while D C represents the force P. If we confine ourselves to small displacements it is evident that the whole length of the string between A and B will not vary much.

But force P : tension T : : C D : A D.

Now the tension T is supposed to be constant, while the length A D of the same string is constant also. If therefore C D be doubled the force P will be doubled, and in fine if C D increase in any proportion the force P will increase in the same proportion. Now C D is the double of E D, and E D represents the distance to which the string has been plucked from its position of rest. Here then we have a force which is proportional to the displacement.

It will also be seen that if we double the length of the string A D B while the displacement E D remains the same, then the ratio of C D or E D to A D will be halved, in other words, the ratio of the force P to the tension T will be halved. Therefore if we double the length of the string while the tension remains the same, the force P, corresponding to a definite displacement, will be halved. On the other hand, the mass to be moved (that of the string) will be doubled, so that for a string of double length the force is halved while the mass is doubled, in other words, the ratio between the force and the mass is diminished four times, and hence (Art. 132) the time of vibration which $\propto \sqrt{\frac{\text{mass}}{\text{force}}}$ will be doubled.

In the next place it is easily seen that the time of vibration varies as the radius of the string. For the mass of the string is proportional to the square of the radius, and (other things being the same) the time of vibration $\propto \sqrt{\text{mass}}$, and therefore $\propto$ radius.

Thirdly, the time of vibration varies inversely as the square root of the stretching weight.

For it will be easily seen from Fig. 49 that if we have two similar strings and if T, or the tension, in the second is four times as great as in the first, then P, or the force in the second, will be four times as great as the force in the first, and the time of vibration, which $\propto \sqrt{\frac{\text{mass}}{\text{force}}}$ will be only half in the second of what it is in the first.

Lastly, the time of vibration varies as the square root of the density of the string. This is evident, for we know that the time $\propto \sqrt{\text{mass}}$, and that the mass varies as the density of the string, hence also the time of vibration varies as the square root of the density.

These various laws are susceptible of an easy and striking verification by means of an arrangement devised by M. Melde of Marburg. The nature of this will be seen from Fig. 49*a*. It consists of a string passing over a pulley and weighted at one end, with its other end attached to a vibrating tuning-fork.

If the attachment be such that the fork vibrates backwards and forwards in the direction of the string, then every time the prong moves towards the string it gives the string an opportunity of forming a loop or ventral segment. In this case the string will have one ventral segment, or make a semi-vibration for every complete vibration of the fork. If, on the other hand, the attachment be such that the fork vibrates backwards and forwards at right angles to the direction of the string, there will be a complete vibration of the string for each complete vibration of the fork.

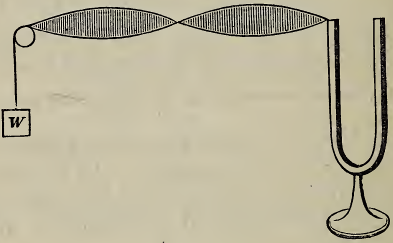


Fig. 49*a*.

Thus the string will vibrate twice as rapidly in the latter position as in the former. If therefore we have a string of such length, material, and tension that it vibrates in one loop and without a node when attached in the first of these two ways to the fork, it ought, in accordance with Law (1) of this article, to split itself into two parts when attached to the fork in the last of the two ways, in order that it may have the opportunity of vibrating twice as fast. Or if, as in the figure, it vibrates in two parts in the first position, it will vibrate in four parts in the second.

Again, if in any arrangement a string when attached to a tuning-fork vibrates in one loop and without a node, if we diminish fourfold the stretching weight W, we shall find that in order to coincide with the tuning-fork it will now vibrate in two parts, thus proving Laws (1) and (3) of the present article.

156. Wind Instruments.—In these instruments it is not the substance of the tube, but the column of air which it contains, that is the cause of the sound.

An organ-pipe is a very good illustration of this kind of instrument. Its mode of action will be understood by reference to Fig. 50.

The mouthpiece is fixed at one end of the tube; of this tube a portion only is represented in the figure; the other end we shall suppose to be closed.

As a current of air is made to enter by the mouth, it strikes against the upper lip, and the effect of this is to cause the air to issue from *b o* in a pulsatory manner.

In an organ-pipe of this kind, the primary note is that of which the semi-wave length is twice the length of the pipe, and the air in the pipe is agitated in such a manner that the stratum at the closed top of the pipe is at rest, while the stratum of air at the mouthpiece has the greatest amplitude of vibration.

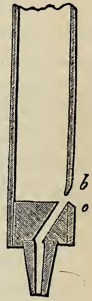


FIG. 50.

We may indeed suppose that there are a number of different small pulses produced by the air striking against the lip of the pipe, but of these one will be strengthened by the column of vibrating air in the pipe, and exalted into a musical sound.

To find which, let us suppose that one of these pulses is just in the act of striking the air *upwards* into the pipe. This blow will be transmitted by the air to the top of the pipe, and will then be reflected back to the lip. Now if when it reaches the lip the pulse be in the act of moving *downwards*, its motion will evidently be strengthened by the blow from the air of the pipe. In fact, the pulse which is strengthened is that which performs half a complete vibration in the time that the blow takes to ascend up to the top of the tube, and come back again to the lip.

We have said that in such a pipe it is the vibration of the column of air which causes the sound, in proof of which, if an organ-pipe be filled with any other gas, we have quite a different sound. The reason of this is, that the complete time of vibration of the column of gas is twice that which the undulatory motion requires to travel up to the top of the tube and back again. Now this is different for different gases. Hence the velocity of sound in different gases may be found by filling an organ-pipe of known length with these gases, and estimating the pitch of the sound which is produced.

What we have said refers to a shut pipe. In an open pipe there is a stratum of greatest amplitude of vibration at each end, and the semi-wave-length of the sound produced by an open pipe is equal to the length of the pipe, so that it is only half of that produced by a shut pipe of the same length. We shall again return to the subject of organ-pipes when we have considered the longitudinal vibrations of rods.

157. Vibrations of Rods.—Let a series of rods of wood be fixed at one end, and be free to move at the other: then we may have two kinds of vibratory motions of such rods.

Transverse Vibrations are produced by striking the rod or passing a bow over it.

Longitudinal Vibrations are produced by rubbing it up and down with a piece of cloth with particles of resin on it, or with the moistened finger.

It is found that *the number of transverse vibrations made in a second by a rod is directly as its thickness and inversely as the square of its length.* It is also found *that the number of longitudinal vibrations of a rod in one second is inversely as its length, whatever be the diameter of the transverse section.*

A little consideration will convince us of the truth of these laws. Suppose that we study the transverse vibrations of two similar rods, the one twice as long as the other. Let each be deflected through the same distance from its position of rest. Then by (Art. 68) we may conclude that the force called into action by the displacement is eight times greater in the shorter than in the longer rod. On the other hand the mass to be moved is double in the case of the longer rod, so that altogether the ratio of the force to the mass is sixteen times less in the longer than in the shorter rod. Therefore since by the formula (Art. 132) the time of vibration $\propto \sqrt{\frac{\text{mass}}{\text{force}}}$, this time will be four times greater in the case of the longer rod.

A precisely similar mode of reasoning will show us that the number of transverse vibrations of a rod made in one second will be directly as its thickness, inasmuch as for a double thickness the relation of the force to the mass is increased four times.

With regard to longitudinal vibration, a rod must be regarded in the same way as we should a column of air. A blow given to a rod fixed at one end and free at the other, say one tending to compress its particles, is transmitted along the rod to the fixed end, where it is reflected back. When however it makes its reappearance at the free end it is as a force tending to elongate this end, being thus in the opposite direction to the original blow. This state of things in its turn runs along the rod to the fixed end and back again to the free end. When at the free end it is now at last similar to the original blow, so that a complete cycle, representing a complete wave, will only take place in the time which the blow requires to go twice backwards and forwards along the rod.

If however the rod be fixed at both ends, then the time of a complete vibration will be the time that a blow takes to go once, and not twice, backwards and forwards along the rod.

If therefore we have two similar rods of equal length, one fixed at one end and free at the other, and the other fixed at both ends, the second will vibrate twice as rapidly as the first.

It is easily seen from this that in both cases the time of vibration will vary directly as the length of the rod, and also that the time will be independent of the cross section, because if we double the cross-section we double not only the mass moved, but also the force of restitution (Art. 65), and therefore the proportion between the two which determines the time remains unaltered.

158. From what has been said it is very easy to see in what way a rod in longitudinal vibration is capable of having nodes.

Here of course the essential requisite is that in whatever way the rod splits up as to vibrations the various sections shall all vibrate in the same time. Let us therefore consider three cases, namely, (1) a rod fixed at both ends; (2) a rod fixed at one end and free at the other; and (3) a rod free at both ends.

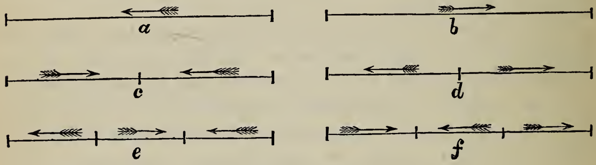


FIG. 50*a*.

In the above diagram Fig. 50*a* we have an illustration of the way in which rods fixed at both ends are capable of breaking up into nodes. In the first line we have the rod vibrating as a whole and without any node.

In the second line we have a node in the middle, the pulses alternately tending to press together the nodal point or to pull it out.

In the third line we have the whole rod divided into three equal parts and therefore possessing two nodes, and in like manner the rod might be subdivided into 4, 5, 6, &c., equal parts. Also the time of vibration for one node is half that for the fundamental vibration, while the time for two nodes is one-third of that for the fundamental vibration, and so on.

In diagram Fig. 50*b*, we have an illustration of the way in which rods fixed at one end and free at the other are capable of breaking up into nodes. *a* and *b* represent the fundamental

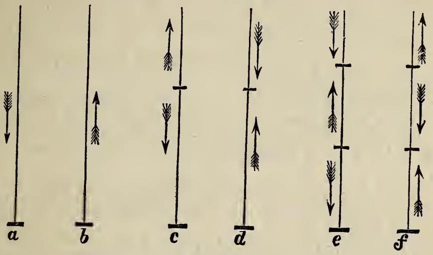


FIG. 50*b*.

vibration of such a rod. In *c* and *d* we have its vibration with one node, which will make its appearance at one third of the whole length from the free end of the rod. For in this case the upper segment forms a rod fixed at one end and free at the other, while the under segment forms a rod twice as long as the upper one, but fixed at both ends, and hence the time of vibration will be the same for both segments.

In *e* and *f* we have two nodes; the upper free segment is one-fifth of the whole rod, and the other two are double of it in size. Here also the three segments will vibrate in the same time.

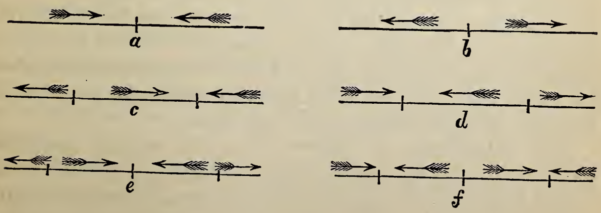


FIG. 50*c*.

Finally in diagram Fig. 50*c*, we have an illustration of the method in which rods free at both ends are capable of breaking up into nodes as regards their longitudinal vibrations

In figures *a* and *b* we have a representation of the fundamental vibration of such a rod. Here we have a node in the middle—that is to say, a rod free at both ends vibrates as two rods of half the length fixed at one end and free at the other.

In figures *c* and *d* we have two nodes each one-fourth of the whole length from the end. Finally, in figures *e* and *f* we have three nodes by which the whole rod is divided into $1 + 2 + 2 + 1 = 6$ parts, the rods free at one end being only half the size of the rods fixed at both ends, by which means the vibrations of all the various sections take place in the same time.

It has already been remarked that a column of air vibrates after the same laws as those which regulate the longitudinal vibrations of rods.

Thus an organ-pipe shut at one end is similar to a rod fixed at one end, is in fact a rod of air, and its time of vibration (Art. 156) is the time that a pulse takes to travel twice up and down the organ-pipe, just as the time of vibration of the rod is that occupied by the pulse in travelling twice backwards and forwards along the rod.

In like manner an organ-pipe open at both ends will vibrate like a rod free at both ends, and will therefore have a node in the middle, so that its fundamental note will be the same as that of a shut pipe of half the size.

In fine, the analogy between such rods and organ-pipes is complete. An organ-pipe open at one end will therefore be capable of breaking up into nodes precisely in the same manner as a rod fixed at one end and free at the other, while an organ-pipe open at both ends will split up into nodes precisely after the manner of a rod free at both ends.

159. Vibrations of Plates.—A plate may be made to vibrate by drawing a bow across its edge.

The following law governs the vibrations of such bodies ; *In plates alike in other respects, the number of vibrations per second varies directly as the thickness of the plate, and inversely as its area.*

Gongs, cymbals, and bells are instruments in which the sound is produced by vibrating plates or masses, while in a drum the sound is produced by a vibratory membrane.

160. Communication of Vibrations.—If a musical note is traversing the air in the presence of an instrument capable of sounding the same note, this instrument seems to take up the note and give it out of its own accord. This may be frequently observed in the case of a piano, which rings to a sound; or, again, the string of a violin may be made to vibrate by sounding a tuning-fork which gives out the same note.

Now since an undulation of sound is a kind of energy, and since energy cannot be created, it follows that this undulatory energy must be absorbed by the string in order that it may be given out again by the string on its own account. In fact, when a string takes up a note in this manner, there is a communication of the energy of the sound-wave from the air to the string; but when we strike the same string, there is a communication of the energy of the sound wave from the string to the air. We thus see that *a string when at rest absorbs that particular kind of undulation which it gives out when struck.* It will be afterwards seen that there is a similar law in the case of radiant light and heat.

161. Determination of Number of Vibrations.—One of the simplest machines for measuring the number of vibrations corresponding to a given sound is that of Savart. In this machine a toothed wheel B is made to revolve very fast, and there is a card E at one end, which is so fixed as to be struck by each of the teeth of the wheel B as it revolves with great velocity. Thus, if the wheel B revolves three times in a second, and has 100 teeth, the card will be struck 300 times in a second, and will therefore emit a musical note.

At the side of the wheel there is an indicator which shows how many revolutions have been made by the wheel, from which we can calculate the number of vibrations in a given time. What we have to do, therefore, is to increase the velocity of revolution until we get the required sound. We should then keep the speed of the apparatus constant for a given time, say 30 seconds, and meanwhile note on the indicator how many revolutions have been made; we shall thus obtain the number of vibrations in one second corresponding to the sound.

The instrument now described affords us an easy method of determining the velocity of sound in air, which can be practised in an ordinary sized room.

First of all let us take a tuning-fork and find the number of vibrations which it makes in one second by means of Savart's machine, or any similar instrument; next let us take a long cylindrical vessel and fill it with water to such a height that when the tuning-fork is held over its mouth the column of air between the water and the fork vibrates in unison with the fork. A little practice will enable us to decide the height very exactly, because it is accompanied by an unmistakable exaltation of sound—we have in fact made an organ pipe.

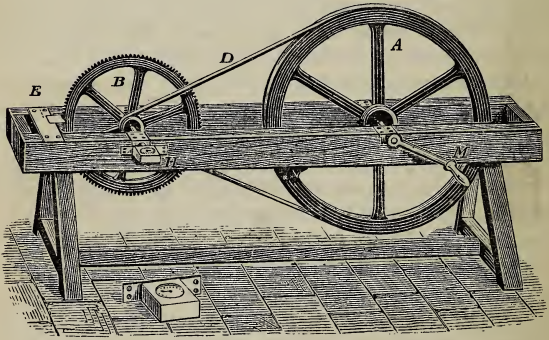


FIG. 51.

Now we know that in this case the blow given by the fork to the air will have travelled twice down to the water and back again during the time of a complete vibration of the fork (Art. 156). If therefore we measure the length of the column and multiply it by four we find the length which the blow has travelled in air during one vibration of the fork, and hence if we have previously ascertained how many such blows are delivered in one second we can find the velocity of sound.

Suppose, for instance, that a given tuning-fork is found by means of Savart's machine to vibrate 550 times in one second, and that it vibrates in unison with a column of air six inches long. The sound-blow will therefore travel over four times six inches, or two feet of air during one vibration, and as there are 550 of these in one second, it will travel 1,100 feet in one second.

162. Graphical representation of Vibrations.—One of the best methods of making vibrations apparent is that of M. Lissajous, which is represented in Fig. 52. The essentials of the arrangements are, in the first place, a tuning-fork with a small mirror attached to one arm, and a small counterpoise, in order to balance the mirror, to the other.

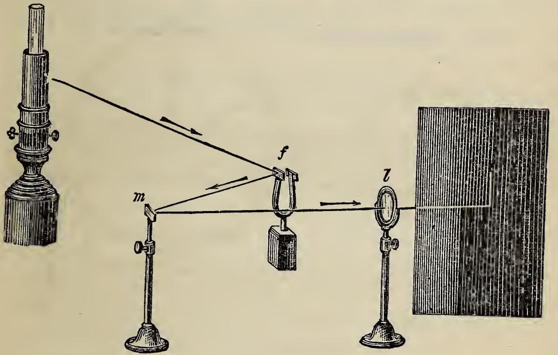


Fig. 52.

A ray of light from a hole in a darkened chimney of a lamp is made to strike this mirror, and is reflected from it towards another mirror *m*; it then strikes a lens *l*, which is so arranged as to throw upon a screen a small luminous point, being the image of the hole in the dark chimney of the lamp from which the light originally proceeds. Thus, if the tuning-fork be at rest, we shall simply have a luminous point on the screen ; but if it has been put into vibration, the mirror will of course, move along with it, and the result will be that the luminous dot of light will oscillate on the screen up and down with each vibration of the mirror. But these oscillations will be so rapid that the eye will merely perceive on the screen a luminous line of light, on the same principle that when a burning brand is twirled rapidly round we see a continuous circle of light. Now if, while the tuning-fork is in a state of vibration, we make it rotate, a curved or sinuous bright line will appear on the screen instead of the straight line we have mentioned, the amount of sinuosity depending on the relation between the rapidity with which the tuning-fork vibrates, and that with which it is made to rotate. We are thus furnished with a visible representation of the vibrations of the tuning-fork.

CHAPTER V.

HEAT.

LESSON XX.—TEMPERATURE.

163. IT is proposed in the present chapter to discuss that form of molecular energy which we term heat. As, however, this word is used to denote two kinds of energy—one of which is capable of residing in a body, while the other kind traverses space with an enormous velocity—we shall at present confine our attention to the first of these, leaving the last, or radiant energy, to form the subject of another chapter. It will be convenient to consider, in the first place, the various effects of heat upon matter; secondly, the laws regulating the distribution of heat through space; and thirdly, the relation between heat and other kinds of energy.

164. Temperature.—In the first place, let us consider temperature. This word is used to denote the state of a body with respect to sensible heat. To illustrate the meaning of the word, let us suppose that two substances, such as a quantity of water and of mercury, each contain heat to such an extent that when they are brought intimately together there is no transference of heat from the one to the other, but each keeps what it has; then these two substances are said to be of the same temperature. But if, when the water and the mercury are shaken together, the water parts with some of its heat to the mercury, then the water is said to be of a higher temperature than the mercury; while, on the other hand, if it receives heat from the mercury, it is said to be of a lower temperature than the mercury.

165. Bodies in general expand through Heat.—Let us take a brass ball and a ring, such that at the ordinary temperature of the air the ball will pass through the ring. Now if we heat the ball intensely by a flame, owing to the expansion occasioned by heat we shall no longer be able to force it through the ring. Next let us take a bladder that is nearly but not quite filled with air, and place it beside the fire ; the air within the bladder will soon expand through the heat, so that the bladder will appear to be quite full of air.

There are, however, exceptions to the law of expansion. Thus water between the temperatures of 0° C. and 4° C. contracts instead of expanding through an increase of heat, while after 4° C. it begins to expand in the usual way, at first very slowly, but as the temperature rises, with increasing rapidity.

166. Measurement of Temperature by Thermometers.—As we can only perceive heat through its effects upon bodies, we must make use of some one of these as a means of measuring it. The expansion caused by heat is probably the effect most convenient for this purpose ; and if the same body always expanded to the same extent for equal increments of temperature, there would be no difficulty in measuring temperature exactly by this means ; but this is ar from being the case. Thus a gramme of water will occupy the same volume at 0° C. and at 8° C., so that in this case we cannot correctly estimate the temperature by means of the volume. In fact water near its freezing-point (0° C.) is undergoing very rapid molecular changes, and in general liquids near their freezing-points, or solids near their melting-points, are not well fitted to be used as the means of measuring temperature by their change of volume. On the other hand, a gas such as atmospheric air, which is incapable of being condensed into a liquid by the most extreme cold, is admirably adapted for the purpose of measuring temperature as far as accuracy is concerned. Nevertheless, an air ther mometer is a most inconvenient instrument for ordinary use. A mercurial thermometer is best adapted for general purposes, being very convenient and tolerably accurate, although when extreme accuracy is desired it ought to be corrected by means of an air thermometer.

167. Mercurial Thermometer.—This instrument is constructed on the principle that mercury expands more than glass. In order to make a mercurial thermometer, let us take a glass tube, having a narrow capillary bore with a bulb blown at one end of it, the other end being in the meantime open so that the bulb is filled with air. Let the bulb next be heated over a lamp, in consequence of which the air in the bulb will expand, and part will be driven out at the mouth of the tube. Next, before the bulb begins to cool, let the mouth of the tube be plunged beneath the surface of a vessel filled with pure mercury. During the process of cooling, the air left in the bulb will contract, and the pressure of the atmosphere will cause the mercury to rise in the tube until part of it gets into the bulb. Having by this means got some mercury into the bulb, the next operation is to boil the mercury in the bulb until not only the bulb but also the capillary tube is filled with the vapour of mercury. When this has been accomplished, let the end of the tube be once more plunged into the basin of mercury. As there is now no air in the tube or in the bulb, but only vapour of mercury, when this cools there will be a vacuum, and the mercury into which the instrument is plunged will be driven up by the atmospheric pressure until it fills the bulb. When the bulb and tube have by this process been filled with mercury, the tube is then hermetically sealed; and when the instrument has cooled, it will be found that the mercury will fill the bulb and part of the tube, the other part being left empty.

If we heat an instrument of this kind, the glass of the bulb will expand through heat, and likewise the mercury; but the mercury will expand more than the glass, and the consequence will be that the mercurial column will rise in the stem. In like manner, if the instrument be cooled, the mercury will contract more than the glass, and the column of mercury will fall. If the capillary bore be fine enough, a large rise of the column of mercury may be caused by a comparatively small elevation of temperature, and a thermometer may thus be made to indicate differences of temperature with very great delicacy.

168. Determination of Fixed Points.—Having thus constructed our instrument, the next operation is to mark off on the stem the heights of the mercurial column corresponding to the freezing and the boiling-points of water. To ascertain the freezing-point, the instrument is plunged into some *melting* ice, where it is allowed to remain for about a quarter of an hour. A mark is then scratched on the stem at the termination of the mercurial column. This point denotes *zero* of the centigrade scale.

The next thing is to determine the boiling-point of water, and here it must be borne in mind that this point is not constant like the freezing-point, but varies with the pressure of the atmosphere; indeed it is well known that water will boil at a much lower temperature in an exhausted receiver than in the open air.

Let us suppose, for the sake of simplicity, that this pressure at the moment when the experiment is made is exactly equal to that of a column of mercury 760 millimetres in length, and having the temperature of the freezing-point of water; in other words, let the barometric height be 760 millimetres. Let us now immerse the thermometer in steam arising from pure water boiling under this pressure, and mark off as before the termination of the mercurial column. The point so marked will denote 100° on the centigrade scale.

In marking off this point it is necessary that not only the bulb but also the stem of the thermometer up to the very point marked should be exposed to the steam, and in order to do this properly we make use of an instrument which is represented both externally and internally in Fig. 53. The thermometer tube is inserted into a thick piece of india-rubber, which is made to cover tightly the top of the instrument, and the stem is lowered until the mercurial column just appears above this cover when the water is boiling; all the stem is thus exposed to the action of the steam. The bulb of the thermometer is not plunged into the water, but remains suspended above it; and the steam is conveyed first up through an interior chamber, and then down again, until it finally leaves the exterior vessel through the aperture C. The whole of the thermometer is thus well surrounded by the steam, and by a cylinder which has the temperature of the steam.

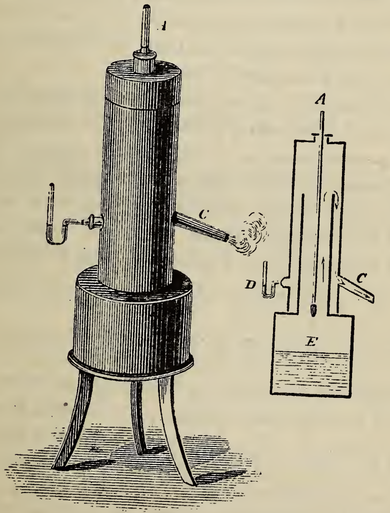


FIG 53.

169. Graduation.—Having thus ascertained and marked off the two fixed points, the next thing is to graduate the instrument. If the bore of the capillary tube be of equal size throughout, the divisions denoting degrees will all be of equal length; and if the centigrade scale be adopted, there will be just one hundred of these spaces between the freezing-point and the boiling-point marks. The glass stem of the instrument should therefore be etched accordingly, the freezing-point of water being *zero*, or 0°, and the boiling-point 100°. In general the graduations are extended from somewhat below the freezing-point to somewhat above the boiling-point, those below the freezing-point being reckoned negative, as, for instance, $-1°$ $-2°$, and so on.

We have here adopted the centigrade scale, which is that now chiefly used by men of science; but besides this there is the scale of Fahrenheit, which is very much used in this country, and finally there is that of Réaumur, which is extensively used throughout Germany.

In the Fahrenheit scale, the freezing-point of water is termed 32° and the boiling-point 212°. A centigrade degree is therefore greater than a Fahrenheit degree in the proportion of nine to five. In a Fahrenheit thermometer a temperature 32° below the freezing-point is termed zero, while one ten degrees lower is called $-10°$, and so on. To reduce centigrade to Fahrenheit the following formula is used: $F = \frac{9C}{5} + 32$, while to reduce Fahrenheit to centigrade we have $C = (F - 32)\frac{5}{9}$. The use of these formulæ will be seen from the following examples.

Example I.—Find the degree of Fahrenheit which corresponds to 45° Cent. *Answer.*—Here $\frac{9C}{5} = \frac{9 \times 45}{5} = 81$, and hence $F = 81 + 32 = 113°$.

Example II.—Find the degree centigrade which corresponds to 86° Fahr. *Answer.*—Subtracting 32° from 86°, we find that the temperature is 54° Fahr. above the freezing-point. Hence $C = 54 \times \frac{5}{9} = 30$.

Example III.—What degree Fahr. corresponds to $-40°$ C.? *Answer.*—The temperature is $40 \times \frac{9}{5} = 72°$ F. below the freezing-point, or 40° F. below the zero of Fahrenheit. Hence $-40°$ C. $= -40°$ F.

In the scale of Réaumur, the freezing-point is reckoned *zero*, and the distance between it and the boiling-point is divided into 80 parts.

To reduce centigrade to Réaumur, we have therefore the following expression : $R = \frac{4C}{5}$; while to reduce Réaumur to centigrade we have $C = \frac{5R}{4}$.

170. Corrections to a Mercurial Thermometer.—Even when a mercurial thermometer has been constructed in the best manner possible, being correctly pointed off, and having the volume of the bore between the two points accurately divided into equal parts, there are nevertheless certain corrections which must be applied in order to get the true temperature.

The first of these is for the *change of zero*. Let us suppose that immediately after graduation 0° C. exactly denotes the temperature of melting ice, and that in the course of half a year we plunge the instrument once more into melting ice ; its temperature will not now be denoted by 0° but perhaps by $+ 0^\circ\!\cdot\!3$. This is expressed by saying that zero has risen three-tenths of a degree, and all thermometers are subject to a rise of this kind, more especially if they have been recently made. The cause is probably to be sought for in the sudden cooling of the bulb when it is blown and filled. The glass is, in fact, comparatively unannealed, and its particles are in a state of constraint, the tendency of the bulb being gradually to contract in size, but with greater rapidity at first. On account of this the mercury is pushed up the tube, and in consequence the reading of the instrument corresponding to a fixed temperature, will get higher and higher. To correct for this source of error, the thermometer ought to be plunged occasionally into melting ice, and its reading noted. The amount of alteration thus becomes known ; for instance, if we find that the zero has risen two-tenths of a degree, this amount has simply to be deducted from the readings at all temperatures in order to obtain a correct result.

Besides this permanent change there is also a *temporary change produced by suddenly heating and cooling the instrument.* Thus, if a thermometer be plunged into boiling water and then suddenly withdrawn, its zero will be found to have fallen, and its reading in melting ice may now be — 0°·1. In the course of three weeks or so the instrument will probably have overcome this temporary change of zero, and the freezing-point will again have risen to what it was before.

On account of this effect produced by heating the instrument, the freezing-point of a thermometer should always be first marked off, and the boiling-point afterwards ; for if the freezing-point be determined immediately after the instrument has been in boiling water, the determination will be unquestionably erroneous.

In the next place, there is a *correction on account of the position* in which the instrument is held. If the fixed points have been determined in a vertical position of the instrument, then it must always be used in a vertical position ; if these have been determined in a horizontal position, then the instrument must be read horizontally.

The reason of this is, that at the same temperature a thermometer, especially if the column of mercury be long, will give a lower reading in a vertical than in a horizontal position, since the hydrostatic pressure of the column of mercury will not only tend to compress the mercury, but also to enlarge the bulb. In like manner a thermometer will read lower in an exhausted receiver than in the open air, for the *effect of the air pressure upon the bulb* has a tendency to squeeze it together, and to force the mercury up the stem, so that when this is withdrawn the column will fall.

Finally, if the bulb of a thermometer be plunged into a medium of high temperature, such as boiling water, while the stem remains exposed to the air, which is of a much lower temperature, the reading of the instrument will not denote the true temperature of the water. It would have done so had the whole of the stem, as well as the bulb, been exposed to the water, but this is not the case. If the stem from 0° to 100° be exposed to air at the temperature of the freezing-point, while the remainder is immersed, along with the bulb, in boiling water, the temperature denoted will only be 98°·4 instead of 100°. The exact reading, will, however, depend to some extent upon the nature of the glass of which the instrument is made.

When these corrections have been applied, a mercurial thermometer, well graduated, may be considered to be a tolerably good, though not a strictly accurate, instrument.

171. Other Thermometers.—A thermometer filled with alcohol instead of mercury is sometimes used for very low temperatures, since mercury freezes much sooner than alcohol. Such a thermometer is also often used for a self-registering **minimum** thermometer, in order to give the lowest temperature which has been reached. For this purpose there is a small glass index immersed in the column of alcohol.

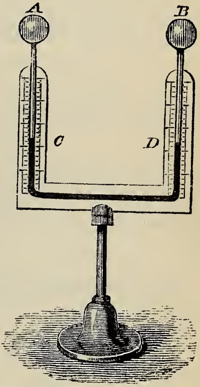


Fig. 51.

First of all, in setting the instrument this index is brought to the extremity of the column, and the instrument is then placed in a horizontal position. Should the temperature rise, the alcohol will expand and flow past the index; but should the temperature fall, the alcohol will contract and carry the index with it rather than admit of the concave capillary surface of the column being broken. The lowest temperature reached is thus registered.

In Professor Phillips' **maximum** thermometer, part of the column is separated from the main body of the mercury by a little air. The instrument when in use is laid in a horizontal position. When the mercury expands the pressure of the air pushes this broken column on before it, but the column does not recede when the mercury again contracts. The highest temperature reached is thus registered.

Leslie has constructed an instrument called the **Differential Thermometer**, for showing the difference in temperature between two neighbouring places. This instrument is represented in Fig. 54. In it two bulbs, A and B, filled with air, are connected together by a bent tube, the lower part of which is filled with some coloured liquid.

This liquid will have both its extremities at the same level if A and B are of the same temperature, but if A is hotter than B its air-pressure will therefore be greater than that of B, and the liquid will be driven up the stem of B by the expansion of the air in A; the motion will be reversed if B be hotter than A. Such an instrument may be made to indicate differences of temperature with very great delicacy.

Lesson XXI.—Expansion of Solids and Liquids through Heat.

It has been already stated (Art. 165) that the effect of heat upon bodies is in general to produce expansion, and we shall now proceed to a separate study of this effect in the three states of matter—the solid, the liquid, and the gaseous.

172. Expansion of Solids. Linear Expansion.—In the first place, let us consider the change in length of a solid rod or bar through heat. Several methods of estimating this have been proposed. The plan which pervades all these methods is to fix the bar at one end, and, having heated it, to note the alteration in the position of the other end. This alteration may be magnified by optical means, such as a microscope, or by mechanical means, as, for instance, by an

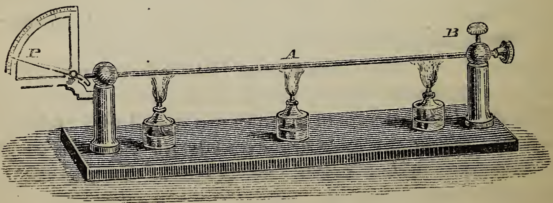


FIG. 55.

arrangement of levers. The preceding figure will illustrate how a comparatively small expansion may be rendered visible by mechanical means.

A rod A is fixed at one end by a screw B, while the other is pressed against by the small arm of a lever, the other arm of which (P) forms a pointer. This pointer moves along a graduated scale, and any increase in the length of the rod will push the short arm of the lever to the left, and hence the pointer will travel to the right. It is evident that by having the pointer sufficiently long a very small expansion may thus be rendered visible. Before proceeding further it may be well to define what is meant by the "**coefficient of expansion.**"

Suppose, for instance, that we have a brass rod, the length of which is unity at 0° C.; then at 1° C. its length will be found to be 1·000018 ; hence ·000018 is the linear coefficient of the expansion of brass for 1° centigrade.

173. In the following table we have the linear expansions of some of the most important solids, the range of temperature being that between the freezing and the boiling-point of water :—

		Length at 100° of a rod whose length at 0° = 1·000000.
Glass (mean result)		1·000853
Copper	„	1·001716
Brass	„	1·001880
Soft iron	„	1·001198
Cast iron	„	1.001090
Steel	„	1·001136
Lead	„	1·002818
Tin	„	1·001959
Silver	„	1·001923
Gold	„	1·001441
Platinum	„	1·000870
Zinc	„	1·002976

It ought, however, to be borne in mind that in such determinations as these we are not always sure of the chemical purity of the bar upon which our experiments are made.

Again, even in those substances which have the same chemical composition, the molecular structure may be very different, in consequence of the treatment they have undergone. Thus, for example, we find that the expansion of the various kinds of glass ranges from ·000776 to ·000918, probably on account of their difference in chemical composition ; and that steel tempered yellow has for its expansion ·001240, while untempered steel has only ·001080, the difference here being probably due to the treatment which the steel has undergone in the operation of tempering.

174. Cubical Expansion.—Hitherto our object has been to find how much a solid in the shape of a bar or rod has increased in length through the application of heat ; but let us now suppose that we wish to estimate its increase of volume. Suppose, for instance, that we have a solid cube, the side of which is *unity*, and that through the effect of heat this side has become = 1·00001, the volume of the solid will now be $(1·00001)^3$ = 1·00003 nearly. Thus while the linear expansion is ·00001, the cubical is ·00003, or three times as great as the linear.

There are different methods of determining experimentally the cubical expansion of solids. One of these is to weigh the solid at different temperatures in a liquid of which the specific gravity at these temperatures is accurately known.

As an example of this method, let us suppose that a solid weighs 600 grammes in vacuo, and only 400 grammes in a fluid at 0° C., of which the specific gravity is 1·2, while it weighs 406 grammes in the same fluid at 100° C., for which temperature the specific gravity of the fluid is known to be 1·16 : find what is the cubical expansion of the solid between these two temperatures ?

Now, since the loss of weight of a solid in any fluid is equal to the weight of the bulk of that fluid which it displaces, 600 — 400 = 200 grammes must be the weight of the fluid whose specific gravity is 1·2, which is displaced by the solid at 0° C. But according to the metrical system, had the specific gravity of the fluid been 1·0, the 200 grammes of weight would have exactly corresponded to a volume equal to 200 cubic centimetres; as, however, the specific gravity is 1·2, the volume will be less in this proportion, and hence the volume displaced at 0° C. will be $\frac{200}{1·2} = 166·6$.

In like manner, at 100° C. the loss of weight being 194 grammes and the specific gravity 1·16, we have the volume displaced $= \frac{194}{1·16} = 167·2$. We thus perceive that a quantity of the solid, of which the volume was 166·6 at 0° C., has a volume equal to 167·2 at 100°, showing a cubical expansion $= \frac{167·2}{166·6} - 1 = ·0036$ between these two points.

175. We have already (Art. 174) adduced a reason why the cubical expansion of a solid should be three times as great as its linear expansion. Let us now see whether our conclusion is correct as a matter of fact. In the following table a comparison is made between the linear and the cubical expansion of the same substances, the two sets of expansions being independently determined, and it will easily be seen that in all cases the cubical is as nearly as possible equal to three times the linear expansion.

COMPARISON OF LINEAR AND CUBICAL EXPANSION.

Substance.	Mean Linear Expansion between 0° and 100°.	Mean Cubical Expansion between 0° and 100°.
Glass	·000853	·002540
Copper	·001716	·005127
Lead	·002818	·008900
Tin	·001959	·006900
Zinc	·002976	·008900
Iron	·001204	·003546

176. Remarks on the Expansion of Solids.—(1) The connection which the table given above exhibits between cubical and linear expansion presupposes that a solid expands equally in in all directions, so as always to preserve a similarity of figure. This is not, however, in general the case with crystals, which expand unequally in different directions; we cannot, therefore, deduce for these bodies the linear from the cubical, or the cubical from the linear expansion.

(2) In the second place, although solids in general expand through heat, yet there are exceptions to the rule, one of the most notable being Rose's fusible metal, which after a certain point contracts instead of expanding if the temperature be increased.

(3) In the next place, solids in general expand more rapidly at high than at low temperatures. Thus a certain glass, of which the cubical expansion between 0° and 100° is at the rate of ·0000258 for 1° C., will, between 0° and 300° C., expand at the rate of ·0000304.

177. Expansion of Liquids.—As a liquid must always be contained in a solid vessel, the expansion of liquids may be said to be of two kinds—either **apparent** or **real.** The apparent expansion of a liquid through heat means the apparent increase of volume of a liquid, contained in a vessel that expands through heat to a smaller extent than the liquid which it contains. Again, by real expansion we mean the true expansion of the liquid, without reference to the containing vessel. It is real, not apparent, expansion which we shall now consider. There are various methods of finding the true expansion of a liquid. One of these is the method by thermometers, in which the liquid is used to fill the bulb of a thermometer, of which the internal volume or capacity is supposed to be known for the various temperatures used in the experiment.

When the thermometer has been filled with the liquid whose expansion we wish to determine, it is exposed to various temperatures, and for each of these temperatures the height of the column of liquid in the stem of the thermometer is accurately noted. Thus, knowing the capacity of the thermometer at the various temperatures, we come to know the volume occupied by the liquid at these temperatures, and from this the amount of its expansion may be easily deduced.

Another method may be mentioned, which has recently been used by Matthiessen with great success.

A long rod of glass has its linear expansion determined at various temperatures with great exactness, and a short piece of this rod is broken off, the cubical expansion of which is reckoned to be three times its linear expansion. Now knowing the cubical expansion of this piece of glass, we have the means of knowing its volume at various temperatures. The piece of glass is next weighed at various temperatures in the liquid which we wish to examine.

To illustrate this method of observation, suppose that we have determined that one piece of glass has a linear expansion equal to ·00090 between 0° and 100° C.: hence its cubical expansion between these limits will be ·00270, and hence a piece of glass whose volume at 0° C. is unity will have at 100° C. a volume equal to 1·0027. Now let us suppose that at 0° C. this piece of glass loses one gramme of its weight in the fluid in which it is weighed, while at 100° C. it only loses 0·96 of a gramme. We thus learn that at 0° C. a volume equal to unity of the fluid in question weighs one gramme, while at 100° C. a volume equal to 1·0027 of the same fluid weighs 0·96 of a gramme. It would, therefore, require a volume of the fluid at 100° C. $= 1{\cdot}0027 \times \frac{100}{96}$ to weigh a gramme, or, in other words, a volume equal to 1·0444 of the fluid at 100° C. will weigh the same and contain as many particles as a volume equal to unity at 0° C. The expansion of the fluid between 0° and 100° C. will therefore be ·0444.

178. In estimating the expansion of mercury Regnault has adopted a different method from either of these which we have now described. The principle of his method consisted in filling a U-shaped tube with mercury, one limb being kept at a low and the other at a high temperature. The mercury in the heated limb was, of course, specifically lighter than that in the other; and hence, since the two columns were connected, and therefore balanced one another hydrostatically, the hot column necessarily read higher than the cold one. In fact, in such a case the heights would vary inversely as the densities, so that by knowing the heights the densities might be determined. Now when we know the density of mercury at two temperatures, we have the means of deducing its expansion between these two temperatures.

By this means Regnault has obtained the following result:—

True temperature as determined by an air thermometer (*t*).	Whole expansion from 0° to t° of a volume of mercury equal to unity at 0°.
0	·000000
10	·001792
20	·003590
30	·005393
40	·007201
50	·009013
100	·018153
150	·027419
200	·036811
250	·046329
300	·055973
	·065743

It will be seen from this table that the rate of expansion of mercury varies with the temperature. Thus for the fifty degrees between 0° and 50° the expansion is ·009013, while between 300° and 350° it is ·009770.

179. Expansion of Water.—We shall now allude to a peculiarity which water exhibits with respect to its expansion. This liquid, it is well known, freezes at 0° C.; but if heat be applied to water at its freezing-point, the water does not expand, as might be imagined, but contracts until the temperature 4° C. is reached, and from this temperature it continues to expand.

Water thus exhibits a point of maximum density at 4°, and the fact may be easily shown by means of the following apparatus devised by Hope.

It consists of a glass vessel filled with water at the ordinary temperature (Fig. 56), and having in its sides holes through which two thermometers are inserted—one near the top and the other near the bottom of the instrument. The middle of the instrument is surrounded by an outer receptacle, into which a freezing mixture is put. As the temperature begins to fall from the effects of the mixture, the lower thermometer is at first much affected, and the upper one hardly at all. The reason of this is, that the water, being cooled by the freezing mixture, grows specifically heavier, or contracts, and therefore descends, and is replaced by lighter and warmer water from below. The lower thermometer is, therefore chiefly affected, and this will continue until the water attains the temperature of 4°, at which point the lower thermometer will cease to fall. After this point, the water being still cooled, but growing now specifically lighter, since 4° is its point of maximum density, will ascend, and the upper thermometer will begin to fall rapidly, and continue to do so until it reaches the freezing-point.

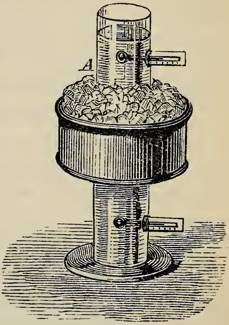


FIG. 56.

180. The following table exhibits the volume of water at various temperatures between 0° and 100°, the volume at 4° being taken as unity :—

Temperature.	Volume
0° C.	1·00013
4	1·00000
10	1·00027
20	1·00179
30	1·00433
40	1·00773
50	1·01205
60	1·01698
70	1·02255
80	1·02885
90	1·03566
100	1·04315

181. It will be noticed from this table that the rate of expansion of water for the same increment of temperature is greater as we approach the boiling-point. M. Pierre has made many experiments on the rate of expansion of various liquids at 0° C., and he finds that those which have high boiling-points expand less at this temperature than those which boil at a low temperature.

This prepares us to believe that the expansion of very volatile liquids must be very great, especially of such liquids as carbonic acid, which can only be retained in the fluid state at ordinary temperatures under very great pressure.

Now this is found to be the case. Thilorier has remarked that liquid carbonic acid expands more rapidly than any gas; and Drion has proved that sulphurous acid at 130° C. expands about one-hundredth of its volume for 1° C., or about ten times as much as water.

182. We have thus the following laws for the expansion of liquids :—

(1) *Liquids expand more than solids for the same increment of temperature.*

(2) *Liquids expand more rapidly at a high than at a low temperature.*

(3) *Those liquids expand most rapidly of all which can only be kept in the liquid state through the application of intense pressure.*

LESSON XXII.—EXPANSION OF GASES. PRACTICAL APPLICATIONS.

183. It has been stated (in Art. 90) that the pressure of a gas is proportional to its density, provided there is no alteration of temperature; let us now discuss the influence of temperature upon the pressure of a gas. The true connection between temperature and pressure was first discovered by Charles. It may be stated as follows :—Suppose that we have a quantity of gas which is confined in a vessel of invariable volume, of which the temperature is allowed to vary.

Now let P denote the pressure of the gas against a square unit of surface of this vessel at 0° C.; then at t° C. the pressure will be P $(1 + a\,t)$, when a = ·003665 nearly. For instance, let P denote 760 millimetres of the barometric column, and let the temperature rise from 0° to 20°; then the pressure will become 760 $(1 + 20 \times {·003665}) = 815{·}708$ millimetres nearly. Or, again, let the pressure at 0° be unity, and let the temperature rise from 0° to 100°; then the pressure will rise to 1·3665.

184. We have here stated the relation between the temperature and the pressure of a gas, of which the *volume* is kept constant; but we can easily convert the expression into another, which will give us the relation between the temperature and the volume if the *pressure* be kept constant. Thus, for instance, if a bladder half filled with gas be warmed at the fire the volume will increase, while the pressure upon it will remain constant; this being that of the atmosphere which presses upon the outside of the bladder.

Now the increase in the volume of the bladder for 1° C. may be found as follows: Let P denote the pressure at 0° when the volume is V; then if this volume be kept constant we have seen that the pressure at t° will be P $(1 + {·003665}\,t)$. But we have seen that by Boyle's law the pressure of a gas varies inversely as its volume; if, therefore, when the gas has attained the temperature t° and the pressure P $(1 + {·003665}\,t)$, we allow its volume to increase from V to V $(1 + {·003665}\,t)$, we shall reduce its pressure in this same proportion, which will therefore now become P,—that is to say, its pressure will be the same as it was before the heating commenced. Therefore, if the gas be heated under a constant pressure, and if V denote its volume at 0°, its volume at t° will be V $(1+{·003665}t)$, the same multiplier serving to denote the increase of pressure if the volume be constant, and the increase of volume if the pressure be constant.

The following example will illustrate the variation of volume with temperature.

Example: A bladder, which at 0° contains 900 cubic centimetres of air, has its temperature increased to 30° C., the pressure under which the gas exists meanwhile remaining constant: what will now be the volume of the gas in the bladder?

Answer: Its volume will be 900 (1 + 30 × ·003665) = 998·955 cubic centimetres.

185. It is an important fact that the co-efficient ot expansion, or ·003665, is as nearly as possible the same for all gases, so that if at 0° C. we have unit volumes of atmospheric air, hydrogen, oxygen, carbonic acid, and other gases, and if the temperature change so as to become successively 20°, 30°, 50°, 100° C., the volumes of these various gases will continue equal to one another at each of these temperatures. Thus at 50° they will all have the volume 1·18325, while at 100° their volume will be 1·3665.

Or again, if we have at 0° equal volumes of various gases all under unity of pressure, and if the temperature be increased while the volume remains unaltered, the pressures of the various gases will continue equal to one another at each of these temperatures. Thus at 50° C. they will all have the pressure 1·18325, while at 100° their pressure will be 1·3665. Now if we make use of a bulb filled with gas as a thermometer in order to estimate the temperature, either by the increase of volume of the gas under a given pressure, or by the increase of pressure of the gas under a given volume, we have this great advantage over a liquid, that we are independent of the kind of gas with which we fill our bulb, for we have just seen that all permanent gases will give as nearly as possible the same indications. On the other hand, if we have two thermometers filled with two different liquids, and mark them off at 0° and at 100°, the chances are they will not read the same midway between these two points. The advantage of using an **air thermometer** is thus apparent

186. Applications of the Laws of Expansion.—Since all the substances around us are continually changing their temperature, they are, in consequence, subject to a continual change of volume, and this change must be taken account of in all delicate operations. Let us, in the first place, describe how the standards of length, mass, density, and time are affected on account of this change.

187. Standards of Length.—The metre represents approximately the 10,000,000th part of a quadrantal arc of a meridian on the earth's surface. The standard platinum metre of France represents a metre at 0° C.; it will therefore be longer than a metre at any temperature higher than 0°.

The English bronze standard of length, on the other hand, represents a yard at 62° F., so that at a temperature below 62° it will be less than a yard, while at a higher temperature it will be longer. Therefore, when we say that a metre is equal to 39·37079 English inches, we mean that a metre at 0° C. will be equal in length to 39·37079 inches in an English yard at 62° F.; but if the two standards—the French and the English—are compared together at any common temperature, the proportion between the two will be different from the above. The relation between the French and English standards of length is given in Art. 8.

188. Standards of Mass.—Standard weights are in reality standards of mass, since (Art. 34) the weight of a body at the same point of the earth's surface is proportional to its mass.

In France the standard weight, or the gramme, is defined to be the weight of a cubic centimetre of distilled water at the temperature of its greatest density (supposed equal to 4° C.).

The English pound avoirdupois is an arbitrary standard containing 7,000 grs. The relation between the French and English system of weights has already been given (Art. 11).

If we could make all our weighings in vacuo, temperature would not affect our determinations; but since these must necessarily be made in air, and since the density of air depends, among other things, on its temperature, and since the weight of a body when weighed in air is rendered lighter than in vacuo by the weight of air which it displaces, it is necessary in all very accurate weighings to know the temperature of the air.

189. Standards of Density.—The French method of estimating comparative density or specific gravity is now generally adopted by men of science.

In it the specific gravity of solid and liquid bodies is referred to that of water at 4° C., of which the density is reckoned equal to unity. Now we have seen (Art. 11) that a cubic centimetre of water at this temperature weighs exactly one gramme. Hence we know that a cubic centimetre of a substance whose specific gravity, referred to the water unit, is 2, will weigh exactly 2 grammes; and, in fact, that the weight of one cubic centimetre of any substance will denote at the same time its specific gravity. But since substances in general expand through heat, their densities will thus be less at high than at low temperatures; and, also, since they expand unequally, the proportion between the densities of two substances will be different for different temperatures; so that in estimating the comparative density or specific gravity of a substance we must fix upon some standard temperature at which this estimation may be made. 0° C. has been chosen as the temperature for such comparisons; so that when we say that such a substance has the specific gravity of 2·1, we mean that a cubic centimetre of the substance at 0° C. will weigh 2·1 grammes. But in order to know the specific gravity of the substance at any other temperature it will be necessary to know its co-efficient of expansion. Gases, again, are compared together at the temperature 0° C. under the barometric pressure of 760 millimetres of mercury (reduced to 0° C.), the standard here being dry air, of which the density, under these circumstances of temperature and pressure, is reckoned equal to unity.

190. Measurers of Time.—It has already been stated that a long pendulum vibrates slower than a short one; and since a rise of temperature increases the length of a clock pendulum, it will therefore, if uncompensated, increase its time of vibration, and thus appear to make the clock go slow. In like manner an increase of temperature tends to make the balance-wheel of a chronometer oscillate more slowly in hot weather than in cold.

The best method of compensating for the effect of heat upon pendulums is probably that invented by Mr. Harrison. This arrangement, from its form, is called the gridiron pendulum. It is exhibited in the annexed figure, in which the dark lines represent iron and the lighter lines brass or zinc.

Here it is evident from the figure that the iron rods, being attached to the upper cross-pieces, will tend to lower the bob by their expansion, while the brass or zinc rods, being attached to the lower cross-pieces, will tend to raise it.

Now the proportion between the length of the iron and of the brass or zinc rods may be so arranged that the upward and downward expansion shall be precisely equal, in which case the position of the bob will remain unaltered, even although there be a considerable change of temperature.

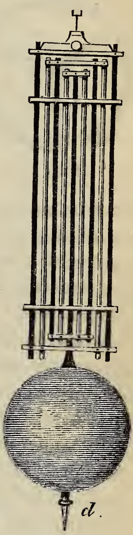


FIG. 57.

191. The compensation balance for chronometers depends upon the circumstance that, if a ribbon or bar be made of two metals of different expansive power firmly attached to each other, this ribbon will, when the temperature rises, bend, so that the most expansible metal shall form the outer or convex surface, and the least expansible the concave or inner surface. On the other hand, should the temperature fall, the most expansible metal will form the inner or concave surface. Now the rim of the balance-wheel of a chronometer is formed (as in Fig. 58) of several separate pieces, which are fixed at one end and free at the other, the free ends being loaded, and each piece is composed of two metals of unequal expansibility, firmly attached to one another, the most expansible being without. Hence, when the temperature increases, the loaded ends will approach the centre, and when it diminishes they will be thrown out from the centre.

But when the temperature increases the radius of the wheel increases also, and the matter of the wheel is from this cause thrown *out from* the centre. Now we have seen that the effect of the temperature compensation is under these circumstances to throw the matter *towards* the centre, and hence the one effect may be made to counteract the other, so as to preserve unaltered the time of oscillation of the balance-wheel.

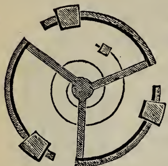


FIG. 53.

192. There are many other instances besides these now enumerated, in which account must be taken of the expansion of bodies. Thus, in estimating the pressure of the atmosphere by means of the mercurial column of the barometer, it is necessary to know the density of the column, and hence we must know its temperature.

Again, in large structures, such as iron and tubular bridges, allowance must be made, so that the materials may have freedom to expand. There is an arrangement for this purpose in the Menai tubular bridge.

In some instances advantage is taken of the fact of expansion. Thus, for instance, in making a wheel the tire is made to fit loosely in a red-hot state, and when it has cooled it grasps the wheel firmly and becomes quite tight.

LESSON XXIII.—CHANGE OF STATE AND OTHER EFFECTS OF HEAT.

193. We have already (Art. 4) spoken of the three conditions of matter—the solid, the liquid, and the gaseous—and stated that very many bodies can be produced in all these three states. Some bodies, however, such as oxygen, hydrogen, and nitrogen, can only be condensed into a liquid form with very great difficulty.

Now, it is an invariable rule that, whenever a solid is changed into a liquid, it is through an increase and not through a diminution of temperature, and in like manner when a liquid becomes a gas it is through an increase, not through a diminution, of temperature; so that when, for instance, we condense hydrogen into a liquid, it is done by cooling, and not by heating the hydrogen.

Let us now, in the first place, consider the passage from the solid to the liquid state; and, secondly, the passage into the gaseous state, or vaporization.

194. Liquefaction.—The passage from the solid to the liquid state may either be gradual or abrupt. Treacle, honey, and sealing-wax are bodies that pass gradually from the one state into the other, so that for a considerable range of temperature these bodies are neither solid nor liquid, but rather viscous or plastic. Ice, on the other hand, is a body that passes very rapidly into water, so that during a rise of temperature, probably not greater than 0·1° C., an unmistakable change of state has been produced; nevertheless, even in this case, we have reason to believe that the change is not absolutely abrupt.

But there is another peculiarity besides greater or less abruptness, which sometimes accompanies this change from the solid to the liquid state: for a large class of substances change their composition in the act of changing their state. Saline solutions are a notable instance of this, and in many of these a larger quantity of salt is retained in solution at a high than at a low temperature, so that, when left to cool, crystals of the salt are deposited. This, therefore, is a case where change of composition accompanies change of state.

A somewhat different change takes place in weak saline solutions, such as sea-water, in which, when the temperature is gradually reduced, the water solidifies as nearly pure ice, separating itself from the salt in the course of congelation.

195. We have stated that the melting-point of ice is not, like the boiling-point of water, dependent upon the pressure; but this, though approximately, is not absolutely correct, for the melting-point of ice is very slightly lowered by an increase of pressure, and the same phenomenon occurs in all cases in which a substance expands in the act of congelation. On the other hand, if a substance contracts when freezing, its melting-point is raised by pressure.

196. Melting-points. — The following table gives the melting-points of some of the most useful substances :—

Substance.	Melting-point (centigrade.)
Mercury	—39°
Ice	0
Phosphorus	44
Spermaceti	49
Stearine	55
Potassium	58
Sodium	90
Sulphur	111
Tin	235
Bismuth	260
Lead	325
Zinc	362
Silver	1000
Gold	1250
Iron	1500

197. Solidification.—Although we cannot retain ice in the solid state above its melting-point, yet we can retain water in the liquid state below its freezing-point, for when allowed to cool in a clean vessel, and at rest, water may remain liquid until about —10° C. In like manner, water in a capillary tube may be kept liquid as low as —20° C. But if water be below the freezing-point, and if a bit of ordinary ice, or anything capable of acting as a nucleus, be dropped into it, solidification commences, and the temperature of the water rises to 0° C.

Solutions exhibit a similar anomaly, for a hot saturated solution of Glauber's salt, when cooled slowly and at rest, will frequently retain all its salt in solution, even at a very low temperature. But when a bit of anything capable of acting as a *nucleus* is thrown in, crystallization immediately commences.

198. Regelation.—A curious property of melting ice was observed by Faraday. If two pieces of such ice, having smooth surfaces, be kept together, they will soon adhere, and will do so in air, or in water, or in vacuo. According to Forbes, this behaviour is due to the fact that the interior of a block of melting ice is in reality a little colder than its surface, just as the interior of a large block of sealing-wax in the act of melting away in a vessel of liquid wax may be supposed to be colder than its surface.

In fact, the temperature of true hard ice is supposed to be somewhat lower than that of ice-cold water, so that in passing from the one to the other there is an intermediate state of viscosity, extending however, in the case of water, through a very small range of temperature.

Now when the smooth surfaces of two pieces of melting ice are put together there is a film of water between them, and we can easily see why this water should freeze, because it is surrounded on this hypothesis by a colder substance on both sides. A new distribution of heat will therefore take place, and the water will become frozen, forming the centre of the block.

199. Let us now discuss the change into the gaseous state. This may be of two kinds :—

(1) Vaporization, or the conversion of a liquid into a gas ;

(2) Sublimation, or the conversion of a solid into a gas.

200. Vaporization.—This sometimes takes place quietly, and without the formation of bubbles, and sometimes in a violent manner, and with the formation of bubbles in the body of the liquid. In the former case it is termed **evaporation,** and in the latter **ebullition.**

Evaporation from the surface of a liquid takes place much more rapidly in vacuo than in air. Suppose, for instance, that we have a volatile liquid under the exhausted receiver of an air-pump, it will evaporate much more rapidly than if it were in the open air. After a time, however, we shall find that the evaporation begun so rapidly will go on less quickly, until at last it will come to a standstill, and no more vapour will rise from the liquid. Dalton was the first to show that the vapour will continue to rise until a definite vapour pressure has been produced in the receiver, and will then cease rising; and further, that this vapour pressure is regulated, not only by the nature of the volatile liquid, but also by the temperature.

Thus, if the liquid be water and the temperature 25° C., vapour will cease to rise when the pressure in the receiver has become 23·550 millimetres; but if the temperature be 35° C., then vapour will rise until the pressure becomes 41·827 millimetres.

On the other hand, if the liquid be ether, we shall at 25° C. have a vapour pressure equal to 525·93 millimetres, while at 35° C. the pressure of its vapour will be 761·20 millimetres.

We have said that if air be present the evaporation will go on much less quickly; nevertheless, the presence of air will not alter the final result. Thus, for instance, if a vessel of water be allowed to evaporate in a receiver full of air, and if the temperature be 25° C., the vapour will, as before, cease to rise when the vapour pressure has become 23·550 millimetres. In fact, at the end of the process there will be the same amount of vapour present in the receiver full of air as there was in the empty receiver, only the result will be much more quickly reached in the latter than in the former case.

If a vessel filled with water be suffered to evaporate in the open air, the process will go on much more rapidly if the air be dry, and also if there be wind or any agitation tending to renew the particles of air above the surface of the water. The reason of this is very obvious; for when evaporation takes place in still air, the air in contact with the surface of water soon becomes nearly saturated with watery vapour, and hence the evaporation proceeds very slowly. In fact, a constant supply of new and dry particles of air is necessary if the process of evaporation is to go on with rapidity.

201. Distillation.—Suppose now that we have two vessels of different temperatures in communication with each other. Let Fig. 59 represent an apparatus of this kind, which is that made use of in distillation.

Heat is applied to the liquid in the vessel at A, and the vapour issuing from it is made to pass through a coil of pipes surrounded by cold water at C. Thus vapour is no sooner formed at A than it rushes along the tube, and is immediately condensed and its place supplied by new vapour; and the rapidity of the process will only be limited by the supply of heat at A and the supply of cold water at C, for if there be a small supply of heat at A there will be a small formation of vapour, and if there be a deficient supply of cold water at C the vapour will not rapidly condense. In fact, the vapour in this arrangement acts as a vehicle by which the heat is drained away from A to C. D is the pipe by which a constant supply of cold water enters the condensing vessel, and at E the heated water is carried off, which being lighter naturally rises to the top. Thus the outside of the coil, or **worm** as it is called, is kept constantly surrounded by cold water.

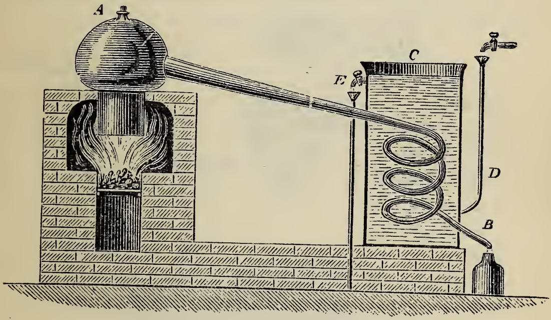


FIG. 59.

The object of distillation is generally to free the liquid from certain impurities with which it is associated. These are left behind in A, and the condensed vapour of the liquid is caught in a vessel at B.

202. Ebullition.—When a liquid, such as water, is heated in the open air, it continues for some time to increase in temperature, and the evaporation becomes more and more rapid. At length bubbles of vapour break out and reach the surface, and the process of boiling or ebullition has begun. When this takes place the temperature ceases to rise, and remains stationary until all the water has boiled away, the only difference being that if the supply of heat be very great the process is very rapid, and if the supply of heat be small the process is very slow. The point at which ebullition commences is called the **boiling-point.**

203. This point, in the first instance, depends upon the *nature of the liquid.* The following table contains the boiling-points under the ordinary pressure of the atmosphere of some of the most important liquids :—

Substance.	Boiling-point (centigrade).
Ether	34·9
Bisulphide of carbon	48·0
Bromine	63·0
Wood spirit	65·5
Alcohol	78·4
Benzole	80·4
Water	100·0
Formic acid	105·3
Acetic acid	117·3
Fousel oil	132·1
Butyric acid	157·0
Sulphurous ether	160·0
Sulphuric acid	337·8
Mercury	350·0

Kopp appears to have traced a definite relation between the chemical composition of certain substances and the temperatures at which they boil.

204. In the next place, the boiling-point of the same liquid depends upon the *pressure*, a liquid boiling at a lower temperature if the pressure be smaller.

This may be easily shown by means of a few simple experiments. First, exhaust a vessel containing ether under the receiver of an air-pump, and it will be found to boil at the ordinary temperature of the air. Secondly, fill a Florence flask (Fig. 60) half full of water, and boil it in the open air until the upper part of the flask be filled with the vapour of water ; let it now be corked tightly and inverted. When it has ceased to boil, pour some cold water upon the flask, and ebullition will again commence. The reason is, that the cold water, by condensing the vapour which fills the upper part of the flask, withdraws the pressure, and thus enables the water to boil at a lower temperature.

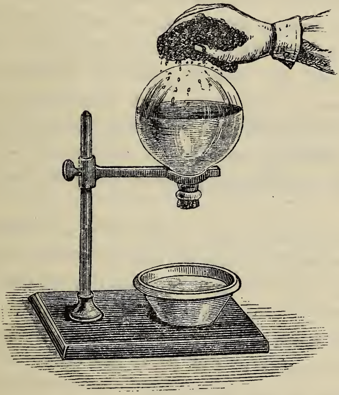


FIG. 60.

It follows from all this that, inasmuch as the atmospheric pressure at the top of a mountain is smaller than at the bottom, so the boiling point of water at the top of a mountain is lower than at its bottom.

Thus at the top of Mont Blanc water boils at 85° C. instead of 100°, a temperature which is too low for culinary purposes. Food cannot, therefore, at such altitudes be cooked in open vessels of water, but people are there compelled to heat water in a close vessel under the pressure of its own vapour, in order to obtain a temperature sufficiently high for cooking purposes.

It has been previously stated (Art. 89) that the height of a mountain can be estimated by means of the barometer, and we may now add that it can also be ascertained by observing the boiling-point of water by means of a thermometer. For, knowing as we do the relation between the boiling-point of water and the atmospheric pressure (Art. 212), an observation of the former enables us to obtain the latter. In fact, we read the boiling-point of water in order to deduce from it the atmospheric pressure, and there is therefore no advantage in using a boiling-point thermometer, except that it is a more portable instrument than an ordinary barometer.

205. In the next place, the *nature of the vessel* affects the boiling-point of the fluid which it contains. Thus it has been found that the boiling-point of water is somewhat higher in a glass vessel than in a metal one. If, however, iron filings be dropped into the glass vessel, or, according to Tomlinson, anything capable of acting as a nucleus, the temperature of the boiling-point is lowered, and ebullition is promoted.

206. It would also appear that the *air dissolved* in the water has some effect upon its boiling-point, and M. Donny, by depriving water as far as possible of the air which it contained, and by enclosing it in a peculiarly-shaped vessel, was able to raise the temperature to 135° C. without ebullition.

207. If the liquid is not pure, but contains *substances in solution*, this will likewise affect the boiling-point. Thus the general effect of salt dissolved in water is to raise its boiling-point.

208. Spheroidal state.—If a drop of water be thrown upon certain surfaces at a very high temperature, it does not adhere to the surface, but moves about and evaporates without boiling. This peculiarity of liquids has given rise to some very curious experiments. For instance, M. Boutigny poured liquid sulphurous acid upon a platinum capsule heated to a white heat; yet this very volatile liquid did not boil, and its rate of evaporation was very slow. Faraday, again, poured upon a red-hot platinum capsule a mixture of ether and solid carbonic acid, which evaporated very slowly, and nevertheless solidified some mercury brought into contact with it.

Want of contact appears to be the explanation of this behaviour, and M. Boutigny has found that in certain cases the light of a taper may be seen between the liquid and the surface.

It is no doubt this want of contact that prevents the heat from reaching the liquid sufficiently fast to cause it to boil, and the difference between contact and non-contact is well exemplified in Faraday's experiment: for when the intensely volatile mixture lay on the red-hot platinum capsule without boiling, there was clearly a want of contact between the two; but when the mercury was thrown into this mixture, it became immediately frozen, because it was brought into intimate contact with it.

209. We have seen that the transition from the solid to the liquid state is gradual in the case of many substances, and that there is an intermediate condition of viscosity in which the substance partakes of the character of both states. The experiments of Cagniard de la Tour, and more especially those of Andrews, lead us to believe that there is an intermediate state between the liquid and the gaseous conditions of matter. Thus, Dr. Andrews finds that if we heat liquid carbonic acid under great pressure in a closed tube, when we reach the temperature of 31° C. or thereabouts, the surface of demarcation between the liquid and the gas becomes fainter and fainter, loses its curvature, and at last disappears.

210. Sublimation.—Generally speaking, the order of things is, that, when the temperature is increased, the solid passes into a liquid, and finally into a gas, but sometimes the solid passes at once into a gas without assuming the intermediate state of liquidity. This is called sublimation, and we have instances of it in arsenic acid and solid carbonic acid, which pass at once into the gaseous state. Snow also slowly evaporates, and thus assumes the gaseous form even at temperatures much below its melting-point.

211. Change of Composition in Evaporation and Condensation.—Sometimes if we heat a mixture of two liquids, or a liquid which has dissolved a quantity of gas, the more volatile component passes off, leaving the less volatile one behind. Thus, when a strong solution of hydrochloric acid in water is heated, the permanent gas at first passes off, leaving a weaker solution behind. In like manner, if chalk be heated, the carbonic acid goes off in the shape of gas, leaving lime behind.

On the other hand, many gases which are permanent by themselves may be brought into the liquid state, or condensed, in virtue of their strong affinity for certain liquids. Thus ammoniacal gas and hydrochloric acid gas have a great attraction for water, and if a jar of either of these gases be held above mercury, and a few drops of water introduced, the gas almost immediately disappears, being absorbed by the water. It is often very difficult to condense gases without making use of a solvent, and there are six which we have only quite recently been able to condense through the joint effect of cold and pressure—namely, oxygen, hydrogen, nitrogen, nitric oxide, carbonic oxide, and marsh gas.

212. Pressure of a Vapour in contact with its own Liquid.—We have seen that when a basin of liquid is allowed to evaporate under a receiver, vapour will rise from it until the vapour pressure in the receiver has reached a certain point, after which there will be no more evaporation. We have also seen that this point depends in the first place upon the nature of the liquid, and in the second place upon the temperature. The pressure thus attained is, in fact, the greatest vapour pressure possible for this particular liquid and temperature, and it is of importance to know for different liquids the maximum vapour pressure corresponding to various temperatures.

This information has been obtained by Regnault. We shall not attempt to describe the various and complicated apparatus which he made use of; rather let us state the most important results which he has obtained.

The following is an abridgment of his results for the vapour of water:—

Temperature (centigrade).	Maximum pressure in millimetres of mercury.	Temperature (centigrade).	Maximum pressure in millimetres of mercury.
0°	4·600	65	186·945
5	6·534	70	233·093
10	9·165	75	288·517
15	12·699	80	354·643
20	17·391	85	433·041
25	23·550	90	525·450
30	31·548	95	633·778
35	41·827	96	657·535
40	54·906	97	682·029
45	71·391	98	707·280
50	91·982	99	733·305
55	117·478	100	760·000
60	148·791		

We are enabled to understand from this table how we may obtain the atmospheric pressure by observations of the boiling-point thermometer. In the first place, it is necessary to bear in mind that in such instruments (Art. 168) the thermometer is not plunged into the water itself, but only into the vapour issuing from it; and in the next place, we must remember that when water boils (Art. 204) its vapour has the very same pressure as the atmosphere. Hence the rule is obvious. Look out on a table similar to the above the pressure corresponding to the reading of the boiling-point thermometer, and this will denote the atmospheric pressure.

Regnault has also ascertained the maximum pressures at various temperatures of other liquids besides water.

213. Density of Gases and Vapours.—By means of Boyle's law we can ascertain how the density of a gas varies with its pressure, and, by means of Charles' law, how its density varies with its temperature. But in order to complete our knowledge of the subject we ought to know the density of various gases at a given temperature and pressure, say at the temperature of 0° C. and the pressure of 760 millimetres of the mercurial column reduced to 0° C.

Gay-Lussac was the first to discern that a connection subsists between the density of gases and their combining chemical equivalents, and that when two gases combine together the volumes in which they combine bear a very simple relation to one another.

Thus, for instance, equal volumes of chlorine and hydrogen combine together without change of volume to form hydrochloric acid gas, which contains one atom of chlorine united to one of hydrogen. Equal volumes (at 760 millimetres and 0° C.) contain, therefore, an equal number of atoms of these two gases.

Regnault has given us the following exact determinations of the weights of a litre of the most important gases :—

Name of Gas.	Density.	Weight at 0° C., and under the pressure of 760 millimetres of mercury reduced to 0° C. at the latitude of Paris.
Air	1·0000	1·293187 grammes.
Oxygen	1·1057	1·429802 „
Hydrogen	0·0693	0·089578 „
Nitrogen	0·9714	1·256167 „
Carbonic acid . . .	1·5291	1·977414 „

214. Recapitulation.—In what has gone before we have considered the effect of heat upon the volume and condition of bodies. We have seen that in general when the temperature of a solid rises it expands in volume, the rate of expansion being greater at a high temperature than at a low one. If sufficient heat be applied the body will pass from the solid to the liquid state, the change being in some cases very abrupt, but in other cases very gradual. In very many cases there will be an increase of volume accompanying this change of condition, but in some bodies there is, on the other hand, a contraction, ice being a notable instance of this latter class. When the liquid state has been completely assumed, the liquid generally increases in volume with any further increase of temperature, and at a greater rate than in solids, while the rate of increase is also greater at a high than at a low temperature. If the process of heating be still continued the liquid will pass into the gaseous form, and a very considerable expansion will take place. Finally, after the liquid has been completely converted into gas, any further increase of temperature will augment the volume of this gas, the rate of increase being in general greater than in liquids or solids.

215. Effects of Heat upon other properties of Matter.—In addition to those effects already mentioned there are many other ways in which this agent influences bodies. Thus we have—

(1) The effect of Heat upon Refraction and Dispersion. Both of these diminish as the temperature increases.

(2) The effect of Heat upon the Electrical properties of bodies. This will be considered when we treat of Electricity.

(3) The effect of Heat upon Magnetism. This will be afterwards discussed when we treat of Magnetism.

Besides these there are other important effects. Thus, in most instances an increase of temperature promotes chemical combination, and when we speak of setting fire to a combustible substance, it is only another way of expressing the fact that a high temperature promotes combination. Occasionally, however, heat promotes decomposition, especially when one of the products of this decomposition is a gaseous body. Thus if limestone be heated lime will be left behind, and carbonic acid will be given off.

Again, the various phenomena of capillarity, such as capillary ascent and curvature, are affected by heat, becoming less marked when the temperature is high. Extensibility, tenacity, and the various properties of solids are likewise affected by heat, and the compressibility of fluids is altered from the same cause. In fine, there is hardly a property of matter unaffected by this species of molecular motion.

LESSON XXIV.—CONDUCTION AND CONVECTION.

216. In the preceding pages we have described some of the most important effects of heat. Let us now consider the laws which regulate the distribution of heat through space.

In the first place, heat from a hot body, such as the sun or a star, proceeds outwards into a medium pervading all space, in which it is propagated with very great velocity (Art. 106). It continues to proceed in the form of radiant heat until it reaches some body, such as our earth, by which it is absorbed, and it is in virtue of this process that we derive our heat from the sun. However, for the sake of convenience, we have agreed to regard radiant energy as a species of energy by itself, and we shall not therefore at present discuss the laws of radiant heat.

217. Conduction forms another well-known mode by which heat is distributed. If one end of a metal bar be thrust into the fire, and allowed to remain in it for some time, the other end will gradually become hot, until at length we shall be unable to touch it. The process by which heat is conveyed to the end of the metal rod is very different from radiation, for it is conveyed very slowly from particle to particle of the rod, until at length it affects that extremity which is farthest from the fire. But if instead of a metal rod we heat a glass or stoneware rod in the fire, the further extremity of this rod will never get very hot, because the substance of which it is formed does not conduct heat so well as a metal.

Organic fabrics, such as wool or feathers, form a still worse class of conductors ; and this is the reason why these substances have been provided by nature as the clothing of animals, for the temperature of an animal is generally higher than that of the surrounding substances, and the heat is not readily conducted outwards through the garment of wool, feathers, or fur with which the animal is clad.

Liquids and gases are very bad conductors, but heat is distributed in them after a different manner, which we call **Convection.**

A bad conductor may be used, not only to keep in heat, but also to keep it out ; for it may either be used to prevent the heat of the body being conducted outwards to those colder substances with which we are brought in contact, or if we wrap flannel round a block of ice it will, in virtue of its bad conducting power, prevent the heat from reaching the ice, and preserve it much better than another covering which might be a better conductor.

We thus see how the very same substance which is best fitted to preserve the heat in our bodies is best calculated to preserve the cold in a block of ice, a purpose for which it is generally used.

218. Conduction in Solids.—The following experiment will enable us to recognise the difference between two bodies in their conducting power.

Let two bars, one of copper and one of iron, be fixed as in Fig. 61, and let a spirit-lamp heat the extremities of both.

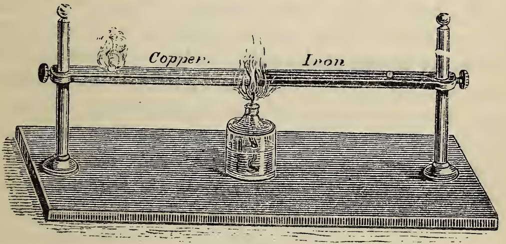


FIG. 61.

The bars will continue at first to grow hotter and hotter, but at length they will settle down into a permanent or stable state with respect to temperature, which they will continue to retain as long as the lamp continues to burn, those parts of either bar near the lamp being hotter than those further away. Nevertheless the copper bar will be hotter than the iron one at the same distance from the lamp, so that a piece of phosphorus will take fire on the copper bar at a further distance from the lamp than on the iron one.

Let us here pause for a moment to consider why it is that when a metal bar has one of its extremities heated in the fire or in a lamp, the other extremity does not *ultimately* attain the same temperature. It would do so if the heat which flows along the bar were not carried off from its surface, but this hot surface radiates into space, and parts also with some of its heat to the surrounding air, and the consequence of this loss is a gradual diminution of temperature as we proceed along the bar from the end which is in the fire to the other extremity.

219. Conductivity.—It is necessary not only to have a clear conception of the conducting power of bodies, but to be able to express this numerically.

This was first done by Fourier, who adopted the following definition.

Suppose, in the first place, that we have a wall one metre in thickness, and that we wish to ascertain the conductivity of this wall. In order to do so, let us imagine that the one side of the wall is kept constantly at a given temperature, while the other side is one degree centigrade hotter. A quantity of heat will, of course, continue to pass across this wall from the hotter to the colder side. Now let us adopt as our heat-unit the quantity necessary to raise one kilogramme of water from 0° C. to 1° C. in temperature. Then *by the conductivity of the wall we mean the number of kilogrammes and fractions of a kilogramme of ice-cold water which will be raised one degree in temperature by the heat which flows in one minute across a square metre of this wall, its thickness being one metre, and the difference in temperature between its two sides* 1° *centigrade.*

MM. Wiedemann and Franz have ascertained the relative conductivity of the different metals, that of silver being reckoned equal to 100. They have obtained the following results :—

Metals.	Relative thermal conductivity obtained from experiments in vacuo.
Silver	100·0
Copper	74·8
Gold	54·8
Brass	24·0
Tin	15·4
Iron	10·1
Steel	10·3
Lead	7·9
Platinum	9·4
Palladium	7·3
Bismuth (in air)	1·8

It has likewise been ascertained by Forbes that the conductivity of iron diminishes as the temperature increases, the rate of diminution being different in different bars.

220. It will be seen that in the definition of Conductivity we take account of the quantity of heat that flows across the substance.

Suppose now that in the experiment of Fig. 61 we have two bars of the same shape and size, and also of the same conductivity, the ends of which we heat by a spirit-lamp to the same extent; finally let the surfaces of both bars be either gilt or covered with a film of the same substance. Now, if we allow the lamp to burn until the various parts of both the bars have settled down into a permanent temperature, we shall no doubt find that both bars are equally hot at equal distances from the lamp. But if we had examined them both at the end of a short time after applying the lamp, it does not follow that the temperature of the two would have been the same at the same distance from the source of heat.

In the one case the bars have attained a permanent state as regards temperature, and the heat which flows along them is not spent in increasing the temperature of the particles, but in making up for that which is carried away from the surface of the bars. Now as both bars have the same surface and the same conductivity, there is no reason why ultimately the distribution of temperature should not be the same in both.

But in the other case, when the bars are examined shortly after applying the lamp, the state of things is very different, for a great part of the heat is consumed in increasing the temperature of the particles of the bar: now it may take much more heat to raise the one bar one degree in temperature than it takes to raise the other to the same extent.

Hence if we take two precisely similar pieces, one of bismuth and the other of iron, and, coating one end of each with white wax, place the other end on a hot vessel, we shall find that the wax will first melt on the bismuth, although iron is the best conductor. The reason is, that it requires more heat to raise iron one degree in temperature than it does to raise bismuth.

221. Safety Lamp.—One of the most important applications of the laws of conduction is the safety lamp devised by Sir H. Davy for the use of miners.

Often in coal mines the atmosphere is so impregnated with combustible gas that a naked flame would cause instant explosion. It is then that the safety lamp will afford the miner sufficient light for his operations without the danger of an explosion. Its action depends on the withdrawal of heat by a wire gauze.

If we lower a surface of wire gauze into an ordinary gas flame it will crush the flame before it, so that there will be none above the gauze. If we now extinguish the flame, retaining the wire gauze in its position, we may relight it above the gauze, in which case there will be no flame below. Thus even although a combustible combination of gases exists on both sides of the gauze, the flame cannot penetrate through the gauze, but remains always on one side of it, the reason being that the mass of metal so cools the flame that combustion cannot spread.

The safety lamp is therefore an ordinary lamp surrounded by wire gauze, so that even if the explosive atmosphere enters the lamp and comes in contact with the flame, the heat cannot penetrate to the outside of the wire gauze sufficiently to communicate the explosion to the outer air. It has recently been shown by Mr. Galloway that the concussion caused by a strong sound, such, for instance, as the noise of blasting operations, causes a disturbance of air which interferes with the action of the safety lamp.

222. Conductivity of Crystals.—It has been shown by De Senarmont that crystals have different conducting powers in different directions. He cut thin slices in various directions out of crystals, and, piercing a small hole at the centre of these slices, he passed a metallic wire through them, along which an electric current was made to pass ; the wire was thus rendered very hot, and the heat spread on all sides along the crystal. Having coated the crystal with wax, the result was a general melting of the wax all round the wire. Had the conductivity been equal on all sides the area melted would naturally have been a circle, but Senarmont generally found it to be an ellipse, and from this he argued that crystals conduct unequally in different directions.

223. Conductivity of Liquids and Gases.—Both of these classes of bodies are very bad conductors of heat. The low conducting power of water may be seen from the following experiment.

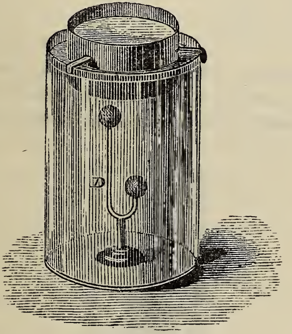


Fig. 62.

Place in a vessel of water (Fig. 62) a differential thermometer, so constructed as to have one bulb near the surface and one near the bottom. Next float on the surface of the water a vessel containing boiling oil, and it will be a long time before the differential thermometer is affected by the source of heat. Professor Guthrie has recently made a series of experiments on the specific thermal resistance of films of liquid all of the thickness of one millimetre, and has obtained the following results :—

Substance.	Specific thermal resistance.
Water	1·0
Glycerine	3·84
Acetic acid	8·38

Substance.	Specific thermal resistance.
Sperm oil	8·85
Alcohol	9·09
Nitro benzol	9·86
Butylic alcohol	10·00
Amylic alcohol	10·23
Oil of turpentine	11·75
Chloroform	12·10
Bichloride of carbon	12·92
Mercury amyl	12·92
Iodide of amyl	13·27

224. Convection.—In the experiment described in last Article it is necessary to apply the source of heat to the surface of the water, and the reason of this is very obvious; for when the heat is applied to the surface, the heated particles, being rendered lighter, remain where they are, and the heat can only reach the differential thermometer by means of conduction. But if the heat be applied to the bottom of the liquid the heated particles will ascend, and their place will

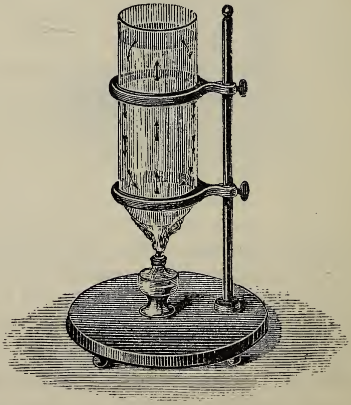


FIG. 63.

be taken by colder particles carried down from above, and the process will continue until the whole liquid is heated. This process, by which heat is conveyed to the various particles of a liquid, is termed Convection.

By introducing a little colouring matter, the direction of the fluid currents may be rendered visible.

Thus (Fig. 63) if we heat a vessel containing water, and drop into it a few fragments of cochineal, we shall find that the ascending currents go up by the centre, while the descending ones come down by the sides, their course being denoted by the arrow-heads in the figure.

225. Freezing of a Lake.—In nature we have several illustrations of convection on the large scale. Let us begin with the case of a lake which is cooled at its surface.

The particles so cooled become specifically heavier and descend, and are replaced by lighter particles from beneath, so that in a short time the whole body of water has been subjected to the cooling agency. This process will go on until the temperature of 4° C. has been reached, which is the point of maximum density of water.

After this the process of convection goes on no longer, and if the surface particles be further cooled they become lighter, not heavier; they do not, therefore, descend, but remain at the top.

Had water contracted down to 0°, and had ice been heavier than water, the ice would have fallen down to the bottom as it was formed, and the whole lake would soon have become one mass of ice. It would in such a case probably have remained frozen all the year round.

But as it is, the body of the water of the lake never attains a lower temperature than the point of maximum density, a temperature which is not destructive to life; while the coating of ice is confined to the surface, and becomes thickened only by the slow process of conduction.

226. Convection Currents in the Sun.—It may here be remarked that convection depends on two things. First of all we must have the force of gravity—an *up* and *down*, for it is in consequence of this force that a body specifically lighter ascends, and were there no gravity there would be no convection. In the next place, the body must expand through heat, for if it hardly expands at all convection will be very feeble, and on this account the convection of mercury is much less than that of water.

Let us illustrate this with reference to the atmosphere of our luminary, where we have every reason to suppose there must be very strong convection currents.

In the first place, there are naturally great changes of temperature occurring in those regions; secondly, gas is a substance which expands greatly through heat; and thirdly, the force of gravity is there very great. We are therefore led to expect in the atmosphere of our luminary storms of terrific violence; and we find that such is really the case, for Lockyer has observed in the sun storms which were travelling at the rate of more than one hundred miles in a second.

227. Trade Winds, &c.—In our own earth we have notable examples of convection currents, for we have the vertical sun shining full upon the equatorial regions of the earth, in consequence of which there is a rising of the rarefied air, and a mounting of it into the upper regions of the atmosphere. The place of the ascending air is supplied by colder air from the poles on both sides, so that we have an under-current sweeping from the poles to the equator, and an upper current of heated air travelling above from the equator to the poles. The under-currents form the **trade winds,** the upper-currents the **anti-trades.**

Now, owing to the rotation of the earth from West to East, the under-current coming from the North Pole, or region of less rotation, into the equatorial regions, or those of greater rotation, will have, from the first law of motion, a tendency to lag behind or to fall to the west at the same time that it advances southward; the under-current from the north will thus become in reality a north-east wind. In like manner the under-current in the southern hemisphere will be a south-east wind. The reverse will take place with the upper-currents or *anti-trades*, for these will travel from a region of greater to one of less rotation; they will therefore be pushed forward in the direction in which the earth rotates.

Hence the return trade which goes north will also go east, that is to say it will be a south-west wind; and the return trade going south will also go east, that is to say it will be a north-west wind. We have thus in the northern hemisphere the trades blowing from the north-east, and the return trades blowing from the south-west; while in the southern hemisphere we have the trades blowing from the south-east, and the return trades blowing from the north-west.

The land and sea breezes are probably due to similar causes. During day the land gets much more heated than the sea, and hence there will be an upper-current from land to sea, and an under-current from sea to land, the latter constituting the **sea breeze.** After sunset, however, the land cools more rapidly than the sea, and we then have an under-current from land to sea, constituting the **land breeze.**

LESSON XXV.—SPECIFIC AND LATENT HEAT.

228. In discussing the laws which regulate the distribution of heat, a very important element is the quantity of heat which a body absorbs when its temperature is raised, and also the quantity which is absorbed when a body changes its state.

The quantity of heat necessary to raise a body one degree in temperature is called its **specific heat.** Thus we define the specific heat of any substance to mean the quantity of heat necessary to raise one kilogramme of the substance 1° C., the *unit* being the amount of heat necessary to raise one kilogramme of ice-cold water 1° C.

Suppose, for instance, that a kilogramme of any substance required as much heat to raise it from 100° C. to 101° C. as would raise $\frac{4}{10}$ of a kilogramme of ice-cold water one degree, then we should say that the specific heat of the substance in question at the temperature of 100° was 0·4.

229. One of the simplest methods of measuring specific heat is the method by mixture, which is best understood by a numerical example.

Suppose, for instance, that 3 kilogrammes of mercury at 100° C. have been mixed with one kilogramme of ice-cold water, and that the temperature of the mixture is 9° C.; find the specific heat of the mercury.

Let x denote the unknown specific heat. Then, since the mercury has been reduced from 100° to 9°, we have thus a loss of 91° in 3 kilogrammes of mercury, which will be represented by $3\,x \times 91$. Also, the gain of heat by the water will be 1×9 (the specific heat of water being unity). Now, as the operation is supposed to be conducted so as not to lose any heat, it is evident that the loss of heat by the mercury will be equal to the gain by the water, and hence $3\,x \times 91 = 9 \therefore x = \frac{9}{273} = \cdot 033$ nearly, from which we see that the specific heat of mercury is only $\frac{1}{30}$ of that of water.

Other methods of estimating specific heat have been devised. In one of these we reckon the quantity of melting ice converted into water in consequence of the hot substance parting with its heat. Another is the method by cooling, for when two substances are exposed to the same cooling influence it is clear that the one with the smallest specific heat will cool fastest, so that the velocity of cooling will afford a means of estimating the specific heat.

230. Specific Heat of Solids.—Dulong and Petit were the first to prove that the specific heat of solids is greater at a high than at a low temperature. They obtained the following results :—

Substance.	Mean specific heat Between 0° and 100° C.	 Between 0° and 300° C.
Iron	0·1098	0·1218
Mercury . . .	0·0330	0·0350
Zinc	0·0927	0·1015
Antimony . .	0·0507	0·0549
Silver	0·0557	0·0611
Copper . . .	0·0949	0·1013
Platinum . . .	0·0355	0·0355
Glass	0·1770	0·1990

From this table we see that the specific heat of platinum remains constant, while that of the other substances increases with the temperature. This is not, however, strictly true, for M. Pouillet has found by another process that the specific heat of this metal increases also with the temperature, although very slowly.

Dr. H. F. Weber likewise has recently made a series of remarkable experiments, which enables him to say that the specific heat of carbon, boron, and silicon rise very rapidly as the temperature increases.

Again, the specific heat of solids depends upon the aggregation of their particles ; and in general, whatever augments the density diminishes the specific heat, and whatever diminishes the density augments the specific heat ; so that it is perhaps owing in part at least to expansion that the specific heat of a substance increases with its temperature.

231. Specific Heat of Liquids.—Irvine was the first to remark that the specific heat of a substance when liquid is generally greater than when solid. Thus ice has only one-half of the specific heat of water.

Again, Regnault has found that the specific heat of water increases with its temperature ; thus the mean specific heat of water between 0° and 230° C. is 1·0204, that of ice-cold water being reckoned equal to unity.

Water has generally been supposed to have a higher specific heat than any other liquid, but Messrs. Dupré and Page have recently shown that a mixture of water with 20 per cent. of alcohol has a specific heat sensibly higher than that of pure water.

232. Specific Heat of Gases.—Since in the operation of determining specific heat the temperature is supposed to change, we shall have two sets of determinations with regard to gases ; for in the first place we may wish to know their specific heat under constant pressure, and in the next place we may wish to know it under constant volume.

Regnault, by means of a set of very laborious experiments, has made the former determination, and has obtained the following results—

1. The specific heat of a given weight of a *perfect gas* does not vary with the temperature or with the density of the gas.

2. The specific heats of equal volumes of the simple and not easily condensed gases are equal, but this equality does not hold for gases easily condensed.

Thus, according to Regnault, the specific heats of equal volumes of the three simple and incondensible gases are as follows :—

Oxygen	0·2405
Hydrogen . . .	0·2359
Nitrogen . . .	0·2368

giving a result sensibly the same for each gas.

233. Influence of Condition on Specific Heat.—The specific heat of substances seems to be greater in the liquid condition than in the solid or the gaseous. This is exhibited in the following table :—

Substance.	Specific Heat.		
	Solid.	Liquid.	Gaseous.
Water	0·5040	1·0000	0·4805
Bromine	0·0833	0·1060	0·0555
Tin.	0·0562	0·0637	—
Iodine.	0·0541	0·1082	—
Lead	0·0314	0·0402	—
Bisulphide of carbon	—	0·2352	0·1569
Ether.	—	0·5290	0·4797

234. Atomic Heat of Bodies.—Dulong and Petit were the first to find that for a large series of simple substances the specific heat of equal weights is inversely proportional to the atomic weight.

If we choose to imagine the atomic weights to denote the relative weights of the atoms of the various elements, this law may be expressed by saying that the same amount of heat will produce the same rise of temperature in all elementary atoms.

The following results have been obtained by Regnault, who made a series of experiments with the view of testing this conclusion :—

Elements.	Specific heat, equal weights.	Atomic weight.	Product of specific heat into atomic weight.
Sulphur	0·1776	32	5·6832
Magnesium	0·2499	24	5·9976
Zinc	0·0955	65	6·2075
Aluminium	0·2143	27·5	5·8932
Iron	0·1138	56	6·3728
Nickel	0·1091	58·5	6·3823
Cobalt	0·1070	58·5	6·2595
Manganese	0·1140	55	6·2700
Tin	0·0562	118	6·6316
Tungsten	0·0334	184	6·1456
Copper	0·0951	63·5	6·0389
Lead	0·0314	207	6·4998
Mercury (solid)	0·0319	200	6·3800
Platinum	0·0324	197	6·3828
Iodine	0·0541	127	6·8707
Bromine (solid)	0·0843	80	6·7440
Potassium	0·1696	39	6·6144
Sodium	0·2934	23	6·7482
Arsenic	0·0814	75	6·1050
Antimony	0·0508	122	6·1976
Bismuth	0·0308	210	6·4680
Silver	0·0570	108	6·1560
Gold	0·0324	196	6·3504

It was thought until recently that carbon, boron, and silicon formed exceptions to the above list, but it has since been ascertained by Dr. H. F. Weber (Art. 230), that they do not, when the determination is made at a sufficiently high temperature.

235. Latent Heat.—We have seen that different substances require different quantities of heat in order to raise the same mass of each one degree in temperature.

But besides this a large quantity of heat is absorbed or rendered latent when bodies pass from the solid into the liquid, or from the liquid into the gaseous state. Thus we may very properly say that water at 0° is equal to ice at 0° *plus* latent heat of liquefaction, or that steam at 100° is equal to water at 100° *plus* latent heat of vaporization.

It cannot fail to have been often observed that a great quantity of heat must be applied to boiling water in order to bring it into steam, and that after all the steam is no hotter than the water. Nevertheless it was reserved for Black to put upon a scientific basis the doctrine of latent as well as of specific heat.

236. Latent Heat of Liquids.—Black's first experiments were upon water, and they were performed in the following manner.

He suspended in a room two similar vessels, one containing melting ice and the other ice-cold water. The temperature of the room was 64° F., and he noticed that in a comparatively short time the ice-cold water had risen to 40° F., while the ice did not reach this temperature until after the lapse of ten and a half hours.

Afterwards Black adopted the following method. He took, let us say, a kilogramme of water at 0° C., and mixed it with a kilogramme of water at 100° C., and the temperature of the mixture was found to be the mean of the two, or 50° C.

In the next place he took a kilogramme of ice at 0°, and mixed it with a kilogramme of water at 100° C., and the temperature of the mixture was only 10·5° C. There is thus a difference in the total heat of the two mixtures sufficient to raise two kilogrammes from 10·5° to 50°, or through a range of 39·5° C.

But the heat of the boiling water was the same in each case, and the only difference was that in the one case we had a kilogramme of water at 0° and in the other a kilogramme of ice at 0°. It would therefore appear that it requires as much heat to liquefy a kilogramme of ice as it does to raise two kilogrammes of water through a range of 39·5° C., so that if we take as our unit of heat that which is needed to raise one kilogramme of water from 0° C. to 1° C. the latent heat of one kilogramme of water will be represented by 79.

The experiments of Person would appear to show that in water the change of state is not quite abruptly assumed, but that heat begins to be rendered latent in ice as low as 2° C. below the freezing-point.

237. Person has obtained the following table exhibiting the latent heat of one kilogramme of various liquids :—

Substance	Latent heat (water = 1).
Water	1·000
Phosphorus	0·063
Sulphur	0·118
Nitrate of soda	0·794
Nitrate of potassa	0·598
Tin	0·179
Bismuth	0·159
Lead	0·067
Zinc	0·355
Cadmium	0·172
Silver	0·266
Mercury	0·035

238. Latent Heat of Vapours.—Black was the first to determine the latent heat of steam, but a more accurate determination has since been made by Regnault. According to him the latent heat of steam at 100° C. is 537 units; so that altogether it requires 637 units of heat, first to heat a kilogramme of water from 0° to 100° and then to evaporate it at that temperature.

The following table embodies the results of Regnault's experiments :—

Temperature.	Total Heat.
0°	606·5
10	609·5
20	612·6
30	615·7
40	618·7
50	621·7
60	624 8
70	627·8

Temperature.	Total Heat.
80°	630·9
90	633·9
100	637·0
110	640·0
120	643·1
130	646·1
140	649·2
150	652·2
160	655·3
170	658·3
180	661·4
190	664·4
200	667·5

Andrews has likewise determined the latent heat of other vapours.

The following table embodies the results of his experiments :—

	Latent heat of equal weights of Vapours (steam = 1).
Water	1·000
Wood spirit	0·492
Alcohol	0·378
Ether	0·169
Bisulphide of carbon	0·162
Oxalic ether	0·136
Formic ether	0·196
Acetic ether	0·173
Iodide of ethyl	0·087
Iodide of methyl	0·086
Bromine	0·085
Perchloride of tin	0·057
Formiate of methyl	0·219
Acetate of methyl	0·206
Terchloride of phosphorus	0·096

239. Remarks on the Latent Heat of Water.—It will be seen that water has a greater latent heat than any other substance, that is to say more heat is spent in rendering liquid a kilogramme of ice than in rendering liquid a kilogramme of any other substance. Again, steam has a greater latent heat than any other gas, that is to say it requires more heat to vaporize a kilogramme of boiling water than to vaporize a kilogramme of any other substance.

These properties of the substance water play a very important part in the economy of nature.

We have already seen (Art. 225) that a lake always freezes at its surface, and that ice formed remains at the top, so that a second layer can only be formed through the substance of the first, and so on. Now the large latent heat of water retards the formation of ice, for this large amount of heat must be taken from a kilogramme of ice-cold water before it can become ice; thus a very large quantity of heat must be carried off from the surface before a lake is frozen to any great depth.

In like manner the large latent heat of the vapour of water prevents the water on the earth's surface from evaporating too fast, and it also prevents the vapour in the air from being precipitated too fast. Were this latent heat very much less the earth would get dry much sooner, and the rainfall, when it took place, would be much more violent than it is at present. In fine, these properties of water are most valuable in toning down the abruptness of the great operations of nature.

240. Remarks on Specific and Latent Heat.—Viewing heat as a species of molecular energy, it has a twofold office to discharge. In the first place, there can be no doubt that the particles of a hot substance are in violent motion, so that when the substance is raised in temperature the energy of its molecular motion is increased.

But the heat of a substance is not wholly spent in this molecular motion, but some of it is spent in moving the particles of the substance away from each other against the force of cohesion; in fact, energy is spent in forcing asunder these particles just as it is spent in removing a stone from the earth and carrying it to the top of a mountain or to the top of a house. Thus there are two quite different offices discharged by heat. One of these is to produce in the particles of the body a species of *molecular motion*, and the other is to pull asunder the particles of the body against cohesion, thus producing a species of *molecular energy of position*.

Now as bodies generally expand through heat, a certain portion of the heat applied to a substance is spent in producing this expansion, and therefore disappears as actual molecular energy in procuring for the particles of the body a position of advantage with respect to molecular force.

Probably, however, on ordinary occasions the greater portion of the heat communicated to a body is spent in motion, and only a small portion is changed into energy of position. There are, however, certain critical occasions on which probably a very large portion of the heat is transferred into energy of position.

This happens when in ordinary language heat is said to be rendered latent. Thus at the melting-point a large quantity of heat may be applied to ice without sensibly raising its temperature, the effect of the heat being apparently to melt the ice.

Now we cannot suppose that all this heat has in this case gone to increase the molecular motion of the particles, otherwise we should expect a great rise of temperature in the water produced. But the energy of the heat, if not spent in producing actual motion of some kind, must have been transformed into energy of position.

We are thus led to imagine that when a body changes its state the heat which is said to be rendered latent is really spent in doing work against molecular force, being thus transformed into a species of energy of position which of course reappears once more as heat when the water comes again to be frozen.

In like manner a very large portion of the heat which must be communicated to boiling water in order to bring it into the state of vapour, is spent in forcing the particles asunder against the force of cohesion, and this potential energy is reconverted into ordinary heat-energy when the vapour is again condensed.

241. Freezing Mixtures and Apparatus.—We thus see how in the change from the solid to the liquid state, or from the liquid to the gaseous state, a large amount of energy of molecular motion is transformed into energy of position. Sometimes this molecular energy is supplied to the body from an external source of heat, as, for instance, when water is boiled in a vessel on the fire ; but sometimes it borrows the heat from its own particles, and this is the origin of freezing mixtures and processes.

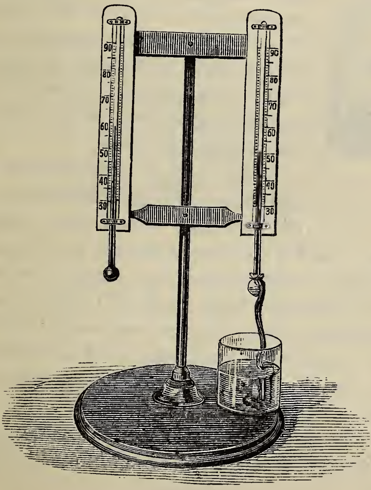


Fig. 64.

Thus if the bulb of a thermometer be covered with a piece of cambric, and if we drop a little ether upon it, the temperature will immediately fall, owing to the cold produced by the evaporation of the ether. Even if we drop water upon a bulb so covered, there is a lowering of the temperature, the amount of depression thus caused being great in dry air, in which the water evaporates rapidly, and small in moist air, in which there is hardly any evaporation ; and meteorologists are in the habit of estimating the hygrometric state of the air by means of the fall of temperature produced by moistening the bulb of a thermometer. The apparatus used for this purpose is shown in Fig. 64. It is called the wet and dry bulb apparatus.

Leslie was the first to freeze water by means of its own evaporation. He took a vessel containing strong sulphuric acid (Fig. 65), which has a great attraction for water, and placed it along with a thin metallic vessel containing water in the receiver of an air-pump. On exhausting the receiver the water rapidly evaporated, and the aqueous vapour was very quickly absorbed by the sulphuric acid. The consequence was a diminution in the temperature of the water, until at last it began to freeze.

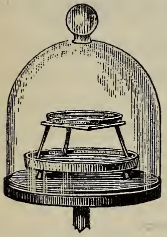


FIG. 65.

A very good arrangement of Leslie's experiment is that devised by M. Carré, of Paris, in which the vapour arising from the water in the receiver is forced by the action of the pump through a vessel containing sulphuric acid. In this case the water actually boils until the cold caused by the rapid evaporation causes it to become a solid mass of ice.

Again, if ether be mixed with solid carbonic acid, gas will escape rapidly from the mixture, and intense cold will be produced.

Mercury may easily be frozen by this means, and Faraday obtained a degree of cold by it which he estimated at -110° Centigrade.

In another kind of freezing mixture we take two solids, or a liquid and a solid, which when mixed together produce a compound which is liquid, and in this case the operation of mixture is generally accompanied by a lowering of the temperature.

Thus if we mix snow and salt together they will liquefy, and the temperature of the solution will be considerably reduced.

LESSON XXVI.—ON THE RELATION BETWEEN HEAT AND MECHANICAL ENERGY.

242. Heat, being a form of molecular energy, is convertible under certain conditions into the other varieties of energy, but we shall here confine ourselves to its connection with mechanical energy, which is the only form we have yet minutely described.

243. Conversion of Mechanical Energy into Heat.—This conversion takes place in the phenomena of percussion, friction, and atmospheric resistance. Thus when the blow of a hammer is arrested by an anvil, its visible energy is changed into heat. Again, it is well known that savages produce fire by rubbing two dry sticks together, and this is a conversion of the energy of motion into heat through friction. We have already shown (Art. 119) that when a body in motion is resisted by the atmosphere, there is a conversion of its energy into heat. Now in all these cases visible energy is absolutely *annihilated* as visible energy, and at the same time heat is *created.* There is, however, no creation or annihilation of energy as a whole, but merely an annihilation of one species, accompanied with the simultaneous creation of another species.

The conversion of mechanical energy into heat is one that can be produced with the greatest possible ease; the difficulty, indeed, is not so much to procure this conversion as to avoid it, and to this intent we use lubricants in order to diminish the friction of machinery as much as possible.

Joule was the first to establish a numerical relation between mechanical energy and heat. He conducted his experiments in the following manner.

He attached a known weight to a pulley (Fig. 66), the axis of which was made to rest on friction rollers with the object of diminishing friction as much as possible. A string passing over the pulley was connected with a vertical axis *r*, so that when the weight fell a rapid rotatory motion was communicated to *r*. Now the shaft was made to work a set of paddles immersed in a fluid in the box B, and a vertical section of one of these paddles is given in the figure. From this it is manifest that in this experiment the fall of the weight was made to agitate the liquid in B, and to heat it through this agitation. It is, in fact, a case of the conversion of mechanical energy into heat.

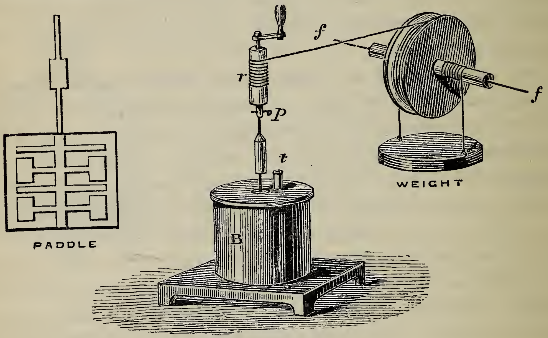


FIG. 66.

By means of a number of such experiments, and others of a similar nature, Joule found that it required the expenditure of an amount of mechanical energy represented by 424 kilogrammetres in order to heat a kilogramme of water one degree Centigrade. In other words, if a kilogramme of water be dropped under gravity from the height of 424 metres the velocity which it acquires will, if wholly converted into heat, raise its temperature one degree Centigrade. Further, if it be dropped from twice this height its temperature will be raised 2° C., if from three times the height 3° C., and so on.

244. Compression of Gases.—Mayer, who at a comparatively early period had divined the law of the conservation of energy, endeavoured to calculate the mechanical equivalent of heat from the heating of gases through condensation ; nevertheless his proof was not quite complete, and it was reserved for Joule to furnish the link necessary to its completion.

When we compress a gas we heat it; but are we at liberty to imagine that the heating produced is the precise equivalent of the work spent in compressing it? Let us answer this question by asking another. Suppose we drop a weight into a large quantity of fulminating powder, the result is the generation of a large amount of heat; but are we at liberty to suppose that all this heat is the mechanical equivalent of the energy of the weight? Clearly not, for the fulminating powder has altered its molecular condition, and in the process of doing so there has been the generation of a large amount of heat. Now when gas is compressed its molecules have been brought nearer together, and hence its molecular state is different. We therefore require to know what portion of the heat developed in the compression of a gas is due to the difference in its molecular state, and what to the mechanical work spent upon the gas.

Now Joule's experiments inform us that in the case of gas the particles are so far apart as to have no perceptible action on each other, so that none of the heat produced by condensation is due to the coming together of mutually attractive particles, but this heat is entirely the equivalent of the mechanical energy spent in the compression.

We see now why a gas suddenly expanded becomes cooled. Suppose, for instance, that condensed air is contained in a vessel similar to the boiler of a steam-engine, and that the vessel has a cylinder connected with it in which a piston works. This piston has above it the pressure of the atmosphere, equal, let us say, to a weight of 1,000 kilogrammes. For the sake of simplicity we may therefore suppose that the atmosphere is done away with, and that instead a weight of 1,000 kilogrammes is placed upon the piston. Now let the condensed air be turned on under the piston, and let us suppose that in consequence the piston is raised one metre in height. A certain amount of work has thus been done by the air in the vessel equivalent to that spent in raising 1,000 kilogrammes one metre in height. This amount of mechanical energy of position has been created, and as a consequence so much heat energy must have disappeared. The air will therefore have become colder in consequence of this expansion.

For a similar reason, when gas is suddenly compressed work is spent upon the gas, that is to say a quantity of mechanical energy is changed into heat, and the gas becomes hotter in consequence.

245. Conversion of Heat into Work.—Suppose in the instance just now given that instead of condensed gas we use steam under a considerable pressure.

Let it be introduced below the piston (Fig. 67), which is raised in consequence up to the top of the cylinder.

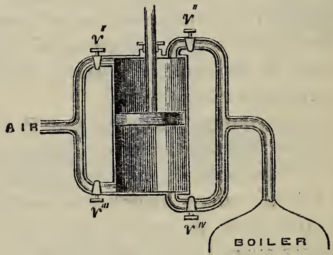


FIG. 67.

Let the supply from the boiler be now cut off by shutting the valve v^{iv}, also let the steam below the piston escape into the air by opening the valve v^{iii}. The piston is now at the top of the cylinder. Next let the steam from the boiler be introduced above it by opening the valve v^{ii}, so as to cause it to descend, and when it has got to the bottom of its stroke let this steam be disconnected from the boiler by shutting v^{ii}, and discharged as vapour into the air by opening v^{i}, so that when the steam is introduced below the piston it will once more mount upward. An alternating motion of the piston in the cylinder may be thus produced, and a large amount of work may be accomplished if the piston-rod be connected with appropriate machinery.

This arrangement is, in fact, the high-pressure steam-engine such as we see in a railway locomotive.

In the low-pressure engine the steam, when once it is cut off from the boiler, instead of being driven out into the air is driven into a vacuum chamber, in which it is cooled by a copious supply of cold water. It is thus condensed and its pressure rendered *nil.*

Thus in the high-pressure engine we have the force of the steam on one side of the piston, and the pressure of the atmosphere on the other, so that the steam must have a higher pressure than the atmosphere, and hence the name of the engine.

But, on the other hand, in the low-pressure engine we have the pressure of the steam on one side, and a vacuum, or nearly so, on the other.

246. It will be noticed that in both engines we obtain useful work only by cooling the steam ; for had the steam not been cooled below the temperature at which it issued from the boiler, we should have been unable to obtain that difference of pressure which keeps the piston going.

In the low-pressure engine the steam is cooled by being brought into contact with cold water in the vacuum chamber of the engine, while in the high-pressure engine it is driven out into the air and cooled in consequence.

Cooling is, in fact, quite essential to the working of any heat-engine ; for as long as all the parts of an engine are at the same temperature it is absolutely impossible to convert heat into work. Heat is only converted into work by being carried from a body at a higher to one at a lower temperature, and even then only a small proportion of the whole heat so carried can be changed into work.

247. Carnot, a French philosopher, who was the first to study this subject, very ingeniously likened the mechanical capability of heat to that of water, remarking that just as water on the same level can produce no mechanical effect, so neither can bodies at the same temperature ; and just as we require a fall of water from a higher to a lower level in order to obtain mechanical effect, so likewise we must have a fall of heat from a body of higher to one of lower temperature.

The laws of this change have been studied by Clausius, Rankine, and Thomson, and from them we learn what proportion of heat may be utilized in a heat-engine.

They begin by showing that the absolute zero of temperature corresponds to about − 270° C., a point which denotes the absolute deprivation of all heat.

Now, if we could imagine a suitable engine, of which the hottest part was at 100° C., and the coldest part, or refrigerator, at −270° C., and if we worked this engine by carrying heat from the hot part to the cold part, under such circumstances all the heat so passing might possibly be converted into mechanical effect, and we should have the full advantage of it. Thus for a quantity of heat sufficient to heat a kilogramme of water 1° C. in temperature we should have 424 kilogrammetres of mechanical energy, and so on in this proportion. But it is obviously impossible to have any part of our engine at an absolute zero of temperature, and therefore we cannot possibly utilize the whole mechanical equivalent of the heat which is carried from the hotter to the colder part. What proportion, then, can we utilize?

This may be expressed in the following way. Let us suppose that the higher temperature of our engine is 100° C., and the lower 0° C.; the former corresponds to an absolute temperature above the zero of temperature of 370°, and the latter to 270°, while the difference between them is 100°.

Now it is found that under these circumstances a proportion of the whole heat carried through the engine, represented by $\frac{370 - 270}{370}$, or $\frac{10}{37}$, may be converted into mechanical effect.

In like manner, if the temperature of the source of heat in the engine be 130° C., and that of the refrigerator 30° C., these will correspond to the absolute temperatures 400° and 300°, and the proportion of heat utilized will be $\frac{400 - 300}{400} = \frac{1}{4}$.

We thus see how to find the proportion of the heat carried through the engine which can be utilized, the rule being to *express the temperature of the source of heat and of the* *refrigerator in the scale of absolute temperatures, and then the difference between these two temperatures divided by that of the source of heat will denote the proportion capable of being utilized.*

This will, however, be only the theoretical limit of utilization, while in practice an engine will fall very far short of this limit, so that probably not one-tenth of the whole heat which passes through an engine is ever converted into mechanical effect.

248. Historic Sketch.—Heat-engines were constructed in an imperfect manner, long before the theory of their action was well understood, but lately they have been greatly improved, and within the last fifty years engines of this kind have played a very important part in the progress of our race.

We have, in the first place, the stationary engine for doing work; secondly, the marine engine, with which steam-ships are fitted up; and thirdly, the railway locomotive.

By means of the first of these the power of production of the human race—their working power—in fact, is very greatly increased, and by means of the two last the power of locomotion is very much facilitated.

Thus we have through the steam-engine not only a much larger power of production, but also a much larger demand for the products of our industry. Steam has, in fact, proved a great civilizing agency, and it promises soon to create a bond of union between all the varieties of the human race.

Hero of Alexandria, about the year 120 B.C. had some idea of the power of steam, and constructed the Eolipyle, to which allusion has already been made (Art. 31).

It is said that in 1543 Blasco de Garay by means of steam propelled a ship of 200 tons burthen in the harbour of Barcelona at the rate of three miles an hour.

Porta, De Caus, and the Marquis of Worcester, at somewhat later dates, independently conceived the idea of utilizing the pressure of steam to raise a column of water, and thus to perform work, but it is doubtful if their conceptions were ever realized in practice.

Much merit is due to Dr. Papin, a Frenchman, who in 1690 applied the motive force of steam to raise a piston, and actually constructed an engine on this principle, which did useful work in a mine.

Newcomen in 1705 conceived the same idea that had occurred to Papin, and his engines continued in use until the time of Watt.

In 1763 James Watt, who was philosophical instrument-maker to the University of Glasgow, received a model of Newcomen's engine to be repaired, and he soon perceived its faults of construction, and was led to those improvements which have made the steam-engine what it now is.

One of these was *the introduction of a separate chamber for condensation of the steam.*

Suppose, for instance, that the steam from the boiler has been introduced below the piston, and that in consequence it has been driven up to the top of the cylinder. In order that it should descend with advantage, it is necessary not only to cut off the connection of the steam under the piston with the boiler, but to reduce the pressure of this steam as much as possible ; in other words, to condense the steam. Now if this condensation takes place in the cylinder a large supply of cold water must be introduced. But this very same cylinder must again be heated before another up-stroke takes place with advantage, as we then wish to realize the full pressure of the steam.

This led Watt to see that the cooling ought to be carried on in a different chamber from the heating, so that when it is wished to condense the steam all that is necessary is to open up a communication between the two chambers, an operation which will immediately reduce the pressure. In fine, the cylinder should always be kept as hot as possible, and the condenser as cold as possible.

In order to keep this separate chamber as cool as possible Watt introduced an arrangement by which the water of injection, after it had become heated by condensing the steam, was pumped out. This was done by means of an air-pump driven by the engine itself, and in order to economize heat the heated water pumped out of the condenser was made to feed the boiler.

Double action was another of Watt's improvements. Before his time, in Newcomen's engines the steam was only introduced below the piston to drive it up, and when at the top the steam below it was cut off from the boiler and condensed, and the piston was then made to descend by means of the pressure of the atmosphere. But by Watt's arrangement the steam was introduced alternately above and below the piston. When it was introduced below in order to drive the piston up, the steam above was shut off from the boiler and condensed, so that there was no counteracting pressure above; and again, when the steam was introduced above in order to drive the piston down, the steam below was shut off and condensed.

Another improvement introduced by Watt was *expansive working*. If the steam from the boiler be introduced below the piston during the full length of the up-stroke, the velocity of the piston will gradually increase until it is suddenly destroyed at the end of the stroke. Work will thus be lost, and the engine will suffer. Now Watt remedied this by cutting off the steam from the boiler before the piston had yet finished its stroke, so that for the remainder of the stroke the pressure of the steam would gradually get less and less, and have only force sufficient to bring the piston to the top of the cylinder without velocity.

249. Horse Power.—Besides the economy of fuel in an engine, another point of interest is the rate at which it works. Thus if it does as much work in a minute as one horse, it is said to be of one-horse power; if as much work as ten horses, of ten-horse power, and so on. In this country an engine is said to be of one-horse power when it will raise 33,000 lbs. one foot high in a minute, this being the average rate of work of the strongest horses.

We have now described the laws which regulate the conversion of mechanical effect into heat, as well as those which regulate the opposite conversion, and it will be noticed that while it is very easy to convert mechanical energy wholly into heat, it is impossible to convert heat wholly back into mechanical energy.

The connexion between heat and the various other forms of molecular energy will be considered when these energies are described.

CHAPTER VI.

RADIANT ENERGY.

LESSON XXVII.—PRELIMINARY.

250. WHEN a substance is heated, it gives out part of its heat to a medium which surrounds it (Art. 106). This heat-energy is propagated as undulations in the medium, and proceeds outwards with the enormous velocity of 186,000 miles in a second. If the temperature of the hot substance be not very great these undulations do not affect the eye, but are invisible, forming rays of dark heat, such, for instance, as are given out by boiling water; but as the temperature rises we begin to see a few red rays, and we say that the body is red-hot. As the temperature still continues to rise the body passes to a yellow and then to a white heat, until it ultimately glows with a splendour like the sun.

It thus appears that we have two kinds of rays, those which do not affect the eye, or rays of **dark heat,** and those which the eye perceives, or rays of **light.**

Let us first proceed to those rays which affect the eye.

251. The term **optics** is given to that branch of physics which treats of luminous rays.

The old idea regarding light was that it consisted of very minute particles given out by a luminous body, but of late men of science have come unanimously to the conclusion that it consists of waves which traverse a medium pervading space. There is thus no addition to the weight of a body which receives light, and no diminution in the weight of one which gives it out, nor is any blow given to a delicately-suspended substance upon which the light of the sun is made to fall.

But while there are many properties of light which can hardly be explained except on the supposition that it consists of waves, there are others which can be studied indifferently under either hypothesis.

252. Suppose now that we have a very small luminous body, in fact a mere shining point, and that light is radiated on all sides by this small body, which forms, as it were, a centre from which waves of light proceed in all directions. If the eye look towards this point, a cone of rays strikes the eye. If the bulb of a thermometer be held near it, a cone or bundle or **pencil** of rays strikes the bulb.

Now just as in a sphere we can geometrically conceive of a single radius, so in the case of the radiant point we can conceive of a single **ray** of light, and we may imagine a vast number of such rays to form a luminous pencil.

If the luminous point be near the eye, the pencil of rays which strikes the eye is *divergent*. On the other hand, the light which strikes the eye from a star or body at a very great distance may be regarded as a *pencil of parallel rays.* We may likewise have a pencil whose section lessens as it proceeds, in which case it is called a *convergent pencil.*

253. Substances are divided into two distinct classes with reference to light : there are those which are **opaque** and those which are **transparent.** The former stop a ray of light, while the latter allow it to pass ; nevertheless no substance is perfectly opaque or perfectly transparent, but a very thin slice of the most opaque substance will allow a little light to pass through it, while a very thick stratum of the most transparent substance will stop some light.

As long as a ray of light moves through the same medium it moves in straight lines, but on passing from one medium to another, one part of the light is **reflected**, or thrown back, and another part enters the medium, but in a different direction from that which it previously pursued ; this bending of the ray is termed **refraction.**

254. Velocity of Light.—Römer, a Danish astronomer, in 1675 was the first to determine the velocity of light from the eclipses of Jupiter's satellites. It so happens that at equal intervals of 42h. 28m. 36s. the first of Jupiter's satellites passes within his shadow, and is thus obscured. Now if light travelled instantaneously from Jupiter to the Earth, we should see this phenomenon precisely at the moment when it took place. But Römer found that when the Earth was farthest away from Jupiter there was a retardation in the time of the occurrence equal to 16m. 36s.

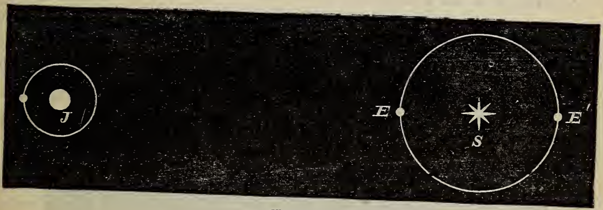


FIG. 68.

Now it will at once be seen (Fig. 68) that the Earth is nearest Jupiter when both are in one line with the sun and on the same side of him, and that the Earth is farthest from Jupiter when both planets are in one line with the sun but on different sides of him, and that the difference of distance in these two cases is the diameter of the Earth's orbit. Hence Römer argued that a ray of light takes 16m. 36s. to cross the diameter of the Earth's orbit. From this it may be calculated that the velocity of light is about 186,000 miles per second.

The velocity of light has also been determined by experiment. Fizeau's apparatus for this purpose will be most easily understood. Suppose we have a toothed wheel, and that a ray of light is made to pass through the interval between two teeth. It is then allowed to proceed to a mirror some distance off, from which it is reflected back precisely in the direction in which it came, so as to return through the same interval between the two teeth of the wheel from which it originally proceeded to the mirror. If, however, the toothed wheel is made to revolve with great velocity, the ray of light, when it comes back from the mirror, may find itself stopped by the next tooth, and may not be able to pass.

Whether this will happen or not will depend upon the time the ray of light takes to go from the toothed wheel to the mirror and back again, and upon the velocity with which the wheel is made to revolve.

M. Fizeau so performed the experiment as to stop the ray of light on its return, and knowing at the same time the rate of revolution of the toothed wheel he was able to estimate the time the ray took to go from the wheel to the mirror and back again, and he thus determined the velocity of light.

255. Intensity of Light.—*If the light from a luminous body fall upon a surface, the quantity of light which the surface receives will vary inversely as the square of its distance from the source*

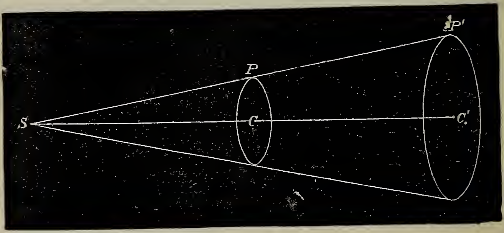


FIG. 69.

To prove this, let S (Fig. 69) denote the luminous source, and let P denote a circular plate upon which the light falls, the distance S C being unity. Also let P′ denote a plate similarly placed, but twice as far from the source, the distance S C′ being equal to 2. Now it is evident from the figure that the same amount of light which would fall on P will, if allowed to pass, spread itself out so as to fall on P′, so that both circles receive the same amount of light from the source. But the figures being similar, P′ is four times as large as P, and hence the light which falls on a portion of P′ equal in size to P will be only one-fourth of that which falls on P; that is to say, the same plate will only receive one-fourth of the light when its distance is doubled, or the light received will vary inversely as the square of the distance.

256. In the next place, *the intensity of the illumination of a plate or screen is proportional to the cross section which it presents to the direction of radiation.* Thus let Fig. 70 denote parallel rays from a far-off source falling upon a plate perpendicular to the plane of the paper, and which we may represent by A B, the line on which it stands upon the paper. First let A B be perpendicular to the direction of the rays; the plane will in this position receive the greatest possible amount of light. Next let it be turned obliquely into the position A B′; it is evident that it will now receive fewer rays, and that the amount received will be represented by A C; that is to say, by the cross section which the plate presents to the direction of radiation.

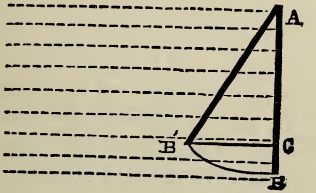


FIG. 70.

This is the reason why the sun is so much less powerful at morning or evening than at noon, at winter than at summer, at the poles than at the equator, for the ground will evidently receive most heat when its surface is perpendicular to the sun's rays.

257. Let us now make a few remarks regarding **intrinsic luminosity,** or inherent brightness, and propose to ourselves the following question:—Will the inherent brightness of a fire be diminished as we recede from it, or would the inherent brightness of the sun be increased could we view it from half its present distance? To this we answer.

that if we could view the sun from half its present distance the apparent area which it would cover would be four times as great as that which it covers at present, and the light which the eye would receive from it would also be four times as great. We should thus get more light from the nearer sun in the proportion in which its size was increased; but if we could cut down or cover over this larger and nearer sun until it became of the same apparent size as the ordinary and more distant sun, then we should get precisely the same amount of light from it as we do at present. The same would take place in the case of a fire; the red body of the fire would grow no brighter as we approached it, but it would grow larger, and our eye would receive more light. If, however, we held a long narrow tube before our eye and viewed the fire through it, we should find no difference in the light that reached our eye until we had gone away so far that the fire did not fill up the field of view looking through the tube.

We thus see that the quality or intrinsic brightness of a luminous body does not vary with its distance, meaning by quality or brightness the light that would reach the eye by looking at the body through such a long narrow tube, and supposing the tube to be always so narrow, and the source of light always so large, that in looking through the tube we should see nothing else but this light.

But if the luminous body should be so distant as not to subtend any perceptible area, but rather to appear simply a luminous point like a star, we should not then be able to judge of its intrinsic brightness.

258. The intensity of light is measured by an instrument called a **photometer**; one of the simplest of these is that devised by Bunsen, who makes a grease spot in a sheet of porous paper.

This spot, if illuminated in front, will appear darker than the surrounding paper, but if illuminated behind it will appear brighter.

Now let a standard light, such as that from a wax candle of known dimensions, be placed behind the paper screen and kept there in a fixed position during the experiment; the greased spot will shine out in consequence, and appear more luminous than the paper. Next place the light to be examined in front of the screen, and move it to such a distance that the grease spot is made as much darker than the paper by the light in front as it is made brighter by the fixed standard light behind, now appearing, in fact, of the same brightness as the paper.

Different lights may thus be compared with one another; for when the grease spot becomes of the same brightness as the paper, or in fact vanishes as a bright spot, it denotes that an amount of light has been thrown from in front upon the paper equal to that thrown upon it by the fixed standard light behind; and if we know at the same time the distance of the luminous object in front from the paper, we are enabled to measure the intensity of its light. Thus if one light causes the grease spot to vanish when placed at the distance of one foot in front of the screen, and another light when placed at the distance of two feet, we should conclude that the luminosity of the latter is four times as great as that of the former, inasmuch as the latter is found to produce the same effect as the former at double the distance, and we know that by doubling the distance the effect upon the screen is diminished four times (Art. 255).

259. We again remark the distinction between the illuminating power of a source of light and the inherent brightness or quality of the light.

The illuminating power is quantitative merely, and refers to the capacity of the light to illuminate a screen at a given distance. But the intrinsic luminosity takes account of the size of the luminous body as well. For instance, a large fire may produce the same illuminating effect as a couple of jets of gas; but the size of the fire is much larger, and if we place the gas between our eye and the fire we shall soon see that, size for size, the gas will give most light.

LESSON XXVIII.—REFLEXION OF LIGHT.

260. From Plane Mirrors.—When a ray of light falls upon a plane and polished metallic surface, it is reflected according to the same law which holds for sound (Art. 142); that is to say, the angle of reflexion is equal to the angle of incidence, while both the incident and reflected rays are in a plane perpendicular to the surface of the mirror.

The truth of this law may be rendered visible to the eye by allowing a beam from the sun, or from an electric lamp, to fall upon a mirror in an otherwise dark room. The path of the ray, both before and after reflexion, will be rendered sufficiently visible by the floating particles of dust which are lit up as they encounter the beam. If the mirror be horizontal, it will be seen that both rays are in the same vertical plane, and also that if the incident ray falls rapidly towards the mirror the reflected ray rises as rapidly on the other side.

If a luminous substance be placed in front of a plane mirror, an image of the substance is seen, as it were, behind the mirror. We know that in reality there is no substance behind the mirror, although the rays of light which reach the eye from the mirror affect it in the same manner as if there were. This image of the luminous substance given by a plane mirror is therefore called a **virtual** image.

The following figure will assist in explaining the laws of the formation of images by plane mirrors. Let A denote a luminous point, and M M a plane mirror, and let the reflexion of the luminous point be viewed by the eye at D D′; also let A A′ and B C denote lines perpendicular to the plane of the mirror.

Now since B D is the reflexion of A B, it follows, from the law of reflexion (Art. 142), that the angle A B C is equal to C B D. But A B C is equal to B A A′, since A A′ and B C are parallel; also C B D is equal to B A′ A for the same reason: hence it follows that B A A′ is equal to B A′ A, and hence that

A M = M A′; that is to say, A′ is as much below the mirror as A is above it.

By similar reasoning it might be shown that the reflected line D′ B′ would, if prolonged, pass through the point A′, making A M = M A′ as before, and in fact all the rays from A which strike the mirror will, after reflexion, appear to proceed from A′. We thus see that the point from which the reflected rays appear to proceed lies as much behind the reflecting surface as the luminous point which is the source of the rays lies before it.

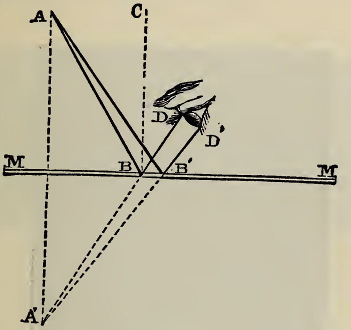


FIG. 71.

We also see how in all cases to construct a figure which will show the apparent form and position of the virtual image of any substance reflected from a plane mirror.

Then let A B C D (Fig. 72) be an irregular body, of which we wish to study the reflexion in the mirror M. From the various points of the body A B C D drop perpendiculars upon the plane of the mirror, and continue them on the other side, until in all cases the lengths behind the mirror are equal to those in front. Then will the figure, formed by joining together the extremities of these lines, represent the apparent position and form of the reflected image.

We are all of us familiar with reflected images from plane mirrors, and it is well known that if a body be close in front of a mirror its image will appear to be close behind, while if the body be a long way in front its image will be a long way behind.

The reflexion of the human figure in a vertical mirror will be erect, like the figure itself, but it will be left-handed, the right hand of the individual appearing as the left in the reflexion.

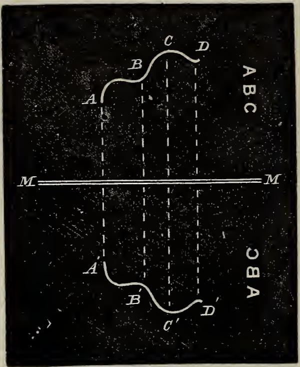


FIG. 72.

In like manner, if letters (Fig. 72) be written upon a wall in front of the mirror from *left to right*, or in the usual way, their reflected image will appear as if they had been written from *right to left*.

All these peculiarities of reflected images are easily understood if we bear in mind the rule that the reflected image of a point is as much behind the mirror as the point itself is in front.

261. Reflexion from Curved Mirrors.—When a ray of light strikes any point of a curved surface, in order to find the direction of the reflected ray we must first of all find the position of the tangent plane to the surface at that point. Now it is well known that for exceedingly small distances around the point of contact a curved surface may be supposed to coincide with its tangent plane, and hence the reflexion at that point will be the same as if it took place from the tangent plane.

The most important case of reflexion from curved surfaces is that in which a concave spherical mirror is struck by a pencil of parallel rays, as in Fig. 73. Let D D′ represent

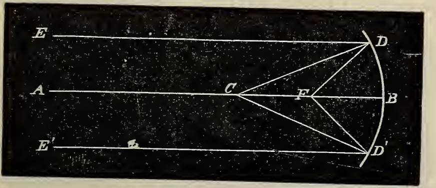


FIG. 73.

a section of such a mirror, and let C be its centre; also let E D, A B, E′ D′ denote parallel rays impinging against the mirror. Let us take the ray E D; now the tangent plane to the sphere at any point is perpendicular to the radius, hence at D it is perpendicular to C D, or C D is the normal to the tangent plane. Hence the ray E D will be reflected in some direction, D F, such that E D C shall be equal to C D F (Art. 260), F being the point where the reflected ray cuts the central line A B, or ray which passes through the centre.

But since E D is parallel to A B, E D C is equal to D C F, hence C D F = D C F, and therefore C F = F D. Now if the beam of parallel rays be confined to short distances from the central line, F D will be very nearly equal to F B. But C F = F D, hence for such rays C F will be very nearly equal to F B; that is to say, the various reflexions from the rays of such a beam will all cut the central line A B in some point, F, half way between C and B: the point F will, in fact, be the **focus** for such rays.

262. Let us now suppose that we have a concave mirror of this kind, and that we point it direct towards the sun. It will at once be seen that the rays which strike the various parts of the mirror from the central point of the sun are all parallel rays, inasmuch as this point is a very great distance off; and in like manner, the rays which strike the various parts of the mirror from any fixed point in the rim of the sun will all be parallel rays for the same reason ; but, on the other hand, the rays from the point in the sun's rim will not be parallel to the rays from his central point.

In fact, the rays from the central point of the sun will refer themselves to one axis of the mirror, and we shall look for their focus in this axis ; while the rays from a point in the sun's rim will refer themselves to another axis of the mirror, and we shall look for their focus in this other axis.

We shall thus have a series of axes all passing through the centre, each axis containing the focus for some one point of the sun's disc, and each focus being at a distance from the centre equal to half the radius. A circular image of the sun will thus be formed ; and furthermore, it will not be a virtual but a real image, so that if a photographic plate were to be placed in the focus of such an instrument we should obtain a likeness of the sun.

263. We have hitherto considered parallel rays or those coming from a body at an infinite, or at least a very great distance. The case will, however, be altered if the rays which strike the mirror are divergent rays proceeding from a body near the mirror.

Thus, in Fig. 74 let L denote a luminous point, from which rays proceed to the concave mirror D B D′. It is evident that the ray which strikes the mirror at D will be reflected in a direction D *f*, such that the angle L D C shall be equal to the angle C D *f*. Had, however, a ray, parallel to the axis L C B, impinged upon the mirror at D, it would have been reflected in the direction D F. In fact, while F is the focus for parallel rays, *f* is the focus for rays proceeding from L, the focus being in this latter case farther from the mirror and nearer its centre.

F, or the focus of parallel rays, is called the **principal focus.**

264. We thus see that if a luminous point be placed at L, the focus of its reflected rays from the concave mirror will be at f, and it will readily appear that if the luminous body be placed at f its focus will be at L, since the only difference between the two cases is that in the first case the rays began their journey at L and ended it at f, whereas in the second they began it at f and ended it at L: in fact, the two points L and f bear a reciprocal relation to each other, and are therefore called **conjugate foci.**

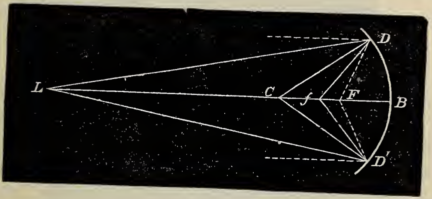


FIG. 74.

We can very easily find the relation between the two foci. For in the triangle L D f we have C D halving the angle L D f, hence $\mathrm{LC} : \mathrm{C}f :: \mathrm{LD} : \mathrm{D}f$, but if the angle D L B is small, L D becomes very nearly equal to L B, and D f to B f, and hence we have for small angles $\mathrm{LC} : \mathrm{C}f :: \mathrm{LB} : \mathrm{B}f$.

But $\mathrm{LC} = \mathrm{LB} - r$ (r being the radius of the mirror), and $\mathrm{C}f = r - \mathrm{B}f$, hence we have $\mathrm{LB} - r : r - \mathrm{B}f :: \mathrm{LB} : \mathrm{B}f$, and hence $(\mathrm{LB} - r)\,\mathrm{B}f = (r - \mathrm{B}f)\,\mathrm{LB}$, or (calling L B, B f D and d) $\mathrm{D}d - rd = \mathrm{D}r - \mathrm{D}d$, that is to say, $\mathrm{D}r = 2\mathrm{D}d - rd$. Lastly, dividing each term by $\mathrm{D}dr$ we have $\frac{1}{d} = \frac{2}{r} - \frac{1}{\mathrm{D}}$, or $\frac{1}{d} + \frac{1}{\mathrm{D}} = \frac{2}{r}$, an expression which gives the relation between the two conjugate foci of a concave spherical mirror.

265. We thus began with strictly parallel rays, which gave us the principal focus, and we then brought our luminous point nearer, so as to be the source of a divergent pencil of rays, which gave us a focus farther from the mirror than the principal focus. Thus none of the foci or conjugate foci which we have been considering have been so near the mirror as the principal focus. Now what will happen if the luminous point be nearer the mirror than the principal focus? This case is exemplified in Fig. 75, where we see that the reflected rays will appear to diverge from some point behind the mirror; in fact the focus will now be virtual, and not real. Thus, for all luminous points further from the mirror than its principal focus we have real foci of reflected rays, but when the luminous point is nearer than the principal focus, the focus for the reflected rays is a virtual one, and appears to proceed from behind the mirror.

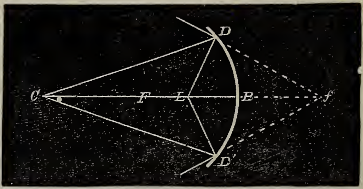


FIG. 75.

The formula $\frac{1}{d} + \frac{1}{D} = \frac{2}{r}$ (Art. 264), is applicable to all these various cases.

1. Let the object be at an infinite distance, so that the rays which fall upon the mirror are parallel rays. Then $\frac{1}{D} = o$, hence $\frac{1}{d} = \frac{2}{r}$, and hence $d = \frac{r}{2}$, or we have the principal focus.

2. Let D be greater than r, or let the object be beyond the centre, then d will be less than r, but greater than $\frac{r}{2}$, or the focus will be between the principal focus and the centre.

3. Let $D = r$, or let the object be at the centre of the mirror, then $d = r$, or the focus is also at the centre, and coincides with the object.

4. Let D be less than r but greater than $\frac{r}{2}$, then d will be greater than r, or the focus will be beyond the centre.

5. Let D be less than $\frac{r}{2}$, or let the luminous point be between the principal focus and the mirror, then d will be negative, that is to say, it must be reckoned on the other side of the mirror, and the focus will be a virtual one.

266. Suppose now that instead of a single luminous point we have a line A B (Fig. 76), and that we wish to find the position and size of its image as given by a concave mirror, of which C is the centre. From the point A draw the line A C D, passing through C the centre of the mirror. The focus corresponding to A will be some point A′, which we already know how to find.

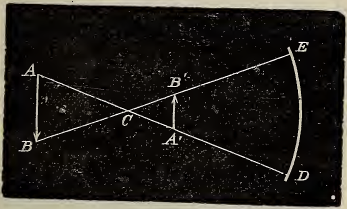


FIG. 76.

Again, from B draw a line B C E, passing through C the centre; the focus of B will in like manner be in some point B′. In fine, A′ B′ will be the image of A B. In the case we have given, the image will be *real* and *inverted*, and it will bear the following relation in size to the object from which it proceeds. Length of object *is to* length of image *as* distance of object from centre *is to* distance of image from centre.

If, however, the luminous point be nearer than the principal focus, the image will be *virtual* and *erect*.

All these peculiarities of the images may easily be studied by means of a small concave spherical mirror, such as that attached to a microscope.

Supposing that a small object is placed immediately in front of such a mirror, its image is at first virtual, and appears to proceed from behind the mirror, and it is also erect. As the object is withdrawn the image increases in size, still remaining erect, until we reach the principal focus, when we are unable to perceive any image. As the object is still withdrawn, a magnified but inverted image may be seen by placing the eye considerably farther away than the object.

When the object is at the centre the image is there also.

Lastly, if the object be beyond the centre, we have an inverted image nearer than the object, and less than it; and if the object be still withdrawn, this inverted image becomes smaller and smaller, until it finally vanishes from the sight.

267. Parabolic Mirrors.—In a spherical mirror the image of a point of light, such as a star, is only brought approximately to a focus: but the case is different if we employ a parabolic mirror, that is to say, a mirror whose surface is that formed by the revolution of a parabola about its axis. In this case, if a luminous point, such as a star, be placed at an infinite distance along the axis, the rays which strike the mirror will do so in lines parallel to the axis, as in Fig. 77.

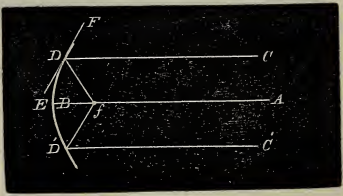


Fig. 77.

Let C D be one of these lines striking the surface at the point D, let E F denote a tangent to the parabola, and let D*f* be the line joining D and the focus of the parabola.

Now it is a well-known property of the parabola that in all such cases the angle C D F is equal to the angle E D*f*, and hence if C D be a line of light incident on the surface of the parabola at D, which surface we may there suppose to coincide with its tangent plane, it will be reflected in the direction D*f*, so that the reflected ray will pass through the focus of the parabola. In like manner, the reflexion of any other ray of light coming from the star will pass through the geometrical focus of the mirror. Therefore in the case of a parabolic mirror and a star placed along its axis, the geometrical focus is not only approximately, but strictly, the optical focus.

On the other hand, if a luminous point were placed in the focus of a parabolic mirror, its rays would be reflected in lines strictly parallel to the axis.

Parabolic mirrors present, therefore, certain advantages over circular mirrors; but, on the other hand, it is difficult to construct them accurately.

268. Convex Mirrors.—We have dwelt on concave mirrors because they are the most important. We shall only state that in convex spherical mirrors the foci are all virtual, the reflected rays appearing to proceed from a point behind the mirror.

LESSON XXIX.—REFRACTION OF LIGHT.

269. Allusion has already been made (Art. 253) to the bending of a ray as it passes from one medium to another. This bending is called the **refraction** of light.

Let us suppose (Fig. 78) that we have a surface of glass, or any similar transparent substance, perpendicular to the plane of the paper, and represented by the line A B, also let C D denote a ray of light passing in vacuo, and incident on this plane at D, and let F D G be a perpendicular to the plane at the same point, then as the ray of light passes from a rarer to a denser medium, it will be bent towards the perpendicular, as we see in the figure. There are two laws which regulate the path of the refracted ray. *In the first place,* C D, D E *are in the same plane with*

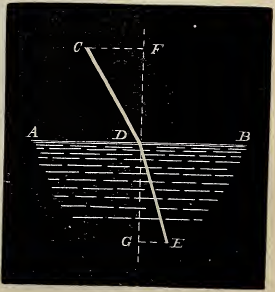


FIG. 78.

F D G, *the normal to the plane at* D ; *and in the next place, for the same medium, whatever be the incidence, the sine of the angle* C D F *or angle of incidence, always bears a fixed proportion of that of* G D E *or angle of refraction.*

270.—This may be expressed as follows: take equal lengths, C D D E, of the incident and refracted rays, and drop C F and E G perpendicularly upon the normal, then for the same substance the ratio between C F and G E will always be the same, whatever be the direction of the incident ray.

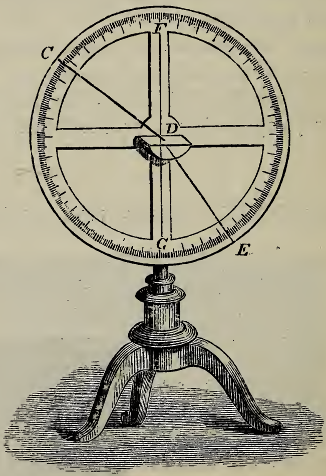


FIG. 79.

Again, if the ray of light, instead of passing from vacuo into the medium, passes out from the medium into vacuo, we have only to suppose the direction reversed; that is to say, if E D be a ray in the substance passing out at D, it will be bent *from*, not *towards* the perpendicular into the direction D C. Another noteworthy point is, that when a ray strikes the surface of a medium at right angles it suffers no refraction, for in this case the sine of the angle of incidence, or C D F, being zero, the sine of the angle of refraction, or E D G, must also be zero, that is to say, the ray will continue in a direction perpendicular to the surface.

Hence also if a ray emerge from a medium in a direction perpendicular to its surface, it will not be bent.

271. The truth of these laws may be illustrated experimentally by an apparatus that may also be used for reflected rays.

It consists of a graduated vertical circle (Fig. 79), having at its centre a small semi-circular cylinder of some refracting substance, the upper surface of which is horizontal. Suppose now that a ray, C D, coming from a small aperture, C, at the circumference of the graduated circle is allowed to strike the refracting substance at D, its centre of figure, this ray will suffer refraction as it passes into the substance, but none as it passes out, because the substance is a semi-circular cylinder of which D is the centre, and hence the ray of light when in the substance will form a radius which will thus be perpendicular to the surface of exit, so that the ray will emerge without a second refraction.

Now let the eye or a screen be so placed as to receive the ray of light at the graduated circle at some point E, next read on the graduated limb the values of the angles C D F and E D G, and taking their sines, it will be found that these will bear a fixed proportion to each other, whatever be the direction of the incident ray C D.

In the next place, it is evident that the incident and refracted rays are in a plane perpendicular to the surface, for these rays are in the plane of the circle, or in a vertical plane, while, on the other hand, the refracting surface is horizontal.

272. Index of Refraction.—We have seen that when a ray of light proceeding from vacuo strikes a plane refracting surface it is bent, so that the sine of the angle of incidence bears for the same substance a constant proportion to that of refraction, that is to say :—

$$\frac{\text{sine of angle of incidence}}{\text{sine of angle of refraction}} = \text{a constant quantity.}$$

This quantity is called the index of refraction for the substance in question. The following are the indices of refraction for some of the most important substances :—

Solids and Liquids.	
Diamond . .	2·47 to 2·75
Phosphorus . . .	2·224
Sulphur	2·115
Bisulphide of Carbon	1·678
Flint glass . . .	1·575
Rock salt	1·550
Rock crystal . . .	1·548
Alcohol	1·374
Ether	1·358
Water	1·336
Ice	1·310

Gases.	
Hydrogen. .	1·000138
Oxygen . .	1·000272
Air	1·000294
Nitrogen . .	1·000300
Carbonic acid	1·000449
Nitrous oxide	1·000503
Olefiant gas .	1·000678
Chlorine . .	1·000772

273. Total Internal Reflexion.—Let us now suppose a ray of light to strike very obliquely against the surface of a refracting medium, so as almost to graze the surface and make an angle with the normal equal to 90°, what would be the path of such a ray after refraction? Thus, suppose that the index of refraction of a substance were = 2, and that a ray entered the substance in this fashion, the sine of the incident angle being = 1.

Hence the sine of the refracted angle would be $\frac{1}{2}$, because $\frac{1}{\frac{1}{2}} = 2$; that is to say, the refracted ray would make an angle with the normal = 30°, since $\sin 30° = \frac{1}{2}$. Therefore rays in all possible directions entering the substance would, after refraction, be compressed within a cone whose sides make an angle with the normal equal to 30° all round. Hence also a ray in such a medium, making an angle with the normal equal to 30° or forming a side of the internal cone of rays, would emerge into vacuo, so as just to graze the surface. But what would happen to a ray traversing such a medium not embraced within the cone, but making an angle with the normal greater than 30°, and therefore having a sine greater than $\frac{1}{2}$? Evidently if it emerged the ratio of 1 to 2 in the sines could not be preserved, for we cannot have a sine greater than unity. Now under such circumstances the ray will not emerge from the medium at all, but there will be total internal reflexion. For all media there is an angle of this kind, beyond which the rays will not emerge into vacuo, but will suffer total reflexion from the surface. This is called the **critical angle.**

274. We often see a reflexion of this kind from the surface of water in a vessel. Thus (Fig. 80) the reflexion of an object at A will be seen at B, being reflected from the surface of the water in the spherical vessel.

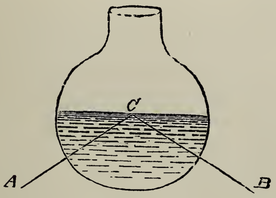


FIG. 80.

Very frequently in hot climates the layers of air in contact with the ground are more heated and less dense than those above them, so that the angle of total internal reflexion is sometimes reached by the rays of light which fall obliquely upon these layers from an object , now such rays after reflexion entering the eye of the distant observer, will give an inverted image of the object as if from the surface of a lake. This phenomenon is known as the **mirage.**

275. Relative Indices.—Suppose that we have, in the first place, a plate of one medium, with its sides parallel.

A ray of light in passing through such a plate would be bent, as in Fig. 81, but would ultimately emerge in a direction, E Q′, parallel to its original direction, P′ D. Suppose now that we have two plates, the upper plate being of a medium less dense than the under, and that we wish to find the relative index of these two media ; that is to say, we wish to estimate the refraction of a ray of light passing from the upper to the under medium.

Now it is found by experiment that the ultimate emergent ray, C Q, will be parallel to P A. Call n_1 the absolute index of the upper medium, that is to say, $\frac{\sin m\,\mathrm{A\,P}}{\sin \mathrm{B\,A}\,m'} = n_1$. Hence $\sin \mathrm{B\,A}\,m' = \frac{\sin m\,\mathrm{A\,P}}{n_1}$. Let n' be the relative index for the two media, that is to say, let $\frac{\sin \mathrm{A\,B}\,n}{\sin \mathrm{C\,B}\,n'} = n'$.

Finally, let n_2, be the absolute index of the lower medium;

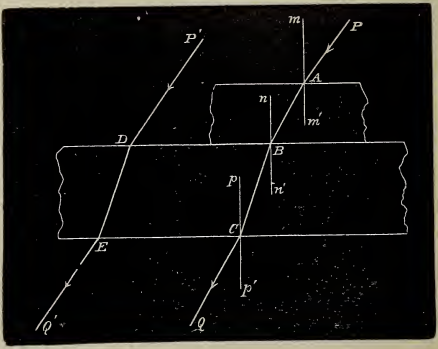


FIG. 81.

that is to say, let $\frac{\sin \mathrm{Q\,C}\,p'}{\sin \mathrm{B\,C}\,p} = n_2$. Hence $\frac{1}{\sin \mathrm{B\,C}\,p} = \frac{n_2}{\sin \mathrm{Q.\,C}\,p'}$.

Now $n' = \frac{\sin \mathrm{A\,B}\,n}{\sin \mathrm{C\,B}\,n'} = \frac{\sin \mathrm{B\,A}\,m'}{\sin \mathrm{B\,C}\,p} = \frac{\sin m\,\mathrm{A\,P}}{n_1} \times \frac{n_2}{\sin \mathrm{Q\,C}\,p'}$; but since P A is parallel to C Q, sin m A P = sin Q C p', hence $n' = \frac{n_2}{n_1}$.

276. Prisms.—For optical purposes transparent substances are often made into prisms, which are chiefly triangular.

Now a triangular glass prism simply means a triangular glass column, and in Fig. 82 this column is supposed to stand perpendicularly upon the plane of the paper on the base *a b c*.

The path of a ray through such a prism is exhibited in the figure.

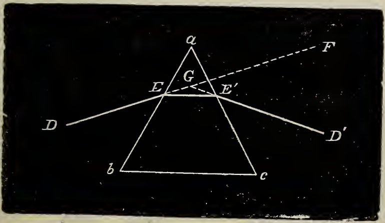


FIG. 82.

Here D E is the incident ray, and is first bent on entering the prism towards the perpendicular, assuming now the direction E E′. In emerging from the prism at E′, it is, on the other hand, bent from the perpendicular in the direction E′ D′. Thus it struck the prism in the direction D E, and finally leaves it in the direction E′ D′, and the angle F G D′, which these two lines make with each other, is called the **angle of deviation.** It can be shown that when the angles D E *b* and D′ E′ *c* are exactly equal, we have that position of the prism which gives us the *minimum deviation.*

It can also be shown that if the angle *a* of a prism be greater than twice the critical angle (Art. 273) of the substance which forms the prism, then the luminous rays will be incapable of passing through the faces of the refracting angle of the prism, as in the figure, but they will instead suffer internal reflexion.

For glass this critical angle is 42°, so that we cannot make use of a glass prism of which the angle is greater than 84°.

A hollow prism whose sides are flat plates of glass, or some similar substance, may be filled with a refracting liquid, so that we may have fluid as well as solid prisms.

Further on we shall see how important the prism has lately become as a method of analysis; in the meantime we see how a ray of light is bent in passing through a prism.

LESSON XXX.—LENSES AND OTHER OPTICAL INSTRUMENTS.

277. Lenses.—These are formed of some transparent substance, and have generally such shapes as are given in the following figure:—

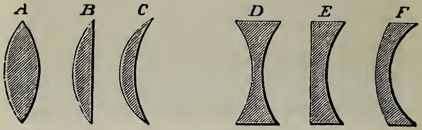


FIG. 83.

A is bounded by two spherical surfaces, and is called a *double convex* lens; B has one spherical and one plane surface, and is called a *plano-convex;* D is in like manner a *double concave,* while E is a *plano-concave;* and, finally, C is a *converging meniscus,* possessing on the whole the properties of a convex lens, while F is a *diverging meniscus* possessing on the whole the properties of a concave lens.

The first three lenses, A, B and C, are converging; that is to say, if a beam of parallel rays falls upon one of these lenses, the beam is made to converge to a focus on the other side. The other three are diverging lenses, inasmuch as they cause a beam of parallel rays to diverge.

The effects produced by lenses depend not only on their shape, but on the materials of which they are composed, and these effects will be most easily understood by referring to the action of a prism on a ray of light. From Fig. 82 it will be seen that the ray of light is bent *towards,* not *from,* the base or thickest part of a prism, and we should therefore expect that a ray of light should be bent towards the thickest part of a lens. Now it will be noticed that the lenses A, B, and C are thickest in the centre, while D, E, and F are thinnest in the centre. Hence if a beam of parallel rays (Fig. 84) falls upon a convex lens such as A, the rays will be bent towards the centre, and made to converge to some focus, F, on the other side of the lens, and this focus will be a real and not a virtual focus.

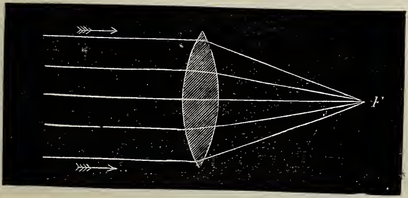


FIG. 84.

On the other hand, if a beam of parallel rays (Fig. 85) falls upon a concave lens it will diverge, as if it proceeded from a focus not real but virtual at F.

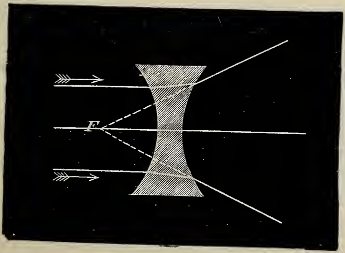


FIG. 85.

278. The most important lens is the double convex. We have already said that if a beam of parallel rays (Fig. 84) fall upon such a lens, it will be brought to a focus at some point F: this focus is called the **principal focus** of the lens, and we may denote its distance from the lens by f.

Now (Fig. 86) let a divergent beam of light strike upon a double convex lens, proceeding from a point at a distance from the lens which we shall call p; it will be bent into a focus on the other side at some distance, p', and the relation between these two distances is obtained by the following formula: $\frac{1}{p} + \frac{1}{p'} = \frac{1}{f}$, f being the distance from the lens of its principal focus.

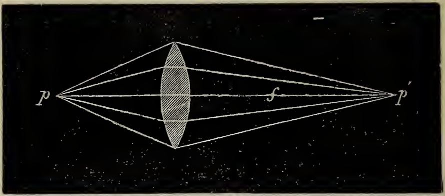


FIG. 86.

We see from this formula that if p = infinity, or if the source of light is at an infinite distance from the lens, then $\frac{1}{p'} = \frac{1}{f}$, and hence $p' = f$; that is to say, the focus becomes the principal focus. As, however, the distance p of the source of light from the lens is diminished, the distance of the focus on the other side is increased, until when the distance of the source becomes equal to that of the principal focus, or $p = f$, we have in consequence $\frac{1}{p'} = 0$; that is to say, p' is infinite, or the rays emerge from the lens parallel, and are not brought to a focus at all.

Still diminishing the distance of the source of light, if p is less than f, $\frac{1}{p'}$ is negative.

This denotes that the beam of light, after it has passed the lens, is not a convergent or even a parallel beam, but is divergent, as if it proceeded from a virtual focus on the same side of the lens as the source of light itself.

In fact the source of light is now so near the lens, and the pencil is in consequence so divergent, that the lens cannot even bend it into a parallel beam, but all it can do is to lessen its divergence. This peculiar action of a convex lens is illustrated in Fig. 87, where *p* denotes the luminous point, and *p′* the virtual focus of the refracted rays.

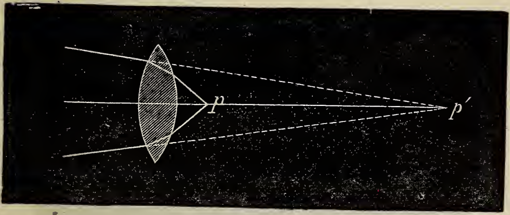


FIG. 87.

It will be noticed that *p′* is greater than *p*, which means that although the lens has not been able to render the beam convergent, it has at any rate diminished its divergence.

279. Images formed by Lenses.—We see by the following figure how to find the position and size of the image of a luminous body formed by a double convex lens :—

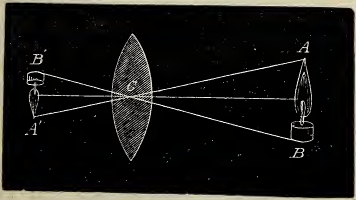


FIG. 88.

For instance, let A B denote a luminous flame further from the lens than its principal focus. Through A draw a line, A C A′, passing through C, the centre of the lens; and through B draw in like manner a line, B C B′, passing also through C. Again, let A denote the focus of the luminous point A as given by the lens, and B′ the focus of B; hence A′ B′ will be the image of the luminous flame A B. It will readily appear from the figure that this image is real and inverted, and that its size may be found as follows :—

Size of image *is to* size of object *as* A′ C is to A C. If however, the luminous flame be placed nearer the lens than its principal focus, the image, as in Fig. 89, will be a magnified, erect, and virtual image.

We thus encounter two sets of phenomena in looking through a lens. In the first place, if we use the lens in order to view an object placed behind it and nearer than its principal focus, we shall see a magnified and erect image of the object. This is the ordinary way of using a lens. If, however, the object be much further away than the principal focus, and if the eye be placed further from the lens than the image of the object, then this image will be perceived as inverted, but small. We may see this action of a lens if we view a distant landscape through it, at the same time placing the eye at a sufficient distance from the lens.

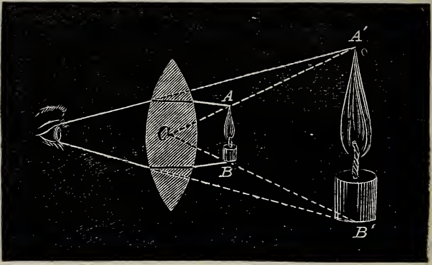


FIG. 89.

280. The chief optical instruments will now be very briefly described. Let us begin with the **Camera Obscura.** This consists of a small chamber or box blackened in the inside, and having a lens placed in front of it.

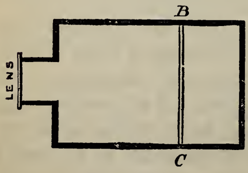


FIG. 90.

This lens receives the rays of light proceeding from the objects outside, and an image of these objects is produced in the chamber at B C. If now a piece of ground glass be placed at B C, this image will be pictured on the ground glass ;

or if a sensitive photographic plate, capable of being affected by light, be introduced instead of the ground glass, we shall be able to obtain an impression of the image upon it, and it is thus that portraits are taken by photography. It will be seen that the various objects whose images we thus obtain should be as nearly as possible at the same distance from the lens, so that all their images may all attain their greatest distinctness on the same plate, B C; for if an object be far off, its image will be brought to a focus nearer the lens than the image of an object close at hand.

281. The Eye.—The eye may be compared to a camera such as we have now described. The front part of it contains a lens, and there is a variable aperture admitting light, which is called the **pupil.** The pupil adapts itself in size according to the amount of light; if the field around be very luminous the pupil contracts, so as to diminish the light which enters the eye; but if there be little light it dilates. Now, just as in the camera an image of the objects in front is formed on a plate, so in the eye an image of the surrounding objects is formed on a membrane in the back part of the eye called the **retina,** and this is connected with a nerve called the **optic nerve,** which finally conveys the impression to the brain.

The eye has a considerable power of adjustment for different distances, so that if it be viewing a near object, the image of which is thrown exactly on the retina, and if then all at once a distant object be viewed, by an exercise of this power the eye is able to adjust itself so as to throw the image exactly on the retina. But sometimes when the lens of the eye is too convex, distant objects will not be distinctly seen, for the focus of their image will be in front of the retina, and only near objects will be distinctly seen; a person in such a case is called *short-sighted.* This defect is obviated by the use of spectacles formed of diverging lenses, which serve to correct the excessive convergence of the lens of the eye.

On the other hand, the lens of the eye is sometimes not convex enough, so that while the images of distant objects are thrown upon the retina, those of near objects cannot be brought to a focus within the eye. In such a case the individual is said to be *long-sighted*, and the defect is obviated by the use of convex glasses, to increase the converging power of the eye.

282. Simple Microscope.—An ordinary convex lens may be used as a means of magnifying small bodies, and its action in this respect will be seen with reference to Fig. 89. Here the eye is placed, let us say, at the point E, and the object to be viewed is placed at A B, somewhere between the lens and its principal focus; the result is that we have an enlarged virtual image of the object, as if it were placed at A′ B′, and this image will be perceived by the eye at E. This virtual image must not, however, be too near the eye, otherwise the eye will not perceive it distinctly; for experience must convince us that an object placed too close to the eye will not be distinctly seen. The virtual image ought not to be nearer the eye than about ten inches or a foot.

283. Telescope.—A telescope for observing a star or distant object consists essentially of two lenses, one an object-glass, and the other an eye-piece. Thus, in Fig. 91, let A B be the distant object which we are viewing, and O the object-glass. This glass will give an image of A B at its principal focus. Let us call this image *a b*. We have now only to view the image *a b* through a simple microscope or eye-piece, just as if it were a real body, and we shall thus obtain a virtual and magnified image A′ B′ of the distant object.

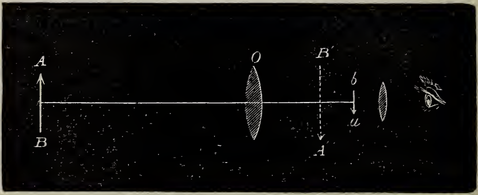


FIG. 91.

Thus the difference between the simple microscope and the telescope is, that in the former we scrutinize by a magnifier the object itself, while in the latter we scrutinize the image of the object.

LESSON XXXI. — DISPERSION OF LIGHT BY THE PRISM.

284. We have hitherto dealt with those properties which are common to all kinds of luminous rays, whether they be white or green or blue or red, but we now come to certain characteristics which enable us to distinguish between different kinds of light.

In the first place, it is necessary to know that those bodies with which we are most familiar give out light, which consists of a great many different kinds all blended together. Therefore, before we can thoroughly examine the character of the light given out by any hot substance, we must devise some means of sifting or separating these various rays from one another, so as to know how many individual rays we have, and what is the intensity of each.

We are all familiar with the magnificent display of colours exhibited by gems, when rays of light are allowed to fall upon them in a particular direction. On such occasions they sparkle with all the colours of the rainbow, and this very allusion bids us ask if the hues of the rainbow be not due to the same cause as the colours of gems. Does not the very name imply the presence in the sky of a multitude of minute spheres of water, such as would on the grass shine forth like innumerable diamonds? Are not all these displays due to the same cause; and if so, what is the cause? The discovery of it is due to Newton, who was the first to show that white light is in reality composed of a great many differently coloured rays, and that these rays are, in their passage through transparent substances, in certain cases separated from each other.

285. The prism gives us the means of separating the variously coloured constituents of a compound ray from one another.

Suppose, for instance, that we have a narrow vertical slit in the shutter of a dark room, through which white sunlight is allowed to pass. Fig. 92 will represent a ground plan of this arrangement; while the prism employed is represented in elevation in Fig. 92*a*.

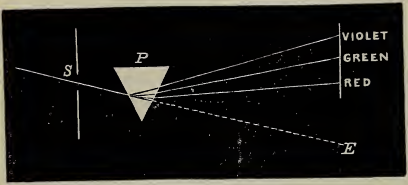


FIG. 92.

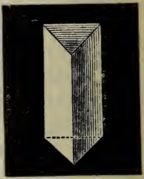


FIG 92*a*.

If we look towards the slit from E without a prism, we shall see it lit up in the usual manner; in fact it will serve as an opening through which we may see the sun beyond. Let us now, however, interpose a vertical glass prism between our eye and the slit. When we have done so, the slit will no longer be visible. If, however, we place our eye somewhere above E (as regards the diagram) we shall now see the light from the slit. But it will not reach us in the shape of a luminous slit as formerly, but will appear as a broad band or ribbon of light of many colours, beginning with red at the one end, and passing gradually and successively through orange, yellow, green, blue, and indigo, to violet at the other extremity.

286. All this may be very easily explained. In the first place, we have already remarked (Art. 276) that rays of light are bent in their passage through the prism, so that the eye, which was formerly placed at E, must now be placed above it in order to see the slit. If all the rays were equally bent by the prism we should still see the slit as before, the only change being one of direction; but the great importance of the experiment consists in the fact that all the rays are not equally bent as they pass through the prism, but rays of one colour are bent differently from those of another colour. This different bending of the various rays is termed their **dispersion.**

If the ray be a red one, it will emerge from the prism bent to a certain extent; if a yellow ray, it will be somewhat more bent; if green, still more, and so on.

If the light which streams through the slit be compounded of many-coloured rays, all the various compounds of this light may be viewed separately by means of the peculiar dispersive action of the prism which we have now described.

If, however, the slit be wide, its position due to one ray will overlap that due to another ray, and the result will be a certain blending together of different rays, and consequent imperfection of the method as far as regards separation of the various constituents of the light of the slit. It is therefore of great importance in all such investigations to make use of a very narrow slit.

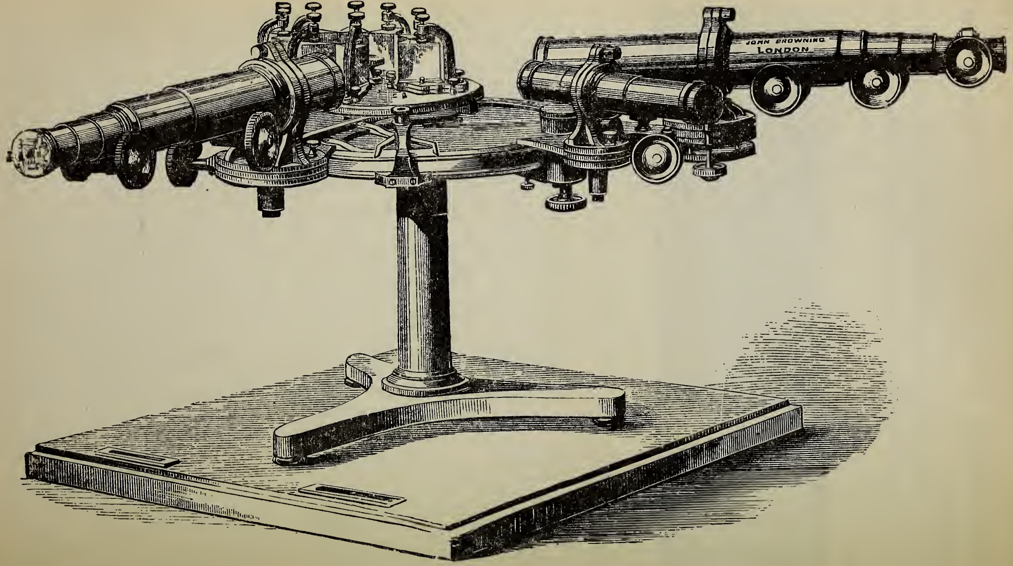
It ought also to be borne in mind that the rays of light which have been separated to a certain extent by one prism will be still more separated by a second one, applied in a proper manner behind the first, and that we shall by this means obtain increased separation for each additional prism which we use.

287. An instrument furnished with prisms for analysing rays of light is called a **Spectroscope,** and the following plate gives a representation of a very powerful instrument of this description belonging to the late Mr. J. P. Gassiot.

The slit to be illuminated by the light we wish to analyse is at the extreme left, the telescope to which it is attached being called the collimator, and its object being to reduce the divergent rays from the slit into parallelism before they fall upon the train of prisms, as it is only in this state of things that we shall get a good result.

The telescope to the right is an ordinary magnifying telescope, intended to magnify the various images of the slit due to the various constituent rays that have passed through the train of prisms.

By an instrument such as this, we may transform the light of the sun illuminating the slit into a broad, many-coloured



Mr. Gassiot's Spectroscope.

ribbon, 10 or 11 feet long—so long, in fact, that only a small part of it can be seen in the telescope at once.

288. But to complete the proof that white light is in reality composed of a number of differently coloured lights blended together, it is necessary not only to decompose a beam of white light into its constituents, but to recombine these constituents once more into white light.

This may be done in several ways. Suppose, for instance, that instead of one prism, we use two prisms turned opposite ways, as in Fig. 93. The first of these prisms will separate the white light into its constituents, and the second will reunite these into white light, so that after the light has passed the combination, it will be a beam similar to that which entered, and pursuing also a parallel path.

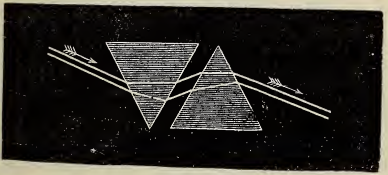


FIG. 93.

But the various colours of the spectrum may be combined so as to form white light without the aid of any optical instrument. For instance, we may take a circular disc, of which the sectors or spaces proceeding from the centre to the circumference are coloured according to the different colours of the spectrum of white light, and in the proper proportion, one being red, another orange, another yellow, a fourth blue, and so on. Now cause this disc to rotate very rapidly, and it will appear to the eye as white, the reason being that, owing to the rotation, the various colours pass so rapidly before the eye that they are blended together, and the impression received is that due to their joint effect; that is to say, the disc will appear white.

LESSON XXXII.—THERMO-PILE.

289. Here it may be asked, how are we to compare together the intensity of light of different colours? and the difficulty of this comparison is magnified if we reflect that there are some rays which are absolutely invisible to the eye, while their heating influence is nevertheless very powerful. Now, how are we to compare together the intensity of a ray of light and of a ray of dark heat? In order to answer this question, let us suppose that the rays of light which we wish to compare are received upon a black screen, which absorbs or stops all kinds of radiant light and heat, and neither reflects back any rays, nor allows any to pass through its substance. What becomes, then, of the energy of these rays, and into what is this converted? We answer, that it is entirely transmuted into absorbed heat; the rays, in fact, are wholly spent in heating the surface upon which they have fallen, and the amount of this heating effect is a true measure of the energy or intensity of these rays, whether they be visible or invisible, coloured or white. If this heating effect be very marked, we may measure it by means of a thermometer, using for the purpose the differential thermometer already described (Art. 171).

290. But it is desirable to have a much more delicate method of measuring heating effect than this. Now this desideratum is supplied by the **thermo-pile**, the principle of which was first discovered by Seebeck.

In this instrument it is an electric current which produces the result, and we must anticipate so far as to explain that when a circuit composed of two different metals soldered together has one of its junctions heated, an electric current will be produced. We ought likewise to state that a magnetized needle will always, if free to move, place itself at right angles to an electric current. Thus in Fig. 94, let B represent a plate of bismuth, and C C C a plate of copper soldered to the bismuth, while *n s* is a magnetic needle delicately swung, and so placed as when at rest to lie along the circuit in the direction of its length. Now heat the junction by a spirit-lamp, and in virtue of the electric current which the heating gives rise to, the marked or north pole of the needle will be pushed forward as in the figure.

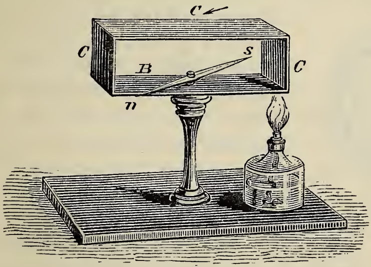


Fig. 94.

291. Now, since our present object is to obtain a very delicate instrument wherewith to measure radiant heat, we must first of all obtain as strong a current as possible ; secondly, we must render it as effective as possible in turning a magnetic needle ; and thirdly, we must have an arrangement by which the smallest motion of the needle may be rendered visible.

To produce a strong current we solder together a number of pieces of antimony and bismuth, as in Fig. 95 ; and if the upper junctions of this arrangement be heated, we have the united effect of the positive currents at all the hot junctions passing through the circuit in the direction of the arrow-heads.

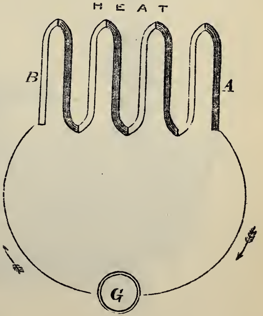


Fig. 95.

But in order to utilize this current we must have a galvanometer in the circuit. The best galvanometer for the purpose is that of Sir W. Thomson. In it we have, as in Fig. 96, a very small magnet attached to the back of a small circular flat mirror, magnet and mirror being delicately suspended by a very fine thread. This arrangement is surrounded by a number of circles of the wire which conveys the current; in fact, the wire of Fig. 95 may be supposed to be left loose, so as to be capable of being wrapped many times round the suspension frame of the mirror and needle, taking care to insulate the various folds from each other. Now, if a current pass through this coil, it will tend to make the needle lie at right angles to the plane of the coil; but, on the other hand, the magnetic attraction of the earth will tend to prevent the needle moving; there will thus be a strife between the force of the current and the attraction of the earth, and the result will be that the needle will only move a small distance before it will be stopped.

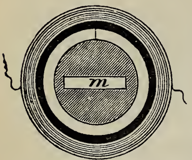


FIG. 96.

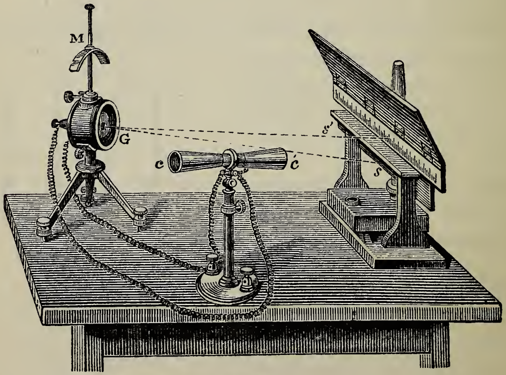


FIG. 97.[1]

The magnetic force of the earth is, however, overcome by means of a large magnet M (Fig. 97), so placed as to counteract the force of the earth upon the small needle *m*, and in consequence of this arrangement a very small current will cause the needle to move through a very large arc. Again, any small motion of the needle and mirror is very much magnified by the optical arrangement which is shown in the figure. Light reaches the mirror *m*, which is placed in the centre of G, from a luminous slit *s*, and after passing through a small lens in the galvanometer this light is reflected back from the mirror, so as to throw on a graduated screen an image (*s*′) of the luminous slit. Now it is evident that a very small change of angle in the mirror will throw the image upon quite a different part of the screen, and by this means we shall be able to detect the smallest change in the position of the mirror.

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[1] This figure is inserted by the kind permission of the late Mr. C. Becker, of Messrs. Elliott Brothers, London.

292. By all these means the current is first of all made as strong as possible; next, the needle is removed as completely as possible out of the directive influence of the earth's magnetism; and lastly, its motions are very greatly magnified. Generally twenty-five bismuth and antimony junctions are used as the source of the current, forming a square surface of about a quarter of an inch each way. These junctions are blackened with lamp-black, so as to absorb all kinds of rays, and upon these blackened junctions the radiant heat is allowed to fall; and this arrangement is often provided with a cone polished in the inside, which catches a larger supply of rays, and reflects them inwards to the sensitive surface.

In Fig. 97 the sensitive pile is furnished on each surface with such a cone, and if a radiating body be placed before *c*, the needle will be driven in one direction; but if before *c*, it will be driven in the opposite direction. The reason of this is very obvious, for the current depends upon the one set of junctions being hotter than the other. If, therefore, the same source of heat be put at the same time before *c* and *c*′, there will be no motion of the needle, since the two sides of the pile will be equally heated; in other words, the needle will be driven in one direction by means of a hot body placed before *c*, and in the opposite direction by means of a hot body placed before *c*′.

293. It will be seen from this how, by means of the pile, we can measure with great delicacy and exactness the heating effect of a beam of radiant heat, whether luminous or non-luminous. Suppose, for instance, that when a source of radiant heat is placed before the pile, the luminous slit is made to move on the screen through twenty divisions of the scale; suppose, again, that when a different source of heat is presented to the pile the index moves over forty divisions, then we should say that the heating effect of the second source was double that of the first; in fine, the heating effect will be proportional to the number of divisions which the index travels over.

294. By the joint aid of the spectroscope and the pile we can analyse a beam of any kind of light. Suppose, for instance, we have a narrow slit lit up by the sun's light. By means of a lens we should be able to throw an image of this luminous slit upon a screen; but if we add a train of prisms we shall, instead of a single bright image, throw upon the screen a long band of variously coloured light, which is the solar spectrum, and by means of which we can tell what kinds of rays are present in the light which comes from the sun.

If we now place a thermo-pile at the different parts of the spectrum, we shall be enabled by this means to estimate the heating effects of all the different rays, so that we shall know not only the different rays that go to make up sunlight, but the proportions in which these various rays are mixed together. There is, however, one weak point in this arrangement, for we cannot be sure that our lenses and train of prisms do not absorb some rays more than others, so that after all we do not learn by means of the spectroscope and pile the true proportion in which the various rays appear in the light which we are examining, for we may suppose that some of the rays are altogether stopped by the lenses and prisms, while others are allowed to pass.

It will therefore be convenient, before we proceed further, to find how various substances behave with respect to different kinds of rays.

295. Dark Heat.—Leslie was one of the first to make experiments on dark heat, and he showed that it possessed many of the properties of light. His source of heat was a cubic vessel containing hot water, and he found that the *dark rays* proceeding from it *were reflected by metals in a manner similar to light-giving rays.* Melloni afterwards employed the thermo-pile, and by its means largely extended our knowledge of these invisible rays. He found that glass, water, alum, and very many of those substances which are transparent for luminous rays, do not allow dark rays to pass; and, on the other hand, he found that rock-salt is a substance which allows all kinds of rays to pass with nearly equal facility. It will, however, be seen further on that even rock-salt absorbs certain rays.

Having found the transparency for heat-rays, or **diathermancy,** as this is termed, of rock-salt, he made a piece of this substance into a lens, and found that by this means the rays of dark heat, after passing through the lens, were brought to a focus after the manner of light rays, from which he concluded that *dark heat is capable of refraction.*

The manner in which his experiment was performed is illustrated by the following figure, in which we see the rays,

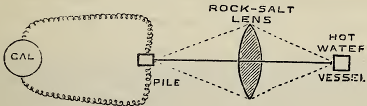

FIG. 98.

after passing through the rock-salt lens, brought to a focus at the sensitive pile; an electric current is thus produced proportional to the amount of dark heat falling on the pile, and this current is read by means of a galvanometer.

If the galvanometer be very sensitive, it will be similar to that in Fig. 97; but it is unnecessary here to repeat the details of the instrument.

Forbes was the first to show that *dark heat is capable of polarization* in the same manner as light, and the labours of other physicists have only served to extend and confirm the likeness between dark rays and luminous ones. It was also shown by Forbes that dark heat is somewhat less refrangible than light.

If, therefore, we have a spectrum[1] in which the violet and blue rays lie to the right, and the red rays to the left, dark rays will be found beyond the red to the extreme left.

296. We have thus briefly attempted to show that dark heat possesses all the physical properties of light, except that it is somewhat less refrangible than light, and is hardly able to pass through glass or water, or the great majority of transparent substances, rock-salt being an exception.

Now it is evident from what has been said, that if we wish to study the spectrum of a heated body, we must employ lenses and prisms of rock-salt, because otherwise a large amount of dark heat will be absorbed.

Supposing, now, that we have a heated strip of coal or carbon of which we wish to obtain the spectrum. By means of a rock-salt lens and prisms, we may throw upon a screen the true spectrum of the rays which issue from this carbon, and they will form (Art. 285) a long ribbon composed of rays of different refrangibilities, the ray of one refrangibility being separated from that of another refrangibility.

Now in order to find how much heat we have of some particular refrangibility, let us take our thermo-pile, and narrowing its sensitive part sufficiently, place it at the various parts of this spectrum, and then read on the galvanometer the heating effect experienced by the pile; we shall thus know, not only what kinds of rays are given out by the heated strip of carbon, but how much heat there is of these various kinds: in fact, we shall know all about the light and heat which the carbon gives out.

297. Let us begin by heating our strip of carbon to a heat below redness: the spectrum will then entirely consist of dark rays wholly to the left of the visible spectrum.

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[1] The frontispiece engraving, giving the spectrum of the sun, stars, and nebulæ, is due to the kindness of Mr. J. N. Lockyer. It will be observed that in it the red rays are towards the right, while the chemical rays are towards the left, their positions being the reverse of what we have supposed them to be in the text.

As the temperature rises, the spectrum of the carbon gradually extends itself towards the red, so that at last, when a low red heat is reached, we have a few red rays along with a much greater number of invisible rays.

As the temperature still continues to rise, in addition to the red rays a few orange and yellow rays are given out, until, when a glowing heat is reached, we have, in addition to the dark rays, most of the colours of the solar spectrum : indeed, the spectrum of the electric light is very similar to that of the sun. But at a very high temperature not only do the rays enter the visible spectrum from the left, but they shoot, as it were, beyond it to the right, a few rays being given out of great refrangibility, lying to the extreme right of the visible spectrum. These rays are equally invisible to the eye with those heat-rays that lie to the left, but they are very different in other respects. They are called chemical or **actinic** rays, and have the power of decomposing chloride of silver ; in fact, they are the rays that are of service in photography. Therefore, if we were to receive the spectrum of the sun or of the electric light upon a photographic plate, we should find that the greatest blackening effect would lie towards the right of the visible spectrum, while there would be none at all for the rays of dark heat towards the left. The properties of these actinic rays and their distribution by the sun over the earth have been extensively investigated by Professor Roscoe, of Manchester, and also by Professor Bunsen, of Heidelberg.

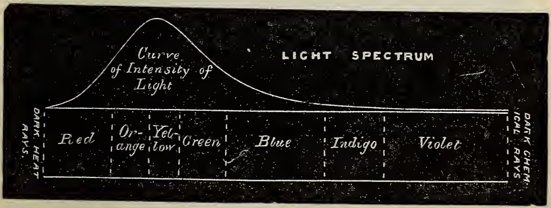

FIG. 99.

298. Thus at a low temperature we have from a black body, such as a strip of carbon, a number of rays of dark heat, but no luminous or actinic rays. As the temperature rises, we have along with a preponderance of dark rays, a few luminous ones of the less refrangible sort, such as the red. As the temperature still continues to rise we have, in addition to the dark rays, a similar proportion of the various rays of the visible spectrum, and a still smaller proportion of the chemical invisible rays which lie to the right.

299. In Fig. 99 we have a representation of the sun's visible spectrum, showing the comparative luminosity at different parts, while in Fig. 100 we have the same spectrum as given by a rock-salt prism, showing the heating effect of the various rays. We see from this that while there is most luminous effect, *as far as the eye is concerned*, about the yellow, yet there is greatest heating effect or true energy of radiation beyond the visible spectrum to the left, while the chemical rays represent in intensity but a very small fraction of the whole effect.

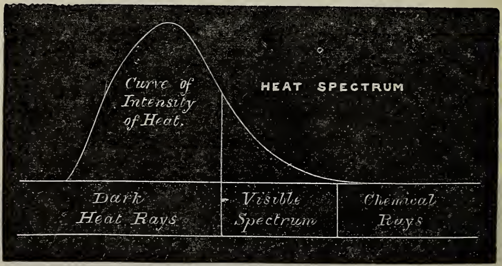

FIG. 100.

In fact, the eye is a very partial judge of the energy represented by a ray, for it is necessary that the rays of which it judges should be able to penetrate to the retina ; but it is questionable whether some of the rays of small refrangibility are able to pierce the eye at all.

Another noteworthy point is, that *the spectrum of carbon is a continuous one.* Thus we get, at a sufficiently high temperature, a continuous band of light,—that is to say, the spectrum does not stop short and commence again, but goes on without interruption; and in this respect carbon is a representative of liquid and solid bodies of which the spectra are generally continuous, like that of carbon.

300. The spectra of gases are very different from those of solid bodies, for they are discontinuous, consisting of one or more bright lines on a dark ground.

Thus the spectrum of ignited sodium vapour (see frontispiece) consists of two bright yellow rays very near one another in spectral position, forming what is called the double line D. In like manner the spectrum of thallium consists almost entirely of one intensely blue line. The light from burning sodium is as nearly as possible mono-chromatic, and all things seen by its light are either black or yellow, for a coloured body will not appear in its true colour unless those colours are present in the light by which it is viewed.

It forms a striking proof of this to put a little bit of metallic sodium into an iron spoon, and heat it over a spirit-flame in a dark room, when it will soon take fire and burn, and everything in the room will either appear black or of a ghastly yellow colour.

We are enabled by means of electricity to obtain even the most refrangible substances in a state of vapour: thus, for instance, when the electric spark passes from iron into the air, the flash seen consists of a few particles of highly-heated iron vapour; and the same holds for other metals. Now we can analyse these sparks by means of the spectroscope, and thus tell the nature of the light which they emit, and we find that they all give a discontinuous spectrum.

In like manner the spectra of the elementary gases are discontinuous, from which we see that bodies in a state of vapour are very different from solids and liquids, as respects the light which they give out.

LESSON XXXIII.—RADIATION AND ABSORPTION.

301. We have hitherto chiefly confined our remarks to the radiation of carbon at different temperatures ; and taking that substance as the type of solids, and comparing its radiation with that from incandescent gases, we have found a very great and striking difference between the two classes of spectra, that of carbon being continuous, while those of gases are discontinuous. We shall now endeavour to connect the radiative properties of bodies with their absorptive properties.

Let us begin with the temperature of boiling water, or 100° Cent. Let us now, therefore, suppose that we have a large thermometer at this temperature hung up in a room having the temperature of melting ice. The thermometer will lose heat in two ways; by *convection* on account of the air which surrounds it, and which is continually carried off and renewed, and also by *radiation*. But in order to confine our thoughts to the latter process, let us suppose that the chamber is a vacuum. Now, in the first place, let the outside of the glass bulb of the thermometer be coated with a thin coating of polished silver, and let us ascertain how much heat it radiates in one minute. Next let the bulb be coated with lamp-black, the same experiment being repeated, that is to say, the thermometer at 100° C. being allowed to cool for one minute in a vacuum chamber of 0°. It will be found that the bulb now radiates in a minute very much more heat than it did when coated with silver. Next, let the glass bulb be left uncovered, and the thermometer will still be found to radiate almost as much as when the bulb was covered with lamp-black. Finally, let it be covered with white paper, and its radiation will still be found to be almost equally great. We are thus entitled to say that at 100° C. a blackened surface, or one of glass or white paper, radiates much more than a surface of polished silver, and we may thus construct a table of the comparative radiating powers of bodies heated to 100° C., at the top of which we may put a lamp-black surface, a surface of glass, and one of white paper, and much lower down one of silver, which is a very bad radiator. Our table of radiating substances for heat of low temperature will therefore stand thus :—

Good radiators . . .	Lamp-black surface.
	Glass ,,
	White paper ,,
Bad radiator	Polished silver ,,

302. Suppose now that the thermometer is at 0°, and is carried into a vacuum chamber of the temperature of 100°, this being the reverse of the previous process ; in the first place, let the bulb, as before, be coated on the outside with a coating of silver ; it will absorb a certain quantity of heat in one minute ; observe how much. Next, blacken the bulb with lamp-black, repeat the experiment, and measure the absorption which takes place in one minute as before ; the absorbing power of the thermometer will now be considerably increased. Again, if the coating be entirely removed, and nothing left above the glass bulb, it will be found that the absorbing power of the glass bulb is almost as great as that of the blackened bulb, and the same result will be obtained if the bulb be covered with white paper. We may thus construct a table of the comparative absorbing powers of various bodies for heat of 100°, at the head of which we may place a lamp-black surface, a surface of glass, and one of white paper, and much further down one of polished silver.

Our table of bodies which absorb heat of 100° will therefore stand thus :—

Good absorbents . .	Lamp-black surface.
	Glass ,,
	White paper ,,
Bad absorbents . .	Polished silver ,,

303. If these two tables, the one of radiators and the other of absorbents, be now compared together, they will be found to be identical ; so that the blackened thermometer at 100° will, in the first case, cool much more rapidly than the silvered one when transferred to the chamber at 0°, on account of its superior radiation, and will also, in the second case, starting from 0°, become heated much more rapidly than the silvered one when transferred to a chamber of 100°. In fine, *good radiators are also good absorbents, bad radiators bad absorbents.* It is worthy of remark, before proceeding further, that surfaces behave very differently in their absorbing power for different rays. White paper and glass, as we have seen, are both very strong absorbents of low temperature heat, while both of them are manifestly non-absorbents of luminous rays.

Extending these considerations to visible rays proceeding from bodies of high temperature, they furnish us with some very interesting and instructive experiments, which will now be described.

Experiment I.—Take a porcelain plate of black and white pattern (the black of the pattern will of course be a strong absorbent of luminous rays, while the white will be a less powerful absorbent). Heat it to a good red or white heat in the fire, and when so heated, take it out and rapidly carry it to a dark place ; the black will appear much more brilliant than the white, presenting a very curious reversal of the pattern.

Experiment II.—Take a piece of polished platinum foil and make an ink-mark upon it. Bring this foil to a red heat with the flame of a Bunsen's burner in a dark room, and the ink-mark will shine out much more brightly than the polished platinum.

Experiment III.—Make a white mark on a black poker with a piece of chalk ; when heated to a good red heat, examine it in the dark, and the chalk will shine out less brightly than the rest of the poker.

These experiments might be multiplied indefinitely, all tending to show that bodies which, when cold, are good absorbents, are, when hot, good radiators ; and the observation may be extended to plates of various thicknesses as well as to mere surfaces. Thus a polished plate of rock-salt absorbs very little heat of low temperature (Art. 295), and when heated to 100° C. it is found also to give out very little heat. In like manner a piece of transparent colourless glass which absorbs very little light, will, when heated in the fire, and quickly examined in the dark, be found to give out very little light; while, on the other hand, a piece of opaque glass treated in the same way will give out a great deal of light.

In like manner a film or stratum of air is well known to absorb little light or heat of any kind, and so when heated it hardly gives out any.

We may now generalize our conclusions by the statement, that *opaque and non-reflecting solid or liquid particles are at once good radiators and good absorbents for most kinds of rays; while, on the other hand, polished metallic surfaces, and more especially films of gas, such as air, absorb and radiate very little either of light or heat.*

304. It has been mentioned incidentally that surfaces or plates do not behave in the same manner with regard to different kinds of rays; let us now dwell at greater length on this point, for it is perhaps the most important of the whole subject. White paper, it was seen, was a strong absorbent for heat of low temperature, while it is evidently not so for luminous rays, for the very reason that it appears white.

In like manner the uncovered glass of the bulb of a thermometer was found to be a strong absorbent for low temperature heat, but it is evidently not so for luminous rays. To prove this, we have only to hold a thermometer in the sun, and we shall be dazzled with the light reflected from its bright mercurial surface, which only reaches the eye after it has twice passed through the glass.

Even within the limits of the visible spectrum we have, as a common occurrence, substances which absorb certain rays and allow others to pass. For what is it that makes the leaves of plants appear green? Is it not that they absorb all the various constituents of sunlight except the green, which they allow to be reflected? In fine, all coloured substances are substances which behave in a partial manner with respect to the visible rays, and if we had no such partial absorption we should be deprived of one great source of beauty in nature.

Coloured glasses afford a very familiar illustration of this selective or partial absorption. A green glass absorbs all, or nearly all, the red rays which fall upon it, allowing the green to pass ; on the other hand, a red glass absorbs nearly all the green, and allows the red to pass. Thus we see that surfaces or plates which behave in one way with respect to dark rays, may behave differently with regard to luminous ones ; nay further, a substance that behaves in one way with regard to a luminous ray of one colour, may behave in a different way with regard to a luminous ray of another colour. We have stated generally that good radiators are good absorbents ; but in view of the fact that bodies select or choose the rays which they absorb, this statement must be extended, and we now assert that *bodies when cold absorb the same kind of rays that they give out when heated.* It will be desirable to give, in the first place, some experimental proofs of this before attempting to explain the principles upon which the statement rests.

305. *Experiment* I.—Rock-salt when heated to 100° gives out that peculiar kind of heat which is greedily absorbed by a cold plate of rock-salt. To prove this, heat a thin plate of rock-salt to 100°, and allow the heat from it to fall upon an appropriate instrument for measuring such heat, but only after it has passed through a cold plate of the same material; now this cold plate will be found to have stopped at least three-quarters of the heat which falls upon it, while it will only stop a very small percentage of any other kind of heat.

Experiment II.—Red glass stops the green rays. Now heat a piece of ruby-coloured glass to a white-heat in the fire ; if examined in the dark it will be found to give out a greenish light, being the same sort of light that it absorbs. Next heat a piece of green or blue glass, which absorbs red rays, and its light, when viewed in the dark, will be found to be particularly red, being, as before, the kind of light which it absorbs when cold.

Experiment III.—Make a spectrum of the electric light after the method already described, and hold burning sodium between the electric lamp and the slit ; it will be found to produce a comparatively dark band in the spectrum. Next stop the electric discharge while the sodium is still burning; the same band will now appear luminous; that is to say, the sodium which, being comparatively cold when compared to the temperature of the electric light, stops one of its rays, gives out when heated this very ray on its own account. All these experiments tend to show, as a matter of fact, that bodies when cold, or comparatively so, absorb the same rays which they give out when heated.

306. Our readers must now permit us to transport them in imagination to a white-hot chamber, kept uniformly at this temperature; such, for instance, as one of those chambers in which glass vessels are annealed. We will suppose it to be shut in closely with walls on all sides, with the exception of a small opening through which we can either introduce anything into the chamber, or if we choose, see what is going on inside.

Let us introduce polished platinum marked with ink, or coal, or black and white porcelain, or red glass, or green glass, or transparent glass, or black glass. When left sufficiently long, until they have acquired the temperature of the walls of the chamber, if we look in through the small hole, we shall see no apparent difference between the light coming from these various substances and that from the walls of the chamber; in fact, everything will appear to be of the same uniform white-heat. If, however, we hastily withdraw these various substances, and, without allowing them time to cool, examine them in the dark, we shall find, as already mentioned, a great variety in the appearances which they present; the colourless glass and the polished platinum will give out very little light, the coal and the black of the porcelain a great deal.

These two facts may be reconciled with one another in the following manner. Let us take the transparent glass; this gives out very little light on its own account, but, on the other hand, it stops very little of that which reaches the eye from the white-hot wall behind it, being eminently transparent for such light. If we suppose that the rays from the wall are as much recruited by the light given out by the glass on its own account, as they are absorbed by its substance, then we shall have an explanation of the fact that the combined radiation of the glass and the wall is no greater than that of the wall itself had there been no glass there. The polished platinum, in like manner, gives out little light on its own account, but when in the white-hot chamber it reflects copiously the light which reaches it from the walls, so that, to an observer viewing it through the small opening, it will have so completely supplemented its deficient radiation by its great reflexion, that altogether it will appear equally bright with the wall itself.

Applying this explanation to the various substances which we have introduced into the white-hot chamber, we see at once why they cause no change in the intensity of the light that reaches the eye placed at the opening; for although, no doubt, it is only in the case of the black substance, such as coal or black porcelain, that all the light comes from the substance itself: yet in the other case, what the substance wants in radiating power it makes up by allowing to pass either through its substance, as in the case of transparent glass, or from its surface, as in the case of polished platinum and white porcelain, what is deficient in its own radiation.

307. But let us now further consider for a moment the red and green glass which we have introduced into this chamber. As we view them from the opening we are at a loss to distinguish which is the red and which is the green, they have so absolutely and entirely lost their colour. Nor have we far to seek for an explanation of this. The red glass absorbs the whitish or greenish rays from the heated chamber behind it, but in return it gives out on its own account an equal amount of rays, and these of precisely the same kind as it has absorbed, so that the light from the wall behind, in passing through the glass, is just as much recruited as it is absorbed, and this equality holds for every individual kind of ray which goes to compose this light, and thus it happens that the combined radiation of the wall and the red glass is precisely the same both in quantity and in quality as if there were no glass.

The same principles apply to the green glass. It absorbs the reddish rays from the wall, but it gives out an equivalent both in quality and quantity for the rays which it absorbs, so that the absorption is virtually cancelled, and the combined result of wall and green glass is, as before, the same as if there were no glass. We thus see that all substances of all kinds, when placed in a room of uniform temperature and allowed to remain until they have attained the temperature of the enclosure, will absorb just as much as they give out; and that this equality between absorption and radiation will hold good for every individual ray of which the heterogeneous radiation of the heated walls is composed. (By individual rays, we mean the various rays into which the whole radiation may be split up by means of the spectroscope.) All that we have now said has been built upon the hypothesis that the substances are in an enclosure, let us say a white-hot one, of the same temperature as themselves, and if we cannot easily command such a field of white heat, yet the centre of a good fire is a very near approximation; and if we introduce into such a fire a number of pieces of variously coloured glass, and exclude from the room all sunlight or gaslight, we shall find their colour vanish when once they have reached the temperature of the fire.

308. Again, it ought to be borne in mind that such bodies as glass lose their characteristic radiating peculiarities only when they remain in such an enclosure, for when taken out of it and viewed in the dark, they resume those peculiarities; thus the colourless glass gives out very little light, the coal and black porcelain a great deal. Indeed, it is only the light from a black body that represents by itself the brightness of the enclosure, and such a body, when taken out and hastily examined in the dark, without allowing it time to cool, will be found to give out rays having a brightness in all respects the same as that of the enclosure in which it was placed, because, being opaque and non-reflective, all the light which it gave out in the enclosure was proper to itself, none having passed through its substance or been reflected from its surface ; it therefore retains this light when taken into the dark, provided its temperature is not in the meantime allowed to fall.

309. Thus we have arrived at the conclusion that the heat from a heated black body represents truly the whole heat due to the temperature of that body, so that were we to heat a piece of coal or black porcelain to the temperature say of 2,000°, we should obtain from it the maximum amount of heat and light which any substance at that temperature could possibly give out ; and not only so, but if we separate from each other by means of a spectroscope the individual rays given out by a black body, any one of these individual rays will represent in intensity the greatest possible amount of light of this description that can be given out by a body at 2,000°.

Viewing, therefore, a black body as the standard or typical radiator, we derive through its means a very simple method of knowing whether or not one body is hotter or colder than another. Taking a white-hot black body, such as the coal in a fire, let us place between it and the eyes a burning light ; now if the combined rays from the coal and the light are more intense than the adjacent light from the coal alone, we may be certain that the light is of a higher temperature than the coal—if less intense, then we may be sure that the light is of a lower temperature than the coal.

We shall improve the accuracy of our determination by employing a spectroscope, in order to analyse the rays from the coal. If we put before the slit of our spectroscope the flame in question, and if in consequence any part of that broad band of variously coloured light which denotes the spectrum of the fire be increased in brilliancy by the flame, then we may be sure that the flame is hotter than the fire ; and if any portion of this broad band be diminished in brilliancy, then we may be sure that the flame is colder than the fire.

Suppose, for instance, that we wish to compare together the temperature of the fire and that of some incandescent sodium vapour. We should, in such a case, place the incandescent sodium vapour between the fire and the slit of our spectroscope. If the sodium vapour were of a higher temperature than the fire, we should see the double line D (Art. 305) brighter than the rest in the midst of a spectrum otherwise continuous, but if it were of a lower temperature, as, for instance, if the sodium vapour were slightly heated in a sealed tube, we should see the double line D darker than the rest in the midst of the continuous spectrum.

310. Suppose now that we were previously ignorant of the chemical nature of the substance which we had placed before the fire, we should at once recognise what it was by the position of these dark lines. Were it sodium, we should have two dark lines corresponding in position to the double line D : were it hydrogen, we should have its own appropriate dark lines ; and we should find that the lines due to one gas always differ in position from those due to another. Kirchhoff, a distinguished German philosopher, has applied these principles with great success in determining the substances which exist in the sun and stars, and he has been followed in this country by Huggins and Lockyer.

311. If we throw upon the slit of our spectroscope an image of the sun or of one of the stars, with the view of obtaining its spectrum, we find a large number of black or dark lines in a spectrum otherwise continuous, and we argue from this that in the sun or stars we start with a solid or liquid substance, or at any rate with some substance which gives us a continuous spectrum, and that between this and the eye we have, forming a solar or stellar atmosphere, a layer of gases or vapours of a comparatively low temperature, each of which produces its appropriate spectral lines, only dark on account of the temperature of the vapours being lower than that of the substance which gives the continuous spectrum. It thus becomes a point of great interest to know whether these black lines correspond in position with the bright spectral lines given out by known terrestrial substances in the state of vapour.

On inquiry we find that they do so, and that we have present in the sun in the state of vapour the following substances amongst others :—sodium, iron, nickel, calcium, magnesium, barium, copper, zinc ; while in Aldebaran, Messrs. Huggins and Miller have detected the presence of sodium, magnesium, hydrogen, calcium, iron, bismuth, lithium, antimony, and mercury, and other elements in other stars.

312. Other Phenomena of Absorption.—We have seen how absorption may become the means of our detecting the nature of the substances which exist in the sun and stars, and we can only now allude very briefly to other phenomena connected with this subject.

313. Absorption of Gases for Dark Heat.—Professor Tyndall has investigated the absorption of various gases for dark heat, and has derived from his experiments the following results :—

COMPARATIVE ABSORPTION OF VARIOUS GASES, EACH OF THE PRESSURE OF ONE INCH.

Air	1	Carbonic oxide . .	750
Oxygen	1	Nitric oxide . . .	1590
Nitrogen	1	Nitrous oxide . . .	1860
Hydrogen	1	Sulphide of hydrogen	2100
Chlorine	60	Ammonia	7260
Bromine	160	Olefiant gas	7950
Hydrobromic acid .	1005	Sulphurous acid . .	8800

From this we may conclude that the absorption of the elementary gases for dark heat is less than that of the compound gases, and we might therefore expect that the absorption of the atmospheric air for heat of any kind should be very small. Nevertheless this conclusion would not be correct ; for Tyndall has shown that the aqueous vapour which is always present in the atmosphere absorbs a very large amount of dark heat, while it allows the rays of the sun to pass with scarcely any diminution.

The result is, that we have nearly the full effect of the sun's rays in heating the earth, but once the earth has been heated this terrestrial heat cannot easily pass out through the aqueous vapour of the atmosphere into empty space, but as it consists of dark rays it is stopped thereby ; thus the aqueous vapour acts like a trap in allowing the sun's rays to pass in and heat the earth, while it prevents the heat of the earth from passing outwards into space. The earth's surface is by this means kept much hotter than it would otherwise be.

314. The laws of radiation explain the deposition of **dew.** In a clear still night the leaves of plants, which are good radiators, give out a great deal of their heat into the upper regions of the atmosphere and into space, and become thereby colder, cooling also the particles of air in contact with them. These particles at last reach a temperature at which they can no longer retain their aqueous vapour, but must deposit it on the leaves ; and this is the origin of dew. Dew is not deposited in a cloudy night, because the leaves get back from the clouds nearly as much heat as they give out ; in fine, it is necessary for the deposition of dew that there should be a free outlook into space, so that the cooling process of radiation should go on without compensation and without interruption.

315. Phosphorescence and Fluorescence.—If we take a tube containing powdered sulphide of calcium, or sulphide of strontium, expose it to the sun's rays, and afterwards view it in the dark, it will be found to emit light for several hours. The luminosity is probably due to some modification in the molecular state of the body which is caused by the sun's rays. The same thing may be observed in many diamonds, in fluor spar, arragonite, chalk, heavy spar, and other minerals, the luminosity in some cases lasting a long time, but in other cases disappearing in a few seconds : this exhibition of light is called **phosphorescence.**

A similar phenomenon happens in certain liquids. Thus if the sun's rays be allowed to strike on a solution of quinine, which is really a colourless transparent liquid, we find that it exhibits a beautiful blue colour, which disappears, however, when the light is withdrawn : this phenomenon has been called **fluorescence.**

Professor Stokes has successfully explained these two phenomena. It appears that when certain rays of light fall upon phosphorescent or fluorescent substances, a change is caused, and the rays are transmuted into others, nearly always of lower refrangibility. It also appears that this is more particularly the case with chemical rays, or those rays of great refrangibility beyond the visible spectrum, so that when such rays fall upon a solution of quinine they are lowered into blue rays, and they thus become visible.

Here, then, we have a means of rendering visible those rays of the solar spectrum beyond the violet, for we have only to throw the spectrum upon a screen washed with a solution of sulphate of quinine, and the screen will be rendered fluorescent, and shine out with a blue lustre far beyond the violet or visible extremity of the spectrum.

The only difference between phosphorescence and fluorescence is one of duration; in the former the effect continues for some time, while in the latter it vanishes as soon as the exciting source of light is withdrawn.

LESSON XXXIV.—ON THE NATURE OF RADIANT ENERGY.

316. All are agreed that a ray of light is a species of energy, that is to say, it represents a motion of some kind; but until recently there have been two different theories regarding the nature of that motion which constitutes light.

Newton led the way in supposing that light consists of exceedingly small particles projected from a luminous body with enormous velocity, while Huyghens supposed it to consist in undulations of an exceedingly rare medium pervading space, and called **ether** or the ethereal medium.

The explanation of the various phenomena given by the last, or undulatory theory, is very much better than that afforded by the Newtonian theory of emission, so that gradually this theory has become obsolete. It has, however, been very difficult to devise experiments which might serve as a crucial test between the rival hypotheses. Such a test has at length been found.

According to the theory of emissions, the velocity of light ought to be greater in water than in vacuo, while according to the theory of undulations it ought to be less. Now Fizeau, who determined the velocity of light by means of a revolving apparatus, determined also that its velocity is less in water than in vacuo. The verdict of this experiment is thus in favour of the undulatory theory.

317. There is a very striking analogy between light and sound that tends in the same direction. Thus we find that a body when *cold absorbs* or *stops* the same *ray* that it gives out when *hot* (Art. 304).

We also see (Art. 160) that a string when at *rest absorbs* or *stops* the same *note* that it gives out when struck, and the analogy is so striking between this behaviour of bodies for sound and light, that we are tempted to believe that light must be a motion similar to sound, that is to say, undulatory, and consisting of various wave-lengths.

318. Assuming, therefore, that light consists of undulations, how can we distinguish between rays of various wave-lengths? We have seen how in sound a difference of wave-length is perceived by the ear; now, in light, how is a difference of wave-length perceived by the eye? We reply, that *colour is for light what pitch is for sound*, and we have evidence that the wave-length corresponding to the red of the spectrum is considerably greater than that of the blue or violet; thus a red ray corresponds to a low note, and a blue or violet ray to an acute one. In fact, the separation accomplished by the spectroscope is in reality the splitting up of a compound beam of light into its constituent wave-lengths.

319. Let us now take those well-known phenomena of light, reflexion and refraction, and show how they can be explained by the undulatory hypothesis; but first let us define a little more fully than we have already done what is meant by the **front** of a wave.

Often at the seaside, where there is a long unbroken beach, we see a wave crest approaching us parallel with the shore. This crest may extend for a considerable distance, and one part of it has the same appearance as another: that is to say, throughout the whole length of this crest the particles have been thrown by the agitation into the same sort of figure at the same moment; they are all, in fact, similarly affected as regards the wave motion.

Such particles form the front of a wave. Generalizing, we may say that the front of a wave consists of all those particles that are in the same phase (Art. 136) or position at the same time; thus the various fronts of the surface waves produced by dropping a stone into the water will consist of circles, while the various fronts of the sound waves proceeding from an explosion in mid-air will consist of spherical surfaces.

In general the wave proceeds in a direction perpendicular to its front.

320. Now, by the help of this conception we may easily explain the leading phenomena of reflexion and refraction by means of the undulatory theory.

Thus let a set of parallel rays (Fig. 101), of which *a b c* represent a front, impinge upon the surface M M, and let us consider the state of things at the moment when the disturbance at *c* has reached the surface at *d*. The ray at *a* has some time since reached the surface and has been reflected in the direction *af*, and in like manner the ray at *b′* has been reflected in the direction *b′ e*, so that *f e d* is the front of the reflected wave. Now, had there been no reflecting surface the front *a b c* would in a given time have advanced through a certain space, keeping parallel to itself, each disturbed point moving onwards at a uniform rate; but as there is a reflecting surface, its front cannot retain its parallelism; nevertheless the reflected front will take up such a position that the disturbed points at *a*, *b*, and *c* will have travelled equal distances in order to form themselves into the new positions, *f*, *e*, and *d*. That is to say, *af*, or the journey of

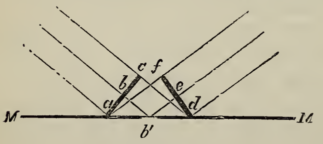

FIG. 101.

a, will be equal to $b b' + b' e$, or the journey of b, and to $c d$, or the journey of c.

But since $af = c d$, and since the angles at c and f are right angles, the fronts being perpendicular to the rays, it is evident that the two triangles $a c d$ and $a f d$ are equal in all respects, and hence the angle $c d a$ = angle $f a d$, from which we see at once that the angle of reflexion is equal to that of incidence.

In fact, we may compare the advancing front $a b c$ to a line of soldiers marching at right angles to their front in the direction $b b'$. At M M they meet with an oblique obstacle which they cannot surmount; but nevertheless each man continues to march with the same velocity as before, but in such a direction as to fall into rank without stoppage, presenting a new front $f e d$, after which the march goes on as before in the direction $b' e$, perpendicular to the front.

321. So much for the reflected ray. In the case of refraction we have slightly to modify our conception. Here the line of soldiers may be supposed able to penetrate the surface, but only with a diminished velocity of march. They have, as it were, got into heavy ground. Thus in Fig. 102, let $a b$ be the advancing front, and M M the boundary of the new ground. It is evident that b is behind a in reaching the new ground by a distance $b b'$. Now a, when reaching the new ground, marches through it in some direction $a c$, with let us say only half the previous velocity, and hence when b has reached the new ground at b', a will have marched over a space $a c$, equal to $\frac{1}{2} b b'$; $b' c$ will in fact represent the new front into which the line will have formed themselves, if we suppose them, after having entered the heavy ground, to march with only half their previous velocity, and in such a direction as to fall into

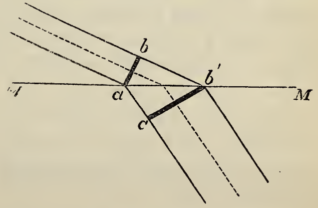

FIG. 102.

rank without stoppage, and they will now continue their march in a direction $a\,c$, at right angles to the front $b'\,c$.

Hence the construction is obvious: $a\,b\,b'$ and $a\,c\,b'$ are both right-angled triangles, but the side $a\,c$ is only equal to one half of $b\,b'$.

Now $b\,b' = a\,b' \sin b\,a\,b'$, and $a\,c = a\,b' \sin c\,b'\,a$: hence $\frac{\sin b\,a\,b'}{\sin c\,b'\,a} = \frac{b\,b'}{a\,c} = \frac{2}{1}$. But $b\,a\,b'$, or the angle which the front makes with the surface, is evidently the same as that which the ray (perpendicular to the front) makes with the normal (perpendicular to the surface); $b\,a\,b'$ is in fact equal to the angle of incidence, and $c\,b'\,a$ is also equal to the angle of refraction.

We thus see that the sine of the angle of incidence divided by that of the angle of refraction, represents, according to the undulatory theory, the relation between the velocity of the ray before and its velocity after entering the surface; and this is what is meant by the index of refraction, so that in the instance just quoted the index of refraction is evidently $= 2$, since the velocity was supposed to be diminished in the ratio of 2 to 1.

322. Therefore, according to the undulatory theory, the velocity of light is less in glass than in vacuo. Indeed we may suppose that the ethereal medium in glass and similar bodies is trammelled to some extent by ordinary matter, so that there is more work to do without more force to do it. For this reason a wave of sound travels more slowly in carbonic acid than in air, and for the very same reason a ray of light may be supposed to travel more slowly in glass than in air.

Now when a ray of light proceeding in vacuo strikes a surface of glass, the motion of the unloaded ether is communicated to the loaded ether of the glass, and we may compare this to a series of small elastic bodies striking a series of large ones. In such a case two things will happen: in the first place, the small bodies will rebound back; and secondly, the large bodies will be pushed forward. Now this is precisely what takes place when a ray of light strikes a polished glass surface; part of the light is reflected back, and part is transmitted through the glass, and thus reflexion and refraction accompany each other.

323. We are apt to imagine that light differs from sound in a fundamental respect, for if a volume of sound be allowed to enter a room from an opening, the sound which enters is not only heard in front of the opening, but considerably to one side; if, however, a flood of light be allowed to enter the same opening, the sides will cast a well-defined shadow, past which the light will not be seen.

Now the reason why sound-shadows are not in general so well marked as light-shadows depends on the circumstance that the waves of sound are much larger than those of light. A wave of sound may be several feet in length, while a wave of light is only $\frac{1}{50000}$ of an inch long.

But if the aperture be sufficiently small, we have an apparent bending of the light-waves, just as we have in the sound-waves; and furthermore, the extent of this bending will be different for rays of different colours, inasmuch as these represent rays of different wave-length.

Thus we obtain a beautiful display of colours by looking at white light through a series of gratings; and again, if a very small opaque circular disc be placed between the source of light and the eye there will be seen a bright spot in the middle of its shadow, and this will be surrounded by a series of coloured rings.

By varying the nature of the experiment, we have often beautifully coloured appearances, which are quite in accordance with the undulatory theory; nevertheless the precise appearance presented can only be foretold by a series of difficult calculations.

324. Interference.—The general explanation of the appearances is, however, very simple. Suppose that we have two waves travelling in the same direction, and that the crest of the one wave coincides in position with the crest of the other, and the hollow of the one wave with the hollow of the other, then these two waves will supplement each other, and unite to produce one wave of a double amplitude.

But if the two waves do not coincide, but if the crest of the one corresponds to the hollow of the other, they will destroy each other, and there will be no light.

Now something of this kind takes place in the phenomena of gratings; and where the waves supplement each other we have a bright spot, and where they destroy each other we have darkness.

325. Newton's Rings.—The same principle may be applied to explain Newton's rings.

To produce these, let us take a piece of plane glass, and lay upon it a lens of small curvature (Fig. 103).

Now if this arrangement be viewed lying on the table, there is first of all a reflected ray, which comes from the lower surface of the lens; and secondly, we shall have a reflected ray from the upper surface of the plane glass on which the lens rests. Also at a certain distance from the centre, all round the point of contact, these two reflected rays, as they travel together to the eye, will have the crest of the one coinciding with the trough of the other, and their effect will be darkness, or we shall have a black ring.

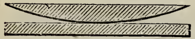

FIG. 103.

A little further out, the distance between the two surfaces being greater, the one wave will be a whole wave-length before the other, and hence the two crests will again coincide; they will therefore supplement one another, and we shall have a bright ring.

Further from the centre the crest of the one wave will again correspond with the trough of the other, and we shall have a dark ring, and so on. The appearance presented to the eye will therefore be a series of rings dark and bright alternately (Fig. 104), and this is the phenomenon known as Newton's rings.

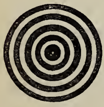

FIG. 104.

326. Colours of Thin Plates.—By a similar method we can explain the colours of thin plates, such as those of a soap-bubble; for in such a case we have the rays of light reflected from the two surfaces of the film interfering with one another, and cancelling or supplementing one another, as the case may be. The effect is, however, different for different wave-lengths, so that while rays of certain wave-length cancel one another, those of another wave-length are allowed to pass, and by this means we have often a magnificent display of colour.

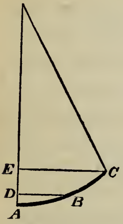

FIG. 105.

327. Objection to the preceding Explanation. — The oscillation of a vibrating particle has been compared to that of a pendulum. Now if the displacement of the pendulum from its lowest point A (Fig. 105) be doubled, or if A C = 2 A B, we have, by a well-known proposition in geometry (provided the displacements are small), A E = 4 A D; that is to say, the pendulum vibrating through the arc B A falls from B to A through a vertical distance D A, while that vibrating through the arc C A = 2 B A falls from C to A through a vertical distance equal to 4 D A. Now the energy of the oscillation is represented by the vertical distance through which the pendulum falls; hence we see that if we double the amplitude of a vibration we increase its energy four times.

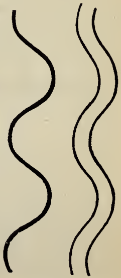

FIG. 106.

Suppose, now, we have two similar waves moving in the same direction, as in Fig. 106. We have said that they will supplement one another, and form one wave of double the amplitude, as in the figure; but by doubling the amplitude of the oscillation, shall we not increase the energy four times, as in the case of the pendulum, which represents oscillatory movements generally?

Can we therefore, consistently with the laws of the conservation of energy, imagine two rays of unit energy to unite so as to form one ray of which the energy is four units?

Let us now consider two rays that are travelling in the same direction, but so that the crest of the one fits into the hollow of the other.

The result, as we have seen, will be a destruction of motion. Now suppose that each of the waves represents unit of energy. Can we, consistently with the laws of the conservation of energy, imagine two rays of unit energy to unite so as to have their energy entirely cancelled ?

We reply, that if either of these two phenomena occurred alone, the laws of energy would present a formidable obstacle to this conception of light. Thus, if two rays of unit energy were to unite into one ray, having an energy equal to four units, without any other compensation ; or if two rays of unit energy were to cancel one another without any other compensation, we might justly imagine that the laws of energy had been broken.

The case is, however, completely altered if we bear in mind that the two phenomena always occur together ; that is to say, if we have two rays of unit energy combining into a ray of energy equal to four units, we have at the same time, and side by side with it, other two rays of unit energy cancelling each other.

There is thus, on the whole, neither a creation nor a destruction of energy, but merely a displacement, and thus the apparent objection to the undulatory theory derived from the laws of energy is entirely removed.

328. Alteration of Wave-length by Motion of Radiating Body.—The reader may have noticed when in a railway station, that if an engine approaches the station at a rapid rate, and whistles at the same time, the note is different as it approaches the station and as it recedes on the other side, being shriller in the first case than in the second.

The reason of this is very obvious, if we bear in mind that the whistle consists in a number of impulses that are rapidly communicated to the air one after another by the engine, and that the note or wave-length consists in the distance between one such impulse and the next. When the engine is approaching the station it gives an impulse to the air, which impulse is propagated in the air towards the station with the usual velocity of sound. But the engine has already advanced some distance in the same direction before it gives the next impulse, therefore the distance between the two impulses will be less than if the engine were at rest, and the note will therefore be shriller.

On the other hand, when the engine is leaving the station, it gives an impulse to the air, which impulse is propagated to the station with the usual velocity, but the engine has already moved some distance in the contrary direction before it gives the next impulse, and the consequence is that the distance between two impulses will now be greater than if the engine were at rest; that is to say, the sound will be more grave.

Thus, when a sounding body is rapidly approaching the ear its note is rendered more acute, while if it be receding from the ear its note becomes more grave; we might therefore expect that when a luminous body is approaching the eye, there will be a general decrease in the wave-length of its light, and that when it is receding from the eye there will be a general increase of wave-length. But in order that this change may be perceptible, the rate of approach or recession of the body must bear a sensible proportion to the velocity of light; in other words, the body must be moving at the rate of at least several miles per second. Now, it is only in the heavenly bodies that we can look for such velocities. Let us therefore suppose that we have brought upon the slit of our spectroscope the image of a star or planet, in the spectrum of which there is an absorption band corresponding to the double line D. If this star be not in motion either towards or from the eye, the position in the spectrum of these absorption lines should agree precisely with that of the bright lines formed by burning incandescent sodium before the slit of the spectroscope; but if the star be moving towards the eye, these absorption lines ought to be slightly displaced towards the most refrangible end of the spectrum, which is that of smallest wave-length. In like manner, if the star be receding from the eye, the absorption lines ought to be displaced towards the red or least refrangible portion of the spectrum. Mr. Huggins has by this means been able to make out the proper motion of several stars in a direction to and from the eye ; and more recently Mr. Lockyer has been able by the same means to detect violent convection currents in the sun's atmosphere (Art. 226).

LESSON XXXV.—POLARIZATION OF LIGHT. CONNEXION BETWEEN RADIANT ENERGY AND THE OTHER FORMS OF ENERGY.

329. It thus appears that we have strong evidence in favour of the undulatory theory of light, but we do not yet know of what particular kind of wave motion a ray of light consists. It may either consist of transversal vibrations (Art. 134), in which the direction of displacement is perpendicular to that of the wave motion, as in a wavelet produced by throwing a stone into water, or of vibrations in the direction of the wave motion similar to those of sound.

It will easily be seen that there is a very marked difference between these two kinds of vibrations. Let us, for the sake of illustration, take a long string, extending horizontally between two points, and strike it rapidly with a vertical stroke, we shall then perceive a wave consisting of a vertical displacement propagated rapidly from one end to the other of the string. Let us now strike it on one side with a horizontal stroke, and we shall see a similar wave consisting of a horizontal displacement propagated rapidly in the same direction.

In both cases the displacement is perpendicular to the direction of motion, but the one is in a horizontal and the other in a vertical plane. A transversal undulation is thus capable of assuming a particular side, or bias, or direction.

Now in a wave of condensation and rarefaction, such as that of sound, there is evidently no capability of assuming a particular side or bias of this kind. This is expressed by saying that a transversal wave is capable of **polarization,** while a wave of condensation and rarefaction is incapable of it.

330. Next suppose that we strike the string of which we have spoken with a vertical stroke, and that we likewise make it to pass between the vertical plates of a frame (Fig. 107), it is clear that these vertical plates will not prevent the vibration taking place; we may, in fact, place a great number of such frames in the path of the wave without interfering with its progress. If, however, we place another frame with horizontal plates (Fig. 108), so as to have one plate on each side of the vibrating string, it is evident that the arrangement will now tend to check the vertical undulation; and if we have a great many such frames, even although the string does not when at rest touch the plates, yet the progress of a vertical wave may be completely checked thereby.

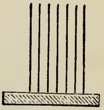
FIG. 107.

Suppose now that we cause a horizontal wave to pass along the string. This wave will be stopped by the frame with vertical plates, or the same which allowed a vertical wave to pass, while it will be unaffected by the frame with horizontal plates, or the same which stopped a vertical wave; in fine, the one frame will stop the vertical wave and the other the horizontal. Now suppose that a mixture of vertical and horizontal waves are being propagated along the string; if we insert in their path a series of frames with horizontal plates we shall obstruct the vertical parts or components of these waves, and if we insert frames with vertical plates we shall stop the horizontal components, and if we insert both kinds of frames we shall stop all motion.

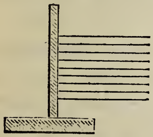
FIG. 108.

331. Polarization by Tourmaline.—A similar phenomenon takes place in rays of light. A ray of ordinary sunlight, proceeding, let us say, in a horizontal line, would seem to consist of transversal waves, and not waves of condensation and rarefaction; but there would be as many vibrations in one plane as in another—in fact the rays would consist of an impartial mixture of horizontal and vertical vibrations, all, however, taking place at right angles to the direction of propagation. Furthermore, there are certain substances which admit of the progress through them of a ray of light of which the vibrations all take place in one plane, while, however, they stop nearly all rays consisting of vibrations in a plane at right angles to the first. Thus let us cut a couple of slices from a gem called tourmaline in a direction parallel to the axis of the crystal, then let us place these two slices together with the axis of each vertical, and in the path of a horizontal ray of sunlight, and we shall find that the ray of sunlight will pass, although with diminished intensity, through the two slices. Suppose, however, that we now turn the one slice round upon the other until the axis of the one is at right angles to that of the other, the one axis being horizontal and the other vertical, then we shall find that the light will no longer be able to penetrate the combination, but we shall have total darkness ; the one slice, let us say, stops all those rays of which the vibrations are horizontal, and the other slice all those with vertical vibrations, and thus between them both they will stop all light, just as a combination of the two frames will stop all vibration in the string.

Now the only possible explanation of this behaviour is *that a ray of light is capable of assuming a side or bias, in which case it must consist of transversal vibrations, and not of vibrations of condensation and rarefaction.* This explanation is due to Young, who along with Fresnel has greatly increased our knowledge of the nature of light.

332. Polarization by Reflexion.—We have just now seen that a ray of light is polarized by being made to pass through a plate of tourmaline cut parallel to the axis, by which we mean that it is only those rays of light the vibrations of which are in a particular direction that are allowed to pass. Polarization is likewise produced by causing a beam of light to be reflected from a surface of glass or water, or any similar substance, at a particular angle for every such substance.

For glass, the ray must make with the surface the angle 35° 25′, and the reflected ray is then polarized in the plane of reflexion.

It is imagined that in this reflected ray the vibrations are all in a direction perpendicular to the plane of reflexion, so that that portion of the incident ray consisting of vibrations in the plane of reflexion has not been reflected at all. If, therefore, we allow an ordinary ray of light first to be reflected from glass at an angle with the surface equal to 35° 25′, and if the reflected ray be again made to impinge upon another surface of glass at the same angle, while however the plane of incidence in the second case is perpendicular to that in the first, we shall have no reflected ray in the latter case at all. For in the first case those vibrations which were in the plane of reflexion were not given out in the reflected ray, which consisted entirely of vibrations perpendicular to this plane. But vibrations *perpendicular* to the first plane of incidence will be *in* the second plane of incidence, which is at right angles to the first, and therefore they will not be reflected from the second surface.

Viewing an ordinary ray of light (see Fig. 102) as composed in equal proportions of light for which the vibrations are in the plane of the paper, and of light for which the vibrations are perpendicular to the plane of the paper, it can be easily shown without entering more deeply into the subject, that the mechanical concomitants of reflexion and refraction, say from a surface of glass, must be different for these two components. For (the plane of the paper being the plane of incidence) it is clear that the direction of vibration for that component which vibrates in a direction perpendicular to the plane of the paper is not altered either in reflexion or refraction, whereas, on the other hand, for that component which vibrates in the plane of the paper the direction of vibration will be different in the incident, the reflected, and the refracted rays.

It is thus easily conceivable that this mechanical difference may in certain instances so act as to cause the reflected ray to be confined entirely to one of these two components,—in other words, to be polarized.

333. Polarization by Double Refraction.—We have seen that a crystal of tourmaline cut parallel to the axis has sides, and that it stops all that portion of a beam of light consisting of vibrations in a particular direction, while it allows to pass that portion consisting of vibrations perpendicular to this direction.

A crystal of Iceland spar does not do this, but there is in it a particular direction or axis, and it trammels those light vibrations which take place in the direction of the axis differently from those which take place in directions at right angles to it, and hence the one set of rays passes through the crystal with a different velocity from the other set.

But the bending of a ray of light by a substance (Art. 321) depends on its change of velocity in passing through the substance. Hence, in a crystal of Iceland spar, the part of a ray consisting of vibrations in the direction of the axis will be differently bent from that part consisting of vibrations perpendicular to the axis; there will therefore be two rays: that is to say, a ray of ordinary light in entering a piece of Iceland spar will generally be split into two, which will travel through the crystal with different velocities. This is what is meant by double refraction.

If, therefore, we look at any small substance through a piece of Iceland spar, we shall generally see two images of it. Now in these two images the light is oppositely polarized.

We cannot here enter more minutely into this subject; suffice it to say that there are many wonderful and beautiful phenomena presented by polarized light, all of which may be explained by the theory which supposes a ray of light to consist of transversal vibrations, or vibrations at right angles to the direction of propagation.

334. Connexion between Radiant Energy and the other forms of Energy.—We have seen that a heated substance parts with some of its energy of heat into space, and that this heat then assumes the form of radiant energy and travels through space with a very great velocity. Ordinary or absorbed heat may thus be converted into radiant energy; and on the other hand, radiant energy may be reconverted into absorbed heat. Thus, when the sun's rays strike upon a black substance they are absorbed, and their energy is spent in heating the substance.

There is, however, no direct transmutation of radiant energy into mechanical effect, or of mechanical effect into radiant energy, but only a transmutation through the medium of absorbed heat. Thus we may use the sun's rays to heat the boiler of a heat engine, from which we may thus obtain mechanical effect, but the radiant energy from the sun must first have been transmuted into absorbed heat.

In like manner, by percussion or friction, a substance may be rendered incandescent, and the mechanical energy of a blow be made to produce radiant light and heat, but the first step of the process is the transmutation of the mechanical energy into absorbed heat, and the second, that of the absorbed heat into the form of radiant energy.

The connexion between radiant energy and the other forms of energy, will be afterwards treated of when these varieties are discussed.

CHAPTER VII.

ELECTRICAL SEPARATION.

LESSON XXXVI.—DEVELOPMENT OF ELECTRICITY.

335. IT was known as early as six centuries B.C. that when amber is rubbed with silk it attracts light bodies, and Dr. Gilbert, in the sixteenth century, showed that many other substances, such as sulphur, sealing-wax, and glass, possess similar properties.

From this very small beginning our knowledge of these phenomena has of late years vastly increased, and we know that this attractive power manifested by rubbed bodies is the result of the development of an agent which we term electricity (from the Greek word ἤλεκτρον, *amber*).

336. Conductors and Insulators.—Suppose we have a metal rod with a glass stem, and rub the glass with a piece of flannel, the glass will in consequence have the power of attracting light bodies, but only at that place where it has been rubbed. Thus the property which the glass has acquired has not the power of spreading itself over its surface. Now we may by various means communicate the same properties to the metal rod to which the glass is attached, and we shall find that the influence has spread over the whole surface of the metal, and is not localized as in the former case. This fact we express by saying that glass is an insulator, or non-conductor of electricity, and that a metal is a conductor of it.

Accordingly bodies have been divided into two classes, as far as electricity is concerned, and the following table exhibits the place in which they stand :—

Conductors.	Non-Conductors or Insulators.
Metals.	Ice.
Charcoal.	Caoutchouc.
Graphite.	Dry air.
Acids.	Silk.
Water.	Glass.
Animals.	Wax.
Soluble salts.	Sulphur and resins.
	Amber.
	Shellac.

The transition from the first to the second class of bodies is not altogether abrupt, but the worst kinds of conductors are to some extent insulators, and even the very best conductor presents some resistance to the passage of electricity.

On the other hand, various circumstances may render a body a conductor; thus glass heated to a red heat is a conductor, although when cold, glass does not conduct.

It is likewise of great importance to make all our experiments on electricity in a dry atmosphere; because, if the air be charged with vapour, particles of water will be deposited on the surface of the apparatus employed, tending to conduct away the electricity.

Indeed the necessity to us of insulators in the study of electricity will readily appear, for if all bodies were conductors, it is evident that we could not confine electricity so as to perform any experiments upon it.

337. Electricity is of two kinds.—Let us suspend a small pith ball by means of a silk thread to a glass support, as in Fig. 109, forming what is called an **electrical pendulum.** Let us now, in the first place, rub a glass rod with silk, and then make the glass rod so rubbed touch the pith ball. After contact has taken place, the pith ball will be repelled by the glass rod. If now we rub a stick of sealing-wax with a piece of flannel or cloth, and bring it near the pith ball, the latter will be attracted to the excited sealing-wax.

Thus a pith ball touched with excited glass will be afterwards repelled by excited glass, but attracted by excited sealing-wax.

In like manner, if we had touched the pith ball with excited sealing-wax instead of excited glass, it would thereafter have been repelled by excited sealing-wax, but attracted by excited glass.

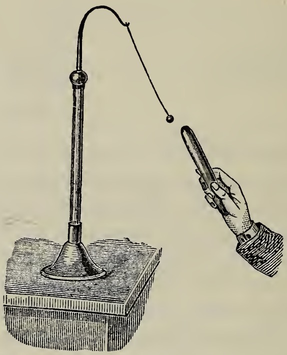

Fig. 109.

This experiment shows us that there are two kinds of electricity, namely, that which we obtain from excited glass, and which we may call **vitreous**, and that from excited sealing-wax, which we may call **resinous** electricity. We see also that when the pith ball has been touched by excited glass, and thus receives part of its electricity, it is thereafter repelled by the glass, and in like manner if it be touched by excited sealing-wax it is thereafter repelled by the sealing-wax, and hence we conclude that *bodies charged with similar electricities repel one another.*

On the other hand, the pith ball, if charged with excited glass, will be attracted to excited sealing-wax, and if charged with excited sealing-wax it will be attracted to excited glass, and hence we conclude that *bodies charged with opposite electricities attract one another.*

338. The Hypothesis of two Fluids.—It is not meant here to speculate upon the nature of electricity, but for convenience' sake we may regard it as a fluid of which there are two opposite kinds, vitreous and resinous, or, as they are more frequently called, **positive** and **negative.** According to this hypothesis every substance may be supposed to contain an indefinite quantity of these two electricities mixed together, and neutralizing one another. By various means the two fluids may be separated the one from the other, but whenever we have a certain amount of positive electricity, there must be somewhere else just as much negative electricity. Therefore in rubbing together two bodies such as sealing-wax and a piece of cloth we do not produce a certain quantity of negative electricity *by itself*, but the sealing-wax becomes negatively electrified, while the cloth on which it is rubbed becomes positively electrified.

In the following table each substance will be positively electrified if rubbed by any substance that follows it in the list, but negatively if rubbed by any substance that precedes it :—

1. Cat's skin
2. Flannel
3. Glass
4. Cotton
5. Silk
6. Wood
7. Shellac
8. Resin
9. Metals
10. Sulphur
11. Caoutchouc
12. Gutta-percha

339. Other modes of developing Electrical Separation.—There are other methods of developing electricity besides that produced by rubbing two bodies together, for it has been noticed that when heterogeneous substances are pressed together, and then suddenly separated from each other, electrical excitement is frequently produced. It has also been noticed by Becquerel that cleavage frequently produces electrical separation, as for instance when two plates of mica are rapidly torn from each other.

Volta was the first to suppose that electrical separation is produced by the contact of heterogeneous metals, and the truth of this has lately been demonstrated by Sir W. Thomson. We shall return to this when we come to treat of the electric current. In all these cases there is a heterogeneity or difference between the two substances or portions of the same substance between which electricity is produced, and it is believed that if two absolutely similar substances were brought together, and then separated or rubbed against each other, we should not be able to obtain any electrical separation.

We must also bear in mind that electrical separation requires energy, so that when an electrical machine is in good action, part of the work spent in turning it is converted into heat, and part into electrical separation.

In fact, in some way or another we must always have spent energy before we can produce electrical separation.

Besides the strictly mechanical sources of electricity there are certain minerals which, when heated, exhibit electrical properties, in which case they are said to be **pyroelectric.**

Tourmaline is a mineral of this kind. It it not the absolute temperature, but only the change of temperature that renders tourmaline electric. Thus if a tourmaline be in a room of any temperature, and if it is completely in temperature equilibrium with this room, it will not be electric. But if taken into a colder medium, it acquires two contrary electric poles, which however vanish when the tourmaline has acquired the new temperature. If it now be brought into the original hotter medium, the tourmaline again acquires electrical poles, but in the opposite direction. Suppose we call these two poles A and B. When taken from the hotter medium into the colder, let us suppose that the pole A is positive and B negative, then when taken from the colder medium and transferred once more to the hotter medium, A will become negative and B positive. It may be asked, what species of energy is spent in this case to produce the electrical separation? to which the reply is, that heat is spent; a small portion of the heat has in fact vanished in producing this separation, and will again appear in the shape of heat when we have recombined the two electricities.

LESSON XXXVII.—MEASUREMENT OF ELECTRICITY.

340. We have seen that there are two kinds of electricity, and that the one never appears without the other. We have also seen that like electricities repel, while unlike attract each other. But to perfect our knowledge of the subject it will be necessary to devise some method of measuring electricity, and of estimating electrical forces.

We have spoken of electricity as a thing which can be conceived of as separate, but we must bear in mind that it is never found disassociated from matter. Our method of determining the quantity of free electricity in a body must therefore be different from that of determining the quantity of matter in the body.

Let us begin by supposing that we have two similar metallic balls, each insulated by a glass stem, and the one charged with electricity, but the other uncharged. Now if we bring these two balls into contact with each other, the electric fluid will diffuse itself in equal proportions over both balls, so that when we separate them, each will have one-half of the original quantity of electricity. We thus see how a charge can be subdivided. Coulomb was the first to investigate the mutual attractions and repulsions of electrified bodies, and he did it by means of an instrument called the **torsion balance.**

It consists of a delicate horizontal needle (Fig. 110), made of some non-conducting substance, such as shellac, suspended by a very fine wire. There is a small disc of copper fixed at one extremity of this needle. There is likewise a vertical glass rod, terminated by a gilt pith ball, which passes through an aperture at *a*. This pith ball is capable of receiving an electric charge. At the bottom of the apparatus is a dish containing chloride of calcium, for keeping the air dry, since moisture would conduct away the electricity which it is wished to measure. Surrounding the cylindrical side of the instrument, at the level of the pith ball and needle, there is a graduated scale. Finally the attachment at the top of the fine thread which sustains the needle is capable of moving independently of the tube, and there is a small circle at the top which registers the angular movement which is thus communicated to the suspended thread. Now let the apparatus be so arranged that the copper disc *n* is just in contact with the pith ball *m*. Next let the rod *a m* be taken out, and let the pith ball receive a charge of electricity. The pith ball will touch *n* and communicate part of its charge to *n*, and the needle will thereafter be repelled by the pith ball, since both are charged with the same kind of electricity. The needle will therefore settle in some position where the electric repulsion tending to drive it from *m* will just balance the force of torsion tending to bring it back to *m*. Now if we suppose that the angle between *m* and *n* as read on the scale is 10°, the force of torsion being proportional to the angle (Art. 67), we may call this force 10, and hence the electric force which counteracts it may be called 10 also. But by moving the top round in the same direction as the hands of a watch, we may, by means of the force of torsion, make *n* approach *m*. Suppose that we twist the suspension round 35°, we shall find

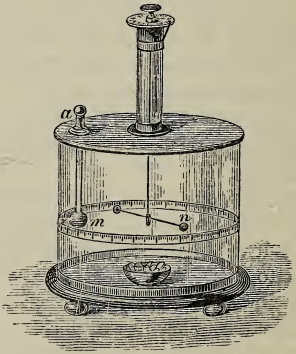

FIG. 110.

by the scale that *m* and *n* are now five degrees apart. Hence the whole torsion will now be denoted by the 35° through which we twisted round the suspension above, *plus* the 5° between *m* and *n* on the scale below; that is to say, there will be 40° of torsion in all, and hence the electric repulsive force will be measured now by 40 instead of 10. Thus, while it takes a force equal to 10 to keep *m* and *n* 10° from each other, it takes a force equal to 40 to keep them at the distance of 5°.

But 10 and 5 represent very nearly the distances of the two electric bodies from each other, so that the force of repulsion is four times as great at half the distance.

We might by a similar method charge the two balls with opposite electricities, and we should then find that when the distance is halved the attraction is increased fourfold. We are thus entitled to conclude that *the attractions or repulsions between two electrified bodies vary inversely as the square of their distance from each other.*

341. Having thus proved the law of variation with distance, the law of variation with quantity can be proved in a similar manner: for if we take out the ball *m*, and cause it to touch a similarly-sized unelectrified ball, its charge will be reduced to one-half (Art 340). If we now replace it, we shall find that the force as measured by the angle of torsion will be reduced to one-half. In this experiment *n* retains its full charge; but had we commenced the experiment with this half-charge both of *m* and *n*, we should have found the electric force only one-quarter of that due to a full charge of both *m* and *n*; that is to say, the half-charge of *m* acting on the half-charge of *n* will produce a result only one-quarter as great as before. In fine, the law of electrical force is similar to that for gravity; that is to say, *the force of electric attraction or repulsion between two electrified bodies varies directly as the product of the quantities of electricity.*

Thus, if we denote for the present occasion by unity the force exercised by unit of positive upon unit of negative electricity, at unit of distance, then the force exercised by

6 units of positive upon 4 units of negative electricity at distance 3 will be denoted by $\frac{6 \times 4}{3^2} = 2\frac{2}{3}$, and so on.

342. We have said that electricities of like kinds repel one another: the consequence is, that electricity only manifests itself on the surfaces of bodies, and would escape from them if it could; but it is prevented by the surrounding atmosphere, which is a very bad conductor.

In proof of this let us take an insulated hollow brass globe, and let it be supplied with two hemispherical brass envelopes capable of fitting tightly upon it, and having glass handles so as to admit of their being separated from the globe. Now in the first place let us electrify the hollow globe, and then let us enclose it in the brass hemispheres. If we now quickly remove the brass hemispheres we shall find them to be strongly electrified, while the sphere will have little or no electricity left in it.

It is another consequence of the repulsive force of electricity, that while on a sphere the charge is uniformly distributed, on a pointed conductor there is an increase of intensity towards the point, so that if the body be disposed to part with its electricity it will do so more rapidly if it be pointed than if it be a sphere.

If we use the term **electric density** to denote the quantity of electricity on unit of surface, we shall find that in a conductor, one part of which is spherical and another pointed, the density is very much greater at the point than at the spherical surface.

An instrument called the **proof plane** is often used to test the relative distribution of electricity over the surface of a body.

It consists of a small disc of copper foil, insulated by a glass rod. This disc is made to touch an electrified body at its various parts, and forms in fact, for the time being, the outer surface of the body at the point where it touches it. We then, by removing the disc, remove the outer surface of the body at this point along with its electricity, which we can test by means of Coulomb's balance.

LESSON XXXVIII.—ELECTRICAL INDUCTION.

343. It has been said that unlike electricities attract, while like repel one another. Now what will happen if we bring near together two insulated conductors, one charged with electricity, and the other not charged? An index to the result will be found in the statement already made (Art. 338), in which a neutral body is regarded as a reservoir of both electricities in a state of union. Suppose now that the conductor in question (Fig. 111) is charged with positive electricity: when it is brought near the neutral conductor it will decompose its fluid, attracting that of the opposite kind to itself, that is to say, negative electricity, and repelling that of a like character with itself, or positive electricity. The end of the neutral conductor next the electrified one will thus be charged with negative, and the opposite end with positive electricity. The state of things will be represented by Fig. 111.

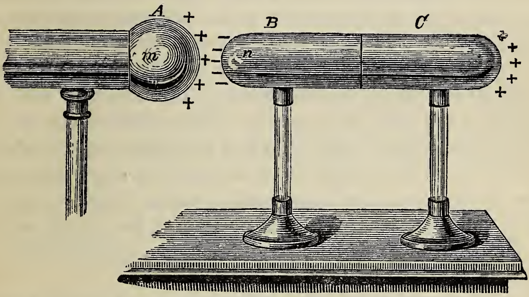

FIG. 111.

Now let us suppose that we have an arrangement by which the neutral conductor may be divided into two, and that we perform this separation, we shall find the half nearest the positively charged conductor to have a negative, and the half farthest from it a positive charge. We may prove this by the pith ball already described (Art. 337), for if we charge such a pith ball positively by touching it with a stick of glass rubbed with silk, we shall find that it will be attracted by that half of the conductor which was next *m*, while it will be repelled by that half which was farthest from *m*, showing that the first half contains negative and the latter positive electricity.

It will be noticed that the electrified conductor *m* has not parted with any of its electricity, but is the same after the experiment as before it. Let us now, however, vary the experiment by slowly bringing the conductors nearer to one another, although not into actual contact; the attraction between the positive electricity of the charged conductor and the negaiive electricity of the neutral conductor which it has decomposed will at length become so great that they will be able to surmount the resistance of the air, and will rush together in the form of a spark. The consequence will be that the conductor *m* will have lost a portion of its positive electricity, and the conductor *n* its negative electricity which *m* had decomposed and attracted towards itself; a positive charge will therefore remain in *n*: in fact, the result will be just the same as if there had been a communication of positive electricity from *m* to *n*.

The action which the charged conductor *m* exerts upon *n* at a distance is called **electrical induction.**

This influence is limited in its extent, and depends upon the distance between the two conductors.

To render this evident, let us suppose the charged conductor *m* to act upon the uncharged conductor *n*, separating the electricities as in Fig. 111.

It is clear that the negative electricity is kept at B by the attraction of the charge at A, while this attraction has to oppose the tendency which the two separated electricities in *n* have to rush together and unite.

Now if the charge at *m* is very small, or very far away, it cannot separate a very great quantity of electricity in *n*, but as the distance decreases, the amount of separation will increase, until at length, as we have already said, a spark will pass between the two conductors. The inductive effect of electricity will afford us a new explanation of the fact that electricity only shows itself at the surfaces of bodies. Thus we may regard an electrified brass globe such as that of Art. 342 as inducing an opposite electricity in the surrounding conductors, including the earth and walls of the room. The two electricities cannot however unite, because the air surrounding the globe is a non-conductor. Nevertheless the electricity of the globe will endeavour to get as near as possible to that of the surrounding exterior, and will thus appear to reside on the surface of the globe.

344. We have spoken as if the one conductor acted upon the other at a distance, but the researches of Faraday lead us to conclude that the action of induction depends upon the substance interposed between the two conductors.

If for instance the space between *m* and *n*, instead of being filled with air, had been filled with sulphur, the same charge in *m* would have separated a greater quantity of electricity in *n*.

The capacity that a substance has for causing induction when interposed between a charged and an uncharged conductor is called its **inductive capacity.**

LESSON XXXIX.—ELECTRICAL MACHINES, ETC.

345. An electrical machine is composed of two parts; first of all we have an arrangement for generating electricity; and secondly, we have one for collecting it.

One of the best known machines is that in which the electricity is produced by a large disc or plate of glass revolving on a horizontal axis (Fig. 112). The axis of the plate passes through wooden supports, and the handle which turns the machine is made of glass. As the glass plate revolves it is rubbed against by two set of rubbers, one above and the other below; these rubbers are generally made of leather stuffed with horse-hair, and press somewhat tightly against the glass. They are coated with an amalgam which is generally made of one part of zinc, one of tin, and two of mercury. These rubbers are placed in electrical communication with the ground by means of a chain.

Now, when the glass disc is turned round, positive electricity is generated in the glass, and negative electricity in the rubbers, and by means of the chain the negative electricity of the rubbers is carried to the ground as fast as it is produced. Our object is to collect the positive electricity, but get rid of the negative.

Surrounding the glass we have two brass rods armed with points, these rods being metallically united to a large metallic surface called the conductor, which is insulated by glass supports. These points act by induction on the positive electricity which we have produced by friction of the glass plate; that is to say, this positive electricity decomposes the neutral fluid of the points, attracting to itself the negative and repelling the positive into the large conductor, which is in metallic contact with the points. The positive fluid on the glass thereupon unites with the negative fluid at the points, and the result of this union is the accumulation of a quantity of positive electricity on the large conductor. (See Art. 343.)

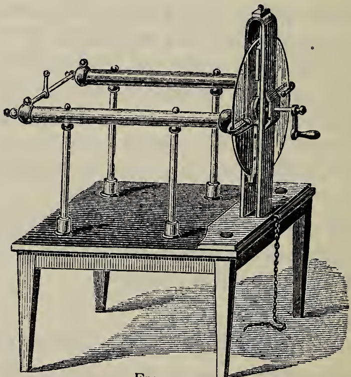

FIG. 112.

346. If when the conductor of an electric machine is charged the finger be placed near it, a spark passes between the conductor and the finger.

The positive electricity of the conductor decomposes the neutral fluid of the body, attracting the negative and repelling the positive to the ground. The negative electricity at the point of the finger thereupon combines by means of a spark with the positive fluid in the conductor, a peculiar sensation being experienced as the combination takes place.

The experiment may be varied by placing an individual on an insulating stool, which is a stool furnished with glass legs. If he now puts himself into electrical communication by means of a chain with the conductor of the machine, he becomes part of that conductor, and another person standing on the ground can take a spark from him just as he could from the conductor itself.

Another very convenient mode of generating electricity is by means of the **electrophorus.**

This instrument consists of a shallow tinned pan or mould, which is filled with resin, so that at the top we have a smooth resinous surface in a mould the outside of which is metallic, and therefore in electric communication with the earth. Besides this there is a moveable metallic cover having a glass handle (see Fig. 112*a*), which can either be brought into contact with the resinous cake or removed from it as desired.

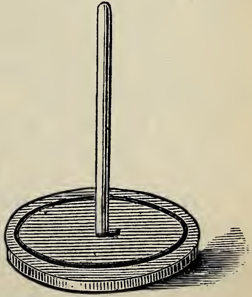

FIG. 112*a*.

When about to be used the resinous surface is smartly beaten by a cat's fur, and by this means negative electricity is developed upon it. The cover is next brought into contact with the excited resin, and the upper surface of the cover is then touched with the finger. It might at first be thought that we should by this means carry off the electricity developed on the resinous surface, but such is not the case. The resin, it must be remembered, is a non-conductor, and the cover does not come into such intimate contact with it as to carry off its electricity by conduction. Instead of this it acts inductively upon it, so that the neutral fluid of the cover is decomposed, the positive electricity being retained on the under side by the inductive action of the resin, while the negative goes away through the finger which touches the cover into the ground. The cover, when removed, is in consequence found to be charged with positive electricity.

As the electricity of the resin has not been conducted away, this operation can be repeated time after time without any apparent diminution of the electricity of the resin, and without the necessity of exciting it afresh.

The tinned mould or **form** in which the resin is placed serves to retain the negative electricity developed on the surface. For this electricity acts inductively on the neutral fluid of the under surface of the metallic form, retaining the positive, but driving the negative into the earth, upon which we may suppose the instrument to be placed.

This positive electricity, which thus resides on the lower surface of the form, binds in its turn the negative electricity which exists on the resinous surface, and prevents it from being easily dissipated.

This is the mode of action of the form while the cover is not on; but when the cover is put on, the electricity of the resin prefers to act inductively upon the cover which is close to it, rather than upon the bottom of the form, which is removed from it by the whole thickness of the cake, and we have then the action which has been already described.

347. Instruments for detecting Electricity.—Let us take a glass jar (Fig. 113), of which the upper and the lower part are both made of metal. Let the upper part terminate in a metal rod with a brass knob, and let this metal rod be brought inside the jar and have two bits of gold leaf or two gilded straws attached to it, hanging loosely. Also let there be inside, at *a* and *b*, two strips of gold leaf communicating with the ground; and, finally, let the air inside be kept quite dry.

Now let us see what will happen if we communicate a charge of electricity to the knob. The straws or the gold leaves, becoming charged with the same electricity by means of the knob with which they are both in contact, will repel each other, and be attracted towards *a* and *b*, and we shall approximately be able to measure the intensity of the charge by means of the amount of repulsion; but we shall not be able by the method now described to tell whether the charge is positive or negative. We can, however, accomplish this in the following way. Suppose that a charge of some kind of electricity has been communicated to the leaves. Let us now bring an excited glass rod, which contains positive electricity, near the knob; this will decompose the neutral electricity of the knob, attracting the negative to itself, and repelling the positive to the gold leaves. If, therefore, the leaves had previously been charged with positive electricity, they will now diverge more widely; but if they had previously been charged with negative electricity, they will be brought together.

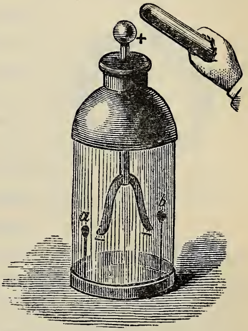

FIG. 113.

The instrument now described is, however, more an electroscope than an electrometer; that is to say, we can perceive, but we cannot easily measure by it. It is called the **gold-leaf electroscope.**

348. The best method of measuring electrical charges is that recently proposed by Sir W. Thomson; its mode of action will be perceived from the following figure:—

A metallic needle C D is arranged so as to be suspended from C by a thread perpendicular to the plane of the paper; that is to say, the metallic needle is all on one side of its point of suspension, or, if balanced, it is balanced by something non-metallic. This needle is kept electrified with a large charge, say of positive electricity, and its suspension is so arranged that it naturally rests between the two insulated half rings of brass, A and B, as in the figure, the attempt to force it round being resisted by the torsion of the thread from which it is suspended. Now if there is no electricity in either A or B the needle will remain at rest: but if B be connected with the earth, and A be positively electrified, it is clear that the needle will be repelled from A, and will move towards B, until the force of torsion is sufficiently great to overcome the electric force. If, however, A be negatively electrified, the needle will be attracted towards A, and will move in its direction until, as before, the force of torsion overcomes the electric force.

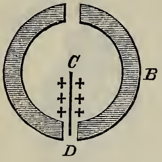

FIG. 114.

Now in such an instrument we can register by suitable means how much, and in what direction, the needle is turned when a small charge has been given to A, and thus we can tell not only the nature of the charge, but also its intensity.

349. Condensers of Electricity.—Suppose we have two insulated circular plates of brass, A and B (Fig. 115), with a glass plate between them. In the first place, let the plate A be removed to a distance, and let the plate B be connected with the conductor of an electric machine. Suppose next

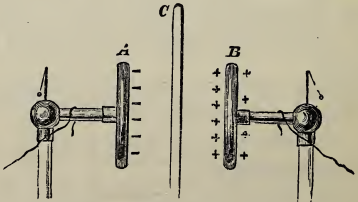

FIG. 115.

that A is made to approach the glass plate on the left-hand side, and is at the same time connected with the ground. The positive charge of B will decompose part of the neutral electricity of A, driving the positive to the ground, and keeping the negative on the side next the glass.

This negative electricity will react on the positive of B, pulling part of it to the side next the glass, and keeping it there. In consequence of this action B will be able to draw a further supply of electricity from the conductor. This further supply will again act in the same way; that is to say, it will induce negative electricity towards the side of A next the glass, and the negative of A will in turn keep this further supply to the side of B next the glass.

In fine, we see how, by an arrangement of this nature, a large quantity of positive electricity will become accumulated on the side of B, next the glass, and a large quantity of negative on the side of A next the glass. B in fact will draw the electricity of the conductor towards itself, or rather to that side of it which is opposite A.

If we now disconnect B from the conductor, and A from the ground, and if A and B are both furnished with electric pendulums (Art. 337), we shall find that there is no appearance of free electricity in A at all, the reason being that it is all held to the side next the glass, and is thus kept from influencing the pith ball. On B, however, there is some free electricity, as is seen by the divergence of the ball, but more electricity that is not free.

If we now separate A and B we shall find that a large quantity of electricity is set free in both, and the pith balls will diverge; this indicates that the respective electricities of the two plates are now not so accumulated on the side next the glass, but are more equally distributed throughout the plates, and can therefore affect the pith balls.

350. The Leyden jar, so called from the town where it was first discovered, owes its effects to electrical condensation. It consists of a glass jar (Fig. 116), the interior of which is coated with tinfoil, as also the outside up to the neck. A brass rod terminating in a knob is placed in electric communication with the inside coating, and is secured by being passed through a cork which closes the mouth of the jar. Thus the jar is furnished with an outside and an inside coating, which are insulated from one another. In order to charge the jar, its outside is put into connexion with the earth, or held in the hand, while the knob connected with the internal coating is presented to the conductor of an electrical machine at work.

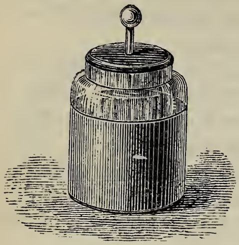
Fig. 116.

Positive electricity thus becomes condensed on the internal coating, and negative on the external coating of the jar. There is, besides, a certain amount of free electricity on the knob which has been placed in contact with the conductor of the machine; but there is no free electricity on the outside coating, the charge outside being wholly disguised owing to its attraction for the charge inside.

The Leyden jar when thus charged may be set upon the table, where it will remain charged for some time if the atmosphere be dry.

The jar may be discharged by means of a discharging rod (Fig. 117). This is held by its glass handles, and one of the knobs is brought into contact with the outer coating, while the other is gradually approached towards the knob connected with the interior of the jar; when the two knobs are near together, a bright spark is seen to pass, accompanied with a report, and the jar is discharged.

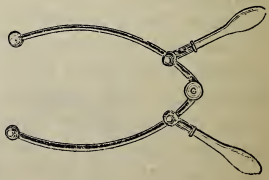
Fig. 117.

If the discharge takes place through the human body, a sharp shock is felt.

If a Leyden jar be allowed to stand for a short time after being thus discharged it is found that it has a small residual charge left in it. This is probably occasioned by the penetration of the electricities into the substance of the jar, and it is found to vary with the nature of the jar and with its thickness, being greater for a thick jar.

351. A number of Leyden jars form what is called an **electric battery.** In such a battery all the outside coatings are metallically connected together, and made to communicate with the earth, while the knobs from the centres of the jars are also connected together and placed in communication with the prime conductor of an electrical machine. It will be necessary to work the machine for a considerable time in order to charge to the full extent a large battery ; but when it is completely charged it forms a very powerful arrangement, Great care is therefore requisite in discharging it, that the discharge may not pass through the body, or the effects might be fatal.

352. The Electric Discharge.—The two opposite electricities often recombine suddenly ; but sometimes their combination occupies an appreciable portion of time.

In the case of frictional electricity, which we are now considering, the combination is more often instantaneous, and is accompanied by a flash of light and also by heat. If the flash be examined by means of the spectroscope, it is found to consist of a small portion of the substance of the terminals vaporized, and in a state of brilliant incandescence, together with a portion of the air between the terminals also incandescent. This spark, or a similar one obtained by the galvanic battery, forms in fact our best means of studying the spectra of ignited metallic vapours.

In the electric spark we have the transmutation of the energy of electrical separation into that of heat.

Bearing this in mind, we are able to estimate the relation between the charge and the amount of heat produced by discharging a jar. Thus let d denote the electric density or quantity of electricity on unit of surface ; now the attraction between the two sides will vary as the square of d, for were the charge on each side reduced to one-half of what it is, the mutual attraction (Art. 341) would be reduced to one-fourth, since we should only have one-half of the original number of electrical elements acting each on one-half the number of opposite electrical elements. But if the mutual force be increased fourfold by doubling the electric density, it is clear that, for the same jar, the potential energy implied in electrical separation, being proportional to the force (see Art. 99), will also be increased fourfold; now since this potential energy is converted into heat when the jar is discharged, this heat will be increased fourfold also.

It is clear, in the next place, that the heating effect of this discharge will be proportional to the surface charged, so that the whole heating effect will be proportional to the square of the density multiplied by the surface. This result is generally put in the following form:—Let h denote the heat produced by a discharge, q the whole quantity of electricity in the jar, s the surface, and d the density; then of course $q=d\,s$, hence $d = \frac{q}{s}$, and *hence the whole heating effect*, or h—which, as we have seen, is proportional to the square of the density multiplied by the surface—will vary as $\left(\frac{q}{s}\right)^2 \times s$; that is to say, as $\frac{q^2}{s}$, or *will be proportional to the square of the quantity divided by the surface.* The electric discharge produces also magnetical and chemical effects, which will afterwards be described when we come to treat of the electric current.

353. Duration of the Spark.—If a disc coloured with the spectral colours in their proper proportion be made to rotate rapidly, we have seen (Art. 288) that it will appear white, because the various colours follow one another so rapidly that the eye blends them all together. Could we, however, manage to view the disc only for an instant, so that no time was allowed for the blending of the colours, then we should see them in their true aspect, and the disc itself would appear to be at rest. Now this will take place if we view a coloured revolving disc by means of the electric spark; the disc will be seen in its true colours, thereby showing that the duration of the spark is exceedingly short.

Sir C. Wheatstone has succeeded in measuring this duration by means of a revolving mirror. A reference to Art. 162 will show that a very small angular motion of a mirror is sufficient to convert the image of a dot of light into a line. If therefore a mirror can be made to revolve through a perceptible angle during the time that a small spark passes, the image of the spark, as given by the mirror, will appear to be lengthened out.

By a method of this kind, Wheatstone found in one experiment that the spark lasted $\frac{1}{24000}$ of a second; and he also found that the electric fluid travelled at the rate of 288,000 miles a second along an insulated wire, a velocity which is greater than that of light.

354. Atmospheric Electricity. — It had long been imagined that lightning was only a manifestation of electricity on a large scale, but this was first proved by Franklin, who by flying a kite during a thunderstorm obtained from the clouds electricity in sufficient quantity to charge a Leyden jar.

Taking advantage of the known laws of electricity, Franklin now proposed to render buildings safe from the effects of lightning. He suggested that all large buildings should be supplied with stout pointed metallic rods, extending into the air to some distance above the highest point of the building, and on the other hand carried down into moist earth. By this means it is found that the destructive effects of lightning may be avoided.

The noise which accompanies the electric flash is probably due to the very sudden expansion by heat of the air along which the current passes, while its destructive effect in rending substances is frequently due to a similar cause, there being a sudden liberation of gas under intense heat, and thus under very high pressure. The flash of lightning consists of the various constituents of the air heated up to incandescence, and were we to analyse the flash by means of the spectroscope, it would no doubt reveal the chemical nature of the substances through which the discharge had passed.

355. Sundry Experiments.—Let us take a hollow brass ball and support it on an insulating glass stand. If we now bring this insulated conductor near an electric machine in action we shall get a spark, but it will be very feeble. If, however, we touch with our finger that part of the conductor which is farthest from the machine, or make a connexion between this conductor and the ground, the spark from the machine will be much more intense.

This illustrates what was said in Art. 346 about the cause of the spark. We have the positive electricity of the machine decomposing the neutral electricity of the hollow ball, and pulling the negative towards itself while it drives the positive as far away as possible. If, however, the ball is insulated, the positive cannot be driven away very far, nor the two electricities sufficiently separated, so that we have a very feeble spark. But if we touch the brass ball, or connect it with the earth, this positive electricity is driven to the earth; the two electricities are thus well separated, and there is a good spark.

In the experiment now described, if we continue to touch the brass ball and at the same time to work the electric machine, a succession of sparks will pass through our body to the earth, and we shall feel a series of shocks. The spark from the electric machine may in truth be compared to a flash of lightning, the difference being that the one is only a few inches, while the other may be a few miles long. We may perhaps compare the thundercloud to the electric machine, or generator of electricity, while the air performs the part of the badly conducting medium, and the earth that of the conducting sphere in the last experiment.

Now suppose that we vary the experiment by attaching a metallic point to the brass ball between it and the machine, while we continue to touch the brass ball and to work the machine as before. We shall not now get a series of sparks from the machine, but only a continuous rush of electricity. In fact, the point will carry off the electricity almost as rapidly as it is produced, and will not allow it to gather so as to form a spark, and we shall not now suffer a series of shocks.

This illustrates the efficacy of the lightning-conductors mentioned in Art. 354; for the pointed rods running into the moist earth will carry off the electricity in a silent manner, just as the point attached to the ball does in the experiment now described. A building well protected by such pointed conductors is consequently safe from the destructive effects of the lightning discharge; for even if these do not succeed in preventing the discharge, they will at least lessen its violence, and prevent it from injuring the building.

356. Connexion between Electrical Separation and the other Forms of Energy.—Having agreed to consider the two opposite electricities as two fluids which have a strong attraction for each other, it is manifest that it will require an expenditure of energy to separate these two attractive fluids from one another, just as much as it will to separate a stone from the earth.

When therefore we have obtained by an expenditure of energy the two electricities in a separated state, we have converted the energy spent by us into a form of potential energy, or energy of position, inasmuch as we have thereby obtained in a separated state two bodies differently electrified, and therefore mutually attracting each other.

In the electric machine it is actual mechanical energy that is converted into that of electrical separation, the electricity produced making it so much harder to drive the machine.

Again, when electrical separation is produced in a tourmaline by the action of heat (Art. 339), it is evidently part of the energy of the heat that is changed into electrical separation; so much heat has in fact disappeared, and so much electrical separation has been created in its place.

As far as we are aware, radiant energy cannot be directly transmuted into electrical separation.

On the other hand, the energy of electrical separation is transmuted into that of visible motion, when two bodies oppositely electrified are allowed to approach each other, and it is transmuted first of all into electricity in motion, and after that into heat, when a spark is allowed to pass between the two oppositely electrified bodies.

356*a*. Electro-Static Standards.—Before quitting this subject it will be desirable to allude to a system of standards that has recently been introduced with the view of bringing electrical science more completely under the domain of mechanical principles. In the first place, let us endeavour to explain what is meant by difference in **electrical potential.** Let us look on electricity as something which changes its place under the action of electrical force just as water flows downwards under the action of gravitating force. Now, in order that water may flow, you must have a difference of gravitation-level, and so in order that electricity may flow you must have a difference of electrical level.

Whenever there is a difference of gravitation-level and a channel of communication opened out, water will flow from a higher to a lower level; and so whenever there is a difference of electrical potential or level between two conductors, and an electrical channel of communication is opened between them, electricity will always flow from the one to the other. In fact, difference of electrical potential is that which causes a flow of electricity.

If we now refer to our standards of energy we find that unit of work (a kilogrammetre) is spent when unit of mass (a kilogramme) ascends against gravity through unit of distance (a metre).

We might therefore define a metre as unit difference of gravitation-level, and say that there is unit difference of gravitation potential, or level, when unit of work is spent by unit of mass in moving from the one level to the other against gravitating force. Of course it is much more easy to obtain any such difference of gravitation-level by trigonometrical measurement; nevertheless in the case of electricity we must define difference of electrical level by reference to the work done in carrying unit of electricity from the one level to the other.

Having thus explained what is meant by difference of electrical potential, we may add that in the system of electrostatic standards it has been found most convenient to adopt the centimetre as the standard of length, the gramme as that of mass, and the second as that of time : the system is therefore sometimes called the Centimetre-gramme-second System of Units.

We shall now give certain definitions founded upon this system :—

(1). The *unit of force* is the force which will produce a velocity of one centimetre per second in a free mass of one gramme by acting on it for one second.

(2). The *unit of work* is the work performed by unit of force when it has moved a body over one centimetre.

(3). The *unit quantity of electricity* is that which exerts the unit of force on a quantity equal to itself at a distance of one centimetre across air.

(4). The *unit difference of potential*, or unit electro-motive force, exists between two points when the unit of work is spent by unit of electricity in moving from the one to the other against electric repulsion.

(5). The *electric density* signifies the quantity of electricity per square centimetre on a charged conductor.

(6). The *capacity* of a conductor is the quantity of electricity necessary to give it unit difference of potential.

(7). The co-efficient by which the capacity of an air condenser must be multiplied in order to give the same capacity when another dielectric is used is called the *specific inductive capacity* of the dielectric.

CHAPTER VIII.

ELECTRICITY IN MOTION.

LESSON XL.—MAGNETISM.

357. There is a certain kind of iron ore which has the property of attracting iron.

This circumstance was known to the ancients; and as the ore was first found in Magnesia, a place in Asia Minor, this gave rise to the term **Magnet.**

It will be afterwards shown how a long steel bar may be made into a magnet much more powerful than any which occurs in nature; the properties of a magnet may therefore be advantageously studied in such a bar.

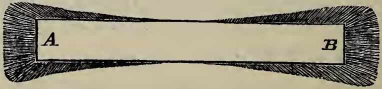

FIG. 118.

358. Properties of a Magnet.—If A B denote a long steel magnet, and a small bit of soft iron be brought near it, the iron will be attracted to it, especially near the extremities of the bar; there will however be no attraction at the middle of the bar.

In like manner iron filings may be made to attach themselves to such a magnet, and they will cluster round the extremities as in the figure, but they will show no disposition to attach themselves to the centre of the bar. The appearance of the iron filings leads us at once to the belief that there are two centres of force in the magnet, one near each extremity, and these two points are called the **poles** of the magnet. Again, if the magnet itself be suspended horizontally by a thread, it will be found to point with its length in a definite direction, to which it will always return, resisting every attempt to twist it away. This direction is, in Great Britain, very nearly north and south, and the pole which points to the north may, for convenience sake, be marked and termed the **marked pole,** to distinguish it from the other. It is sometimes called the north pole of the magnet, but this name is apt to create confusion.

359. Magnetic and Diamagnetic Bodies.—Let us now suppose that we have got a very powerful steel magnet; a small bit of soft iron will, as we have seen, be attracted to either of its poles. This attraction for iron will be very strong if the magnet be powerful, and it has been discovered by Faraday that many other substances will be attracted by such a magnet, only not so strongly as a bit of iron. Thus among the metals, nickel, cobalt, manganese, platinum, osmium, and palladium are magnetic, and so also are paper and sealing-wax among non-metallic substances.

On the other hand, Faraday found that certain substances are repelled by such a magnet; these he termed diamagnetic. Among the metals, bismuth, antimony, zinc, tin, mercury, lead, silver, copper, gold, and arsenic are diamagnetic, being repelled from the poles of a strong magnet. Rock-crystal, phosphorus, and sulphur are also repelled. This repulsion is most marked in bismuth, but it is very much less strong than the attraction exerted between the magnet and iron.

In fact, most bodies are either sensibly attracted or repelled by the pole of a powerful magnet, but its attraction for iron is much stronger than its attraction or repulsion for any other body.

360. Suppose now that we place the marked pole of a powerful magnet near the unmarked pole of another, or what is a better arrangement, have one powerful magnet shaped so that its two poles come close together.

Let A and B (Fig. 119) denote the two poles of such a magnet, and let there be an arrangement by means of which we can suspend different bodies midway between the two poles. If we first of all suspend a needle or slip of iron, it will point **axially**, that is to say, its length will lie in the line joining the two poles, as in the figure.

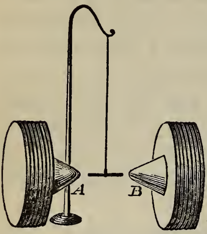

Fig. 119.

If, however, the needle be made of bismuth and not of iron, it will not point axially, but transversely or **equatorially**, that is to say, its length will be in a line at right angles to that joining the two poles, and generally it will be found that *all magnetic substances, or those which are attracted to a single pole, will point axially when placed between two poles, while all diamagnetic substances, or those repelled by a single pole, will point equatorially.* In fact, a magnetic substance placed beween two such poles will endeavour to get as near to these poles as possible, so that it will place itself axially ; while, on the other hand, a diamagnetic substance will keep as far away as possible, so that it will place itself equatorially or transversely.

361. It is instructive to observe what will take place if the substance is suspended in a magnetic liquid instead of in air.

Thus mica is magnetic, and in air will point axially ; and a solution of protochloride of iron is also magnetic, and to a greater degree than mica. Now what will happen if the slip of mica be suspended in a solution of protochloride of iron and then exposed to magnetic influence? In this case, instead of pointing axially, it will point equatorially ; that is to say, *a magnetic substance suspended in a fluid more magnetic than itself will appear to be diamagnetic. And in like manner a diamagnetic substance suspended in a liquid more diamagnetic than itself will appear to be magnetic.*

362. Action of Magnets upon each other.—Having thus described the action of magnets upon bodies in general, let us now proceed to their action upon one another.

Suppose that we swing a small magnet, suspended by a thread, and that we cause a powerful magnet to approach it. We shall find that the marked pole of the small magnet will be repelled by the similar pole of the large one, while it will be attracted by the opposite or unmarked pole. In fine, we have here a law similar to that which holds in electrified bodies, in consequence of which *like poles repel, while unlike poles attract each other.*

Coulomb has applied his torsion balance in order to discover the law of magnetic attraction and repulsion, and he finds that *the forces exhibited vary inversely as the square of the distance.*

Thus in the following figure, if we suspend a small magnet (*a b*), and cause to approach it the pole A of a large magnet (*a* and A being similar poles), we shall first of all have a repulsion exerted between A and *a* proportional to $\frac{1}{(\text{A}\,a)^2}$, and we shall have an attraction between A and *b* proportional to $\frac{1}{(\text{A}\,b)^2}$.

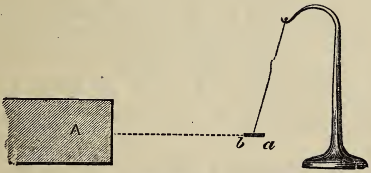

FIG. 120.

The consequence will be that the small magnet will point as in the figure; that is to say, the pole *b* will place itself opposite A, and not only so, but the small magnet, if it be free to move, will have a tendency to rush bodily to the large magnet, its tendency being represented by the excess of the attraction over the repulsion, that is to say, being proportional to $\frac{1}{(\mathrm{A}\,b)^2} - \frac{1}{(\mathrm{A}\,a)^2}$.

363. Magnetic Induction.—While the mutual action of magnets may be compared to that of electrified bodies upon each other, that of a magnet upon a piece of soft iron may be compared in some respects to the action of an electrified upon a neutral body.

We may regard the magnet as decomposing the neutral magnetic fluid of the soft iron, in a manner like that in which an electrified substance decomposes the neutral fluid of the body near it. Thus if the marked pole A of a large magnet be presented to a bit of soft iron, the soft iron becomes temporarily a magnet, having its unmarked pole *b* next the marked pole A of the large magnet, while the marked pole *a* of the small bit of iron will be at the extremity farthest from A. The attraction of A for *b* will therefore be greater than its repulsion for *a*, and hence the bit of iron will be bodily attracted to the large magnet, and will probably fly towards it, and attach itself to its extremity or pole. This is the reason why iron filings attach themselves to magnets, and a bundle of nails or even a heavy iron weight may be held up in this manner by a very powerful magnet.

364. Effect of breaking a Magnet.—It might be supposed that if we broke a magnet A B, we should get two magnets, one containing only the marked pole A, and the other only the unmarked pole B of the original magnet, just as in the experiment of Art. 343 we separated the two electricities of the neutral conductor. Such, however, is by no means the case; for when we break a large magnet into two, it immediately forms two small complete magnets. Thus if we break a magnet A B (Fig. 121) at its centre, a pole B′ is immediately formed to the left hand of the point of rupture, and a pole A′ to the right, so that A B′ and A′ B are two complete magnets.

Thus by breaking a magnet into a number of pieces we make so many separate magnets; and in order to explain this, it has been supposed that all the magnetism of one kind is not concentrated in the marked pole of a magnet, and all that of the opposite kind in the unmarked pole, but

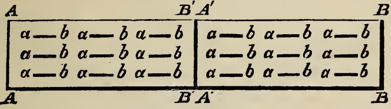

FIG. 121.

that *in each particle throughout the body of the magnet there is a separation between the two magnetisms*, so that the state of things in a magnet may be exhibited by Fig. 121.

Now if a piece of soft iron be brought into the neighbourhood of this arrangement at the left, the marked pole (*a*) of each particle of the magnet will be somewhat nearer the soft iron than the unmarked pole (*b*), and the sum of all the small effects upon the soft iron will be to the same effect as if all the magnetism corresponding to the marked pole were concentrated in the left-hand end of the magnet, and all the opposite magnetism in the other end. Also we see at once that if a magnet consist of an arrangement of this kind, and if it be broken into two parts, each part will become a separate magnet precisely similar to the whole.

365. How to make Magnets.—When soft iron is made to approach a magnet, temporary magnetic poles are readily induced in the soft iron, so that it becomes a magnet for the time being, and while attracted to the magnet, it will attract another piece of soft iron, just as the magnet itself would. Nevertheless all these properties are lost as soon as the soft iron is removed. We are therefore justified in saying that soft iron cannot retain magnetism, and that we cannot have a permanent magnet made of this substance. Hard steel is the substance that can be best made into a permanent magnet.

We may magnetize a steel bar by the following process :—

Let B″ A″ (Fig. 122) be the bar which we wish to magnetize, and let B A, B′ A′ be two powerful magnets, A being the marked pole of the one, and B′ the unmarked pole of the other. Bring the two magnets together, as in the figure, at the centre of the bar to be magnetized, then simultaneously draw them along the bar towards the extremities, moving A towards B″ and B′ to A″. Repeat this process several times, and it will be found that the bar has become a magnet, with A″ for its marked, and B″ for its unmarked pole.

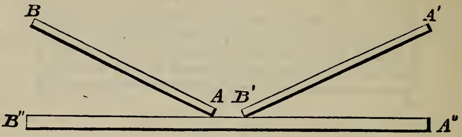

FIG. 122.

366. Effect of Heat on Magnets.—When a magnet is slightly heated it loses part of its magnetism, which is mostly recovered when it is again cooled to its original temperature; but if it be heated beyond a certain limit, the loss of magnetism will not be recovered when it cools, and if heated to redness it loses all trace of magnetic properties of any kind. Soft iron also, when heated to redness, loses the property of being attracted by a magnet.

A similar limit exists in the case of the other magnetic metals, nickel and cobalt, which if heated sufficiently will ultimately lose their magnetic properties.

367. The Earth acts as a Magnet.—If a magnetic needle be suspended horizontally it will point in this country in a direction nearly north and south, the marked pole being about 20° to the west of the north. A vertical plane passing through the poles of such a needle is called the magnetic meridian.

Again, if a truly balanced needle be suspended by a delicate horizontal axle, so placed that the magnet moves in the plane of the magnetic meridian, the marked pole will dip downwards, until the needle makes an angle with the horizon of about 68°. We are therefore justified in saying that were a magnetic needle perfectly free to place itself as it chose, it would be found in a vertical plane passing 20° to the west of north, and with its marked pole dipping downwards and making an angle of 68° with the horizon. The earth, in fact, acts like a gigantic magnet, of which the unmarked pole lies to the north, and the marked pole to the south, in consequence of which the marked pole of a freely suspended needle points in this country approximately to the north.

It is this property that makes the magnetic needle of such value to mariners, who might not otherwise know in what direction to steer.

But the marked pole of a needle will not everywhere and always point as it does in Great Britain at the present moment. Two hundred years ago the needle pointed to the true geographical north in London, while now it points 20° to the west of north. Again, if we travel far to the north we come to a place that is called the magnetic pole, where the needle, if free to place itself as it chose, would point with its marked end vertically downwards; at this place also, were it horizontally balanced, it would have no tendency to turn in one direction more than another; in fact, the unmarked magnetic pole of the earth being there directly under our feet, the needle merely points with its marked pole vertically downwards, and is thus of no use to the mariner.

In like manner, if we travel far south we shall come to the earth's marked pole, where the unmarked end of our needle will point vertically downwards, and where a horizontally suspended needle will settle in any direction. It is unknown in what manner the earth acquired its magnetism, but it has been discovered by Sir E. Sabine that the magnetic properties of the earth are in some way connected with the spots which appear from time to time on the surface of the sun, so that in those years when there are most spots, there are most disturbances of the magnetism of the earth.

We ought to mention that the effect of the earth's magnetism upon a magnetic needle is merely directive; that is to say, the earth twists round a freely suspended needle, but does not attract it bodily otherwise than it does any ordinary non-magnetic substance. The reason is, that the magnetic pole of the earth is very far removed from the poles of the needle, so that the attraction of the earth's pole for the one pole of the needle is not sensibly greater than its repulsion for the other, and therefore the needle is not bodily attracted towards the earth in virtue of its being a magnet.

It will be shown in the following pages that magnetism is very intimately associated with electricity in motion.

LESSON XLI.—VOLTAIC BATTERIES.

368. In the year 1786, Galvani, Professor of Anatomy in Bologna, remarked that convulsions were produced in the leg of a frog when the lumbar nerves were connected with the crural muscles by means of a circuit composed of the two metals iron and copper, and he attributed the effect to electricity inherent in the animal. Shortly afterwards the subject was taken up by the celebrated Volta, who came to the conclusion that the source of electricity in Galvani's experiment was the contact of two heterogeneous metals; and he was soon led by this view to construct a pile, which is the origin of the Galvanic or Voltaic batteries of the present day.

His pile was of very simple construction. He built on an insulated foundation of glass or resin a number of discs placed in the following order:—a disc of copper, one of zinc, and one of cloth or flannel, moistened with acidulated water; then copper, zinc, cloth, as before.

Now in such an insulated pile it may be shown, by means of the electroscope, that the lower half is charged with negative, and the upper half with positive electricity, and that the tension is most powerful at the extremities, so that the lower copper plate is decidedly negative, and the upper zinc plate decidedly positive in its indication. If now the extremities of the pile be connected together by a wire, as in Fig. 123, the two electricities will tend to unite by means of this wire, and there will be a currrent of positive electricity flowing from the upper zinc to the lower copper; or we might say with equal propriety, a current of negative electricity flowing between the lower copper and the upper zinc; but, for the sake of simplicity, we only speak of the positive current. And in order to complete the circuit, the current of positive electricity will flow in the pile itself from the zinc to the cloth, and from the cloth to the copper above it, and so on.

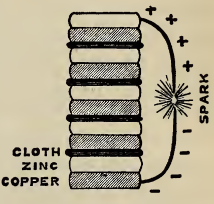

FIG. 123.

Now this combination of the two opposite electricities will be attended by heat, and thus we have a heating effect produced when a voltaic pile has its terminals connected together by means of a wire.

369. Volta afterwards replaced his pile by another arrangement, which he called a crown of cups. This arrangement is exhibited in Fig. 124. Here the zinc and copper plates are connected by wires and placed in glass vessels containing dilute sulphuric acid. This liquid corresponds, therefore, to the acidulated cloth discs placed between the zinc and copper in the original pile; and just as in the pile when the extremities were connected together there was a current of positive electricity proceeding from the zinc to the cloth, and from the cloth to the copper through the pile itself, so here there will be a current of positive electricity proceeding from

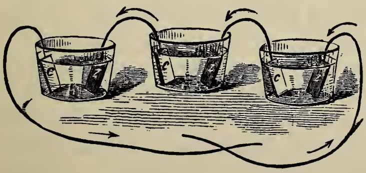

FIG. 124.

the zinc through the liquid of the pile to the copper, as denoted by the arrow-heads.

370. Now Volta explained the effect produced by the voltaic battery, by supposing that a separation of the two electricities is produced by the contact of heterogeneous metals. Thus in the original pile when a zinc and copper plate are laid together, he imagined the zinc to become positively and the copper negatively electrified, and a forcible electrical separation to be thus kept up at the points of contact of the two metals as long as they remain together. He further supposed the total effect to depend on the number of elements in the pile, so that the accumulated action of the elements of a large pile might become very intense.

It is readily seen that this will be the case by referring to Fig. 123. For let us take the middle copper and zinc plates of this figure, which are in contact with each other, and denote the electricity of the copper plate by -1, while that of the zinc plate is $+1$; this difference being forcibly kept up by the contact of these two heterogeneous metals. The conducting cloth conveys the state of this zinc plate to the copper immediately above it, which will therefore be $+1$; and since the zinc is supposed to be two units more positive than the copper in contact with it, the state of the zinc plate in contact with this copper will be $+3$. In like manner the state of the highest zinc plate will be $+5$.

Going downwards now from the middle copper plate, this will communicate its charge (-1) by means of the cloth to the zinc below it, of which the charge will therefore be -1. But the copper in contact with it will be two units more negative on account of heterogeneity, and will therefore be -3; while in like manner the lowest copper will be -5. Thus for five pairs there will be an electrical difference between the top and bottom of ten, or five times as much as that given by a single pair.

Nevertheless Volta appears to have erred, in his ignorance of the laws of energy, by supposing that the mere contact of heterogeneous metals could account for the large amount of energy exhibited by a voltaic battery when its extremities are connected together; for then it generates heat, and is capable of performing mechanical work of various kinds.

It is, in fact, a powerful kind of energy, and this electricity in motion has been described by us in Art. 108 as one of the varieties of molecular energy. Indeed, it is very manifest that, since the tendency of electricity is to equalize itself, in order to procure a continuous stream of electricity we must have a reservoir which is always giving it out, just as much as in order to obtain a continuous flow of water we must have a reservoir of some kind containing water.

Now the mere contact of heterogeneous metals cannot supply us with a constant stream of electricity, for if it could we should at once have a kind of perpetual motion, which is manifestly impossible. We must, therefore, look somewhere else for the source of the energy of the voltaic circuit.

It was seen by those who came after Volta, that in order to produce a continuous current of electricity a quantity of the oxidizable metal of the circuit must, so to speak, be burned, being converted into a metallic salt; and they imagined that the electrical separation of the voltaic pile was due to this chemical action, and not to the mere contact of heterogeneous metals.

371. But very recently Sir W. Thomson has made an experiment, which appears to show that Volta was right in supposing the electrical separation to be caused by the contact of two heterogeneous metals, while he was wrong in imagining that work can be done by the voltaic arrangement without the consumption of some such fuel as zinc. Thus there is probably a sort of electric irritation kept up at the point where the two heterogeneous metals come into contact; and when the battery is at work, this irritation proves a kind of predisposing cause, in virtue of which the potential energy of the fuel zinc is converted into an electric current in the first instance, and after that most frequently into heat. Perhaps it is wrong to suppose that the electrical irritation is entirely confined to the points of contact of the heterogeneous metals, and that nothing is due to the nature of the liquid.

The points of contact of the heterogeneous metals must, however, be regarded as the *chief* sources of electrical separation.

372. The experiment made by Thomson was of the following nature :—

Let *m* denote a metallic needle, moveable round the point *m* in the plane of the paper, and let *c* and *z* denote two semicircular rings—the one (*c*) of copper, and the other of zinc, both being soldered together; then if *m* be charged with positive electricity it will turn from the zinc to the copper, and if charged with negative electricity it will turn from the copper to the zinc.

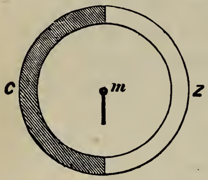

Fig. 125.

This behaviour may be explained by supposing that in consequence of the contact of the two metals, copper and zinc, the copper becomes negatively and the zinc positively electrified. Therefore in a zinc and copper battery this electrical irritation, which is kept up at the junction of the two metals, will cause a current of positive electricity to flow *through the liquid* from the zinc to the copper. Mr. Joseph Thomson of Owens College has recently shown that when a rod of glass is united to one of sealing-wax their surface of contact becomes the seat of a permanent electrical separation in virtue of which the glass will be found to be charged with positive and the wax with negative electricity months after the two have been united together. The electrical separation caused by the contact of heterogeneous bodies is thus seen to be associated with non-conductors like glass and sealing-wax, as well as with conductors like copper and zinc.

373. But whatever may be the true origin of the electrical separation which takes place in a voltaic battery, it is the tendency of the separated electricities to unite that produces the current, so that the intensity of the electrical separation becomes the measure of what we may term the **electromotive force.**

374. Sir C. Wheatstone has devised a means of measuring the electromotive force of different combinations. He found from his experiments that if platinum and an amalgam of potassium be used as the two plates of a battery, the electromotive force or intensity of electrical separation may be called 69. He also found that the electromotive force between a platinum and a zinc plate is 40; while that between a zinc and an amalgamated potassium plate is 29.

We derive from this experiment a very interesting result. Thus if platinum be our negative and zinc our positive element, we get 40 as the value of our electromotive force; again, if zinc be our negative and potassium our positive element, we get 29; but if, instead of going in two steps from platinum to potassium, we go in one step, and make a combination of these two metals, we get 69 at once.

In fine, the *electromotive force between any two metals is equal to the sum of the electromotive forces between all the intervening metals.*

375. The various metals in the following table are classified according to their order in the electromotive series. Thus we have—

Electro-positive.
Potassium.
Zinc.
Cadmium.
Lead.
Tin.
Iron.
Nickel.
Bismuth.
Antimony.
Copper.
Silver.
Gold.
Platinum.
Electro-negative.

That is to say, if potassium be brought into contact with zinc, it will be charged with positive electricity; and, generally, any metal brought into contact with one under it in the series will be positively charged; while if brought into contact with one above it in the series, it will be negatively charged.

376. Constant Batteries. Daniell's.—When a battery consisting of zinc and copper plates immersed in dilute sulphuric acid (Fig. 124) has been some time in action, it is found to be greatly enfeebled. This arises from two causes; for, in the first place, the sulphuric acid is gradually used and converted into sulphate of zinc. In the next place, it is found that the hydrogen, which is set free by the action of the battery, adheres to the copper plate, and that in consequence of this adhesion the force of the battery is very much weakened.

Both of these objections are removed by using a constant battery. This form of battery was first invented by Daniell, and its mode of action will be understood by reference to Fig. 126, which represents a single cell of a Daniell's battery.

We have, in the first place, an outer vessel, made let us say of copper, containing a saturated solution of sulphate of copper, and in this there is a small shelf containing some spare crystals of sulphate of copper to replace those that are decomposed as the action proceeds. There is also an inner cylindrical vessel, consisting of porous earthenware, through which the particles of a fluid may easily pass. This inner vessel contains dilute sulphuric acid, and in it is placed the electro-positive element, consisting of a cylinder of amalgamated zinc.

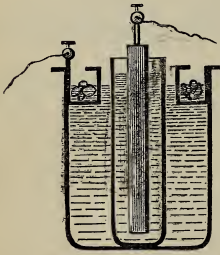

FIG. 126.

The electro-negative element, on the other hand, consists in this case of the copper of the outer vessel. When this battery is in action the amalgamated zinc of the inner vessel is gradually dissolved by the dilute sulphuric acid, and the liberated hydrogen finds its way through the pores of the inner vessel towards the copper plate. It is acted upon by the sulphate of copper, which it decomposes, forming sulphuric acid, copper, and water. The copper is deposited on the copper plate and the acid finds its way into the interior porous vessel, where it replaces that which has been consumed. Also the sulphate of copper, which has been used in this process, is replaced by the spare sulphate from the shelf. Thus both the sulphuric acid of the inner vessel, and the sulphate of copper of the outer, are kept of constant strength, while the surface of the copper itself is kept bright through the deposition of new particles produced from the decomposition of the sulphate. It is found that a battery of this kind will remain constant for a very long time.

Amalgamation of the zinc was first practised by Kemp, who found that zinc so amalgamated is not attacked by acid while the battery is not in action. He also found that with amalgamated zinc the current is both more regular and more intense than it is with ordinary zinc.

377. Grove's Battery.—There are other kinds of constant batteries, and one of the best and most powerful is that of Grove. In it the amalgamated zinc is placed in an outer glass vessel along with some dilute sulphuric acid, and the hydrogen which is liberated during action finds its way into an inner porous vessel, which contains the electro-negative element, consisting of a thin slip of platinum immersed in strong nitric acid. The hydrogen, when it comes in contact with the nitric acid, decomposes it, forming nitrous fumes, and does not therefore attach itself to the surface of the platinum.

378. Thermo-electric Currents.—We have already alluded (Art. 290) to the current which takes place when we heat a junction of copper and bismuth. The existence of this current was first discovered by Seebeck, and it may be easily demonstrated by means of the arrangement of Fig. 94. Thus, if *ns* be a needle, of which *n* is the marked pole, and if the junction at the right be heated by a spirit-lamp, as in the figure, we shall have a current of electricity passing at the heated junction from the bismuth to the copper, its direction being denoted by the arrow-head. Hence (Art. 380) the marked pole of the needle will be deflected as in the figure.

Such a couple is called a **thermo-electric** couple, to distinguish it from an ordinary couple, which may be termed **hydro-electric.** The strength or intensity of the current will of course depend upon the electromotive force of the two metals which form the pile, and we may thus draw up a list of metals such that the positive current shall go across the heated junction from the metal nearest the top to that nearest the bottom of the list. The following is such a list :—

Bismuth.
Nickel.
Lead.
Tin.
Copper.
Platinum.
Silver.
Zinc.
Iron.
Antimony.
Tellurium.

We see that the metals bismuth and antimony are near the opposite extremities of the list, and as they can be easily procured, they are often used in thermo-electric combinations.

A law holds with respect to this series similar to that which held for the series of Art. 375.

Thus in the circuit of Fig. 127, if the wires at the two extremities be copper wires leading to the galvanometer, or instrument for measuring the intensity of the current (see Art. 381), and if we have at A a copper and tin junction, while at B we have a tin and antimony junction, and at C an antimony and copper junction, and if we heat through 1° C. the copper and tin, and also, at the same time, the tin and antimony junction, we shall get a current of the same intensity as if we heat at once the copper and antimony junction through the same temperature range. Only we see from the figure that the latter current will pass along the circuit in an opposite direction to the former, so that the practical effect of heating all the three junctions together to the same extent will be to generate currents which cancel one another.

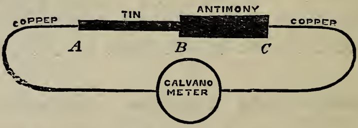

FIG. 127.

379. Within certain limits the strength of the current produced by a thermo-electric arrangement is proportional to the difference of temperature between the two junctions, but when the heat applied is very intense, the current sometimes changes its direction.

This is the case more particularly with a circuit of copper and iron; and Cumming has shown that while at an ordinary temperature the current goes across the heated junction from the copper to the iron, at a red heat it becomes reversed, and passes from the iron to the copper.

LESSON XLII.—EFFECT OF THE ELECTRIC CURRENT UPON A MAGNET.

380. Oersted, Professor of Physics in Copenhagen, discovered in 1819 the connection between an electric current and a magnet, a discovery that has since led to the construction of electric telegraphs between distant places.

In order to represent Oersted's experiment, let us take a horizontally suspended magnetic needle *a b* (Fig. 128), which will of course place itself in the magnetic meridian. Parallel to the length of the needle, and immediately above it, let there be a copper wire through which a current may be made to pass. As long as there is no current passing through the wire, the needle will remain in its ordinary position, but if a current be made to traverse the wire, the needle will take a position nearly at right angles to the current. The behaviour of the needle will vary with the direction and position of the current, and the relation between the two will be best remembered by imagining the observer to lie down in the current, so that the *positive* current enters in at his head and goes out at his feet. If now his face be always turned towards the needle he will find that the action of the current will be to deflect the marked pole *a* of the needle to his right hand.

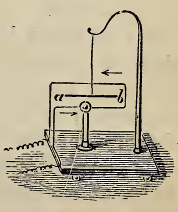

FIG. 128.

Let us apply the rule in the following cases :—

Case 1.—Let the current be above the needle, and go from magnetic north to south.

In this case the marked pole of the needle will be deflected towards the east.

Case 2.—Let the current be above the needle, and go from magnetic south to north.

In this case the marked pole of the magnet will be deflected towards the west.

Case 3.—Let the current be below the needle, and go from north to south.

In this case the marked pole of the needle will be deflected towards the west.

Case 4.—Let the current be below, and go from south to north.

In this case the marked pole of the needle will be deflected towards the east.

381. Galvanometer.—Taking advantage of this action of electric currents upon magnets, we are enabled to construct a very delicate instrument for indicating the existence of such currents, and for measuring their intensity. It is called a galvanometer.

Let us begin by supposing that a current passes in the plane of the magnetic meridian, vertically above and below a delicately suspended needle, as in Fig. 128, the direction of the current being denoted by the arrow-heads. The current above the needle will, by Case 2, cause the needle to turn so as to place its marked pole *a* above the plane of the paper, and by Case 3 the current below the needle will have a similar action. Thus the action of the two currents will supplement each other. Now if the current be coiled many times round about the needle in the same direction, as in the figure, before it is brought back to the battery, and if each of these turns be insulated by enclosing the wire in a non-conductor, then will the various turns of the coil supplement each other, and their united action upon the needle will become very powerful.

382. In this arrangement the struggle is between the directive force of the earth tending to keep the needle parallel to the wires, and the influence of the current tending to bring it into a position at right angles to the wires, and the apparatus will be rendered much more sensitive if we can overcome the directive force of the earth.

This is done by having two magnetic needles of the same strength, suspended as in Fig. 129, the one needle being wholly above the current, and having its poles opposed to the other; such an arrangement will have no directive force, and it is therefore called an **astatic** system of needles.

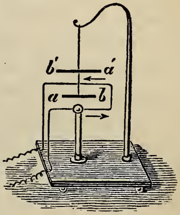

FIG. 129.

Again, it will be seen that the action of the upper current on the upper needle will tend to twist *b'* above the paper, while that of the lower current will be in the opposite direction; the lower current is, however, further removed from the needle than the upper current, and the latter will therefore predominate, and hence the needle will be twisted round so as to place *b'* above the plane of the paper.

Now the lower needle will, as we have shown, be twisted round so as to place *a* above the plane of the paper, and hence the two will be twisted by the current in the same direction; while the directive force of the earth, which opposes this motion, is either altogether cancelled or rendered very small, since the two needles are of the same strength and oppositely placed. Another method of rendering a needle astatic is that exhibited in Fig. 97, and described in Art. 291.

If the needles be not only astatic, but if they be also delicately suspended, and furnished with a mirror so as to reflect a slit of light upon a scale, then, as in Art. 291, the arrangement will be one of extreme sensibility.

The action of a current upon a needle depends upon the quantity of electricity which passes in unit of time. Thus if we double the number of coils round the needle of a galvanometer, without altering their distance from the needle, we double the action of the current upon the needle. In like manner, if without altering the coils or the distance we double the intensity of the current which passes, we also double the action.

A galvanometer thus affords us an accurate measure of the intensity of a current; that is to say, of the quantity of electricity which passes in unit of time.

383. The **tangent compass** is another very useful instrument. It consists of a vertical circle about a foot in diameter, which is placed in the magnetic meridian. A stout wire is carried all round the circumference of the circle, through which we have the means of passing the current whose intensity we wish to measure; the current may thus be supposed to circulate round the rim of the circle. At the centre of the circle we have a small magnetic needle, which will, when there is no current, lie in the plane of the circle, since this has been placed in the magnetic meridian.

But when there is a current it will be deflected, and it may be shown that *the intensity of the current will be proportional to the tangent of the angle of deflection, provided the needle be small compared to the size of the circle.*

384. Electro-magnets.—Let us coil in one direction round a thick cylinder of soft iron a stout insulated copper wire, and when we have completed the operation of surrounding the iron cylinder with a covering of this kind, let us connect the ends of the copper wire with the poles of a voltaic battery.

An electric current will thus circulate round the soft iron cylinder, and the cylinder will become a powerful magnet.

Magnets produced in this way are much more powerful than natural magnets; and a horse-shoe magnet of this kind, furnished with a keeper or cross-piece of iron connecting the poles, might be made so strong as to support a ton or more.

It has been found by Joule that a bar of soft iron is lengthened when made into a powerful magnet, and it has also been observed that at the moment of magnetization it gives out a peculiar sound.

385. Electric Telegraphs.—If an electric current be sufficiently strong, we may carry it, by means of an insulated wire, to a great distance from the battery. At the extremity of the distance we may pass it round a galvanometer, and finally bring it back to the battery. Suppose that the galvanometer is 1,000 miles from the battery, when the circuit is completed the current will pass through the whole length of wire and deflect the needle of the galvanometer. Thus by turning the current off or on we shall be able to put in motion a needle which is a thousand miles away from the battery at which we operate. This is the principle of the electric telegraph; but instead of having two wires—one carrying the current from the battery to the galvanometer, and the other carrying it back from the galvanometer to the battery—the latter is dispensed with, and the body of the earth made to take its place.

The arrangement will be seen by the following figure:—

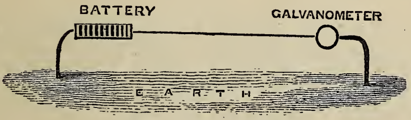

FIG. 130.

Not only is the expense of the telegraph lessened by this arrangement, but the intensity of the current is increased, since it has only to traverse half the amount of wire (see Art. 402).

LESSON XLIII.—ACTION OF CURRENTS ON ONE ANOTHER AND ACTION OF MAGNETS ON CURRENTS.

386. The mutual action of electrical currents was first discovered by Ampère. It is subject to the following laws:—

I. *Two currents which are parallel and in the same direction attract each other.*

II. *Two parallel currents, but in the opposite direction, repel one another.*

III. *When two currents cross at a point they attract each other, if they both tend either towards the point or form it, but they repel one another when they tend in contrary directions.*

In order to understand the bearings of these laws it is not necessary that the reader should trouble himself about the experimental appliances for bringing together currents moving in various directions; suffice it to say, this can be accomplished by means of suitable apparatus.

In Fig. 131 we have the various cases of Law III.

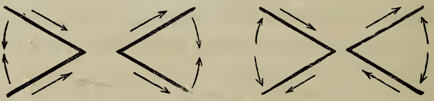

FIG. 131.

In the two left-hand figures we see that the currents are moving in the same direction, either to or from the angle, and hence they attract each other; while in the two right-hand figures the currents are moving in opposite directions with respect to the angle, and therefore they repel one another.

387. Next let us have two currents, *a b* and *c d* (Fig. 132), both movable round *O* as a centre in the plane of the paper.

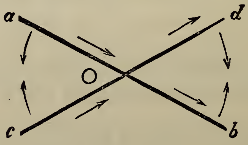

FIG. 132.

There will be an attraction between *a* and *c*, and between *b* and *d*, as in the figure; while, on the other hand, there will be a repulsion between *a* and *d* and between *b* and *c*;

the tendency will therefore be to bring the two currents into the same direction.

388. These laws will in certain cases produce a continuous rotation of currents.

Thus let us take a copper vessel (Fig. 133) and roll round it several coils of insulated wire, through which a current is made to pass; let the direction of this current be denoted by the arrow-heads in the figure. Now let the apparatus be so arranged that while a current is circulating round A, there are also currents passing through the wires *a b* and *a b′*, as in the figure; we have thus two vertical descending currents, *b* and *b′*, near the circular horizontal current which goes round the copper vessel. Now when one of the vertical currents *b* is at A it will be attracted by that part of the circular horizontal current to the left of A, since both tend towards the point A; but the vertical current will be repelled by the portion of the horizontal current to the right of A, since the one current tends towards A, while the other tends from it; the vertical wire *b* will therefore pass in a direction the opposite of that denoted by the arrow-heads.

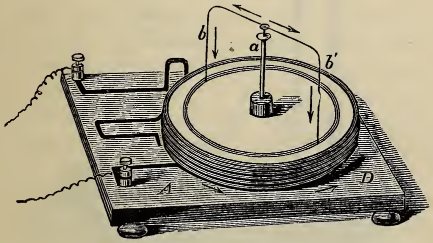

FIG. 133.

The current *b′* will pass in the same direction, and thus there will be a continuous rotation of the vertical currents round their axis in a direction the same as that of the hands of a watch.

389. Action of Magnets on Currents.—Since currents act on magnets (Art. 380), there ought to be a reaction of magnets upon currents ; and to study this we must leave the currents at perfect liberty to move.

In Fig. 134 we have an arrangement of this kind, consisting of a circular vertical current, which is free to place itself in any position.

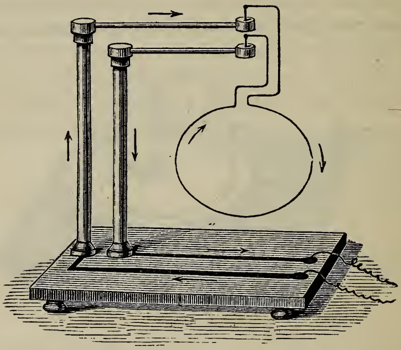

FIG. 134.

Now such a current will place itself so that the plane of the circle shall be perpendicular to the magnetic meridian. Also the descending current will be to the east, and the ascending current to the west.

390. Solenoids.—Suppose now that we construct a system of circular currents in coils, as in Fig. 135, and

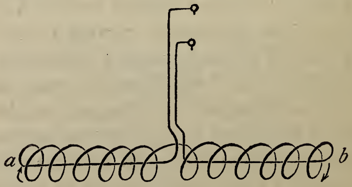

FIG. 135.

suspend it so that it is free to place itself in any direction ; such a current is called a solenoid, and when in action it will place itself so that the axis *a b* shall be in the magnetic meridian. The solenoid will in fact behave as if it were a magnet, and will therefore point to the magnetic north. It will also be found that the currents descend on the east side, and ascend on the west side; in fact, the action of the solenoid is similar to that of a vertical circular current, Fig. 134, and a number of such currents placed in a line constitute a solenoid.

391. Ampère's Hypothesis.—It was suggested by Ampère that we may regard a magnet as a solenoid, each particle of which is traversed by a continuous electric current.

This suggestion explains well all the known relations between magnets and currents, and we may at least receive it as a good working hypothesis.

In this case, if we suppose a cylindrical magnet to be suspended with its marked pole pointing to the north, then the molecular currents will descend on the east side of the cylindrical magnet, flow from east to west on the under side, ascend on the west side, and flow back from west to east at the upper side; in fact, the direction of the molecular currents will be the same in such a magnet as in the solenoid.

392. It is easily seen by this hypothesis why the marked pole of one magnet attracts the unmarked pole of another. For we have here two sets of vertical circular currents all moving in the same direction, so that the various elements of the first set of currents are parallel to those of the second set; the currents will therefore attract each other by the law of Art. 386, that is to say, the two magnets will rush together.

But if the marked pole of one magnet be placed near the marked pole of another magnet, we are presenting to each other two sets of circular vertical currents, one set of which we have twisted round, so that it is opposite in direction to the other set; the two sets of currents will therefore repel one another by the same law.

393. We have alluded in Art. 388 to one way by which a current may be made to produce continuous rotation.

Now a body in rotation is one form of visible energy, and if we set a current to produce this rotation we give it some work to do; it must therefore employ part of its energy in order to do this work, and must in consequence be enfeebled. Therefore if we have two similar voltaic batteries, each charged with the same amount of zinc and possessing the same amount of energy, and if the one is allowed to convert all its energy into heat, while the other, by some arrangement similar to that we have described, performs external work as well, this second battery, in virtue of the external work it has got to do, will generate less heat than the other from the consumption of the zinc; in fact, what is gained in external work done by the battery, is lost in heat generated in the battery.

LESSON XLIV.—INDUCTION OF CURRENTS

394. We come now to the induction of electric currents upon each other. If a conductor be in the neighbourhood of a current which remains constant in intensity and does not change its place, it is found that no current is induced in such a conductor. But Faraday discovered that *at the moment when a current is formed, it produces in a conductor near it a momentary current in an inverse direction to itself. Also, at the moment when such a current is broken, it produces in a neighbouring conductor a momentary current in the same direction as itself.*

In these two cases we suppose the conducting wire to remain stationary and the current to vary, so as to be suddenly generated and suddenly broken; but we may produce similar phenomena by keeping the current constant, and by moving the conducting wire so as rapidly to approach or recede from the current.

If the conducting wire rapidly approach a constant current we have the same effect produced as when the current is rapidly formed; that is to say, an inverse current is generated in the conductor. Again, *if the conductor recede rapidly from a current we have the same* *effect as if the current be suddenly stopped or broken;* that is to say, a direct current will be produced in the conductor.

In fine, a current which is made, or whose intensity increases from any cause, produces an *inverse* current in a conductor; while a current which is broken, or whose intensity diminishes from any cause, produces a *direct* current in a conductor.

395. Magnets may be made to play the part of currents in these phenomena.

Thus, if we have a coil of insulated wire connected with a galvanometer, and if we quickly introduce within this coil a powerful magnet, we shall have a secondary current produced in the coil in a direction opposite to that which is presumed to circulate round the magnet (Art. 391), and this current will affect the needle of the galvanometer. Again, as long as the magnet thus introduced remains stationary in the coil we shall have no action in the galvanometer, but when we withdraw it we shall have an action the reverse of the previous one; that is to say, in the same direction as the currents of the magnet.

We thus perceive that currents are produced in a coil which approaches or recedes from a magnet. Now currents imply energy, and if left to themselves these currents will heat the coil, or by suitable contrivances they may be made to do useful work. From what source, therefore, do we derive the energy of these currents?

396. Let us see what really takes place. As we approach the coil to the magnet a current is generated in it contrary to that of the magnet. There will thus be a repulsion between the coil and the magnet (Art. 392), and we shall be spending energy in bringing them together against this repulsive force.

Again, when the coil is withdrawn from the magnet the currents produced in it are in the same direction as those of the magnet, and hence (Art. 392) the two will attract each other, so that we shall separate the two bodies against an attractive force, and thus energy is spent in the separation.

Thus both in the approach and in the withdrawal of the coil from the magnet energy is spent, and it is this energy which produces the currents in the coil.

If the coil itself were in oscillatory motion in the neighbourhood of the magnet, the energy of this motion would be soon stopped by the influence of the magnet, for as the coil approached or receded from the magnet it would experience a resistance to its motion. Its visible energy of motion would thus be lost, being converted, in the first place, into electric currents, but ultimately into heat.

By means of a suitable apparatus this conversion of mechanical energy into temporary currents, and from that into heat, may be very clearly shown. Thus, if we have a very powerful electro-magnet, and arrange a thick copper disc so as to rotate between its poles, we shall experience intense difficulty in producing this rotation of the disc, and after an enormous expenditure of energy we shall only be able to produce a very slow rotation, just as if the disc were moving in thick honey or treacle. Meanwhile the disc will have become heated, because the energy we have spent upon it ultimately takes the form of heat. This fact was discovered by Joule, who made use of this experiment, among others, to obtain the mechanical equivalent of heat.

397. Induction Machines.—There are two kinds of electrical machines which depend for their action on the laws of induction.

The object of the one set of machines is to transform the energy of work into the energy of electricity in motion, and to get powerful currents without the necessity of a voltaic battery.

The object of the other is to transform the ordinary battery current, which is deficient in tension, into a spark possessing great tension, and resembling in this respect the spark of the electric machine.

398. Magneto-electrical Machines.—The machines for transforming work into electricity in motion go by this name. They consist of a stationary magnet, and of a coil which is made to move rapidly backwards and forwards in presence of this magnet. Under these circumstances an electric current will, as we have seen, be produced in the coil. We may add, that if the motion of the coil be very rapid this electric current will be very powerful. In Clarke's machine two coils connected with one another, and having a core of soft iron in their centres, are made to rotate backwards and forwards before the poles of a powerful horse-shoe magnet. As this coil with its soft iron core approaches one of the poles, an electric current is induced in the coil, the intensity of which is heightened by the soft iron core, which becomes a magnet by induction, and which on this account heightens the current induced by the permanent magnet in the coil.

Now this secondary current will be in one direction when it passes the one pole, and in the opposite direction when it passes the other pole.

There is, however, a **commutator,** the object of which is to make the alternate currents of the coils pass from these coils through a set of wires always in the same direction, and not having their direction reversed as in the coils. The arrangement of the commutator is such that when the current is reversed in one of the coils it is passed in the opposite direction through the wires intended to convey it, and thus the current traverses the wire always in the same direction.

Powerful machines on this principle, if not of this very kind, form a very convenient arrangement for obtaining current electricity, and the electric light (Art. 415) can be produced by them in very great perfection. Of late years very powerful magneto-electric machines have been constructed and applied to a variety of useful purposes.

399. Ruhmkorff's Coil.—In Ruhmkorff's coil the ordinary current is changed by induction into one which possesses very great tension. We have in the centre of this coil a core of soft iron, and a current from two or three pairs of a Daniell or Grove's battery is made to pass round this core in such a manner as to transform it when the current passes into a powerful electro-magnet.

This arrangement forms the interior of the coil, and this interior is inclosed in a thick cylinder of glass. Outside of this glass cylinder, and insulated by it from the primary current, we have the induction coil, consisting of a large quantity of fine wire well insulated, and coiled round the glass cylinder; sometimes 40 or 50 miles of wire are used for this purpose.

In such machines there is generally a self-acting arrangement, by which the primary current is alternately introduced and shut off, and the soft iron core is thus rapidly magnetized and demagnetized.

When the primary current is started the exterior coil is, as it were, rapidly brought into the presence of a strong current, and also of a strong magnet, and a powerful induced current is therefore generated in the exterior coil, the direction of the induced current being the reverse of that of the primary current.

Again, when the primary current is cut off there will be an induced current in the exterior coil, the direction of which will now be the same as that of the primary current. But the secondary current produced when the primary current is broken has more tension than that produced when the primary current is started, so that when the induced current is forced to overcome a great resistance, such for instance as passing through a space of air, it is only the direct secondary currents, or those produced when the primary current is interrupted, that are able to pass; and we have therefore virtually, in a Ruhmkorff's coil, a powerful secondary current always in the same direction as the primary current.

The spark of a Ruhmkorff's machine may be made to pass through more than two feet of air.

LESSON XLV.—DISTRIBUTION AND MOVEMENT OF ELECTRICITY IN A VOLTAIC BATTERY.

400. This subject was first studied by Ohm, a German philosopher, who developed from theory the laws regulating the motion and distribution of electricity in a battery. These laws have since been abundantly verified by experiment, and may therefore be received as at least a near approximation to the truth. In a voltaic battery there are three objects of study: first of all we have the *electro-motive force*, or the effort put forth to establish a current of electricity; secondly, we have the *resistance* to be overcome before such a current can be produced; and lastly, we have the *intensity* of the current which is produced.

401. Electro-motive Force.—Taking these in their order, we have first the electro-motive force. Whatever may be its cause, there is without doubt an electric tension at the poles of a battery, and this may be regarded as the measure of the electro-motive force, inasmuch as it represents the tendency to form a current.

In the first place, *this tension is independent of the size of the plates of the battery but depends upon the nature of the materials used;* in fact, it mainly depends upon the distance of the two metals from one another in the electro-motive series of Art. 375.

Again, the electro-motive force of six cells of Daniell's battery in line will be six times as great as that of a single cell; and, in like manner, the electro-motive force of four cells of Grove's battery will be four times as great as that of a single cell of the same, *so that the electro-motive force varies as the number of cells.*

402. Electrical Resistance.—When we discussed thermal conductivity (Art. 219) we imagined a wall one metre in thickness, one side of which was kept at a given temperature, while the other side was one degree Centigrade hotter, and we measured the conductivity by the quantity of heat which flowed in one minute across a square metre of the wall.

We might in a similar manner measure electrical conducttivity; for we might imagine one side of the wall kept uniformly at a given electric tension, and the other side at an electric tension somewhat different, and measure the quantity of electricity that would in consequence flow in one minute across the wall, and this we might term its electric conductivity. But in the science of electricity it is more convenient to conceive of electric resistance, a quality which is the reciprocal of conductivity, so that the quantity of electricity flowing through a conductor in unit of time, in consequence of an electric difference of tension, will be directly proportional to the conductivity, but will be reciprocally proportional to the electric resistance. In other words, if we denote by intensity the quantity of electricity which passes in unit of time, then we shall have—

$$\text{Intensity of current} = \frac{\text{electro-motive force.}}{\text{resistance}}$$

Or if E be used to denote the electro-motive force of a current, and if R denote the resistance of the circuit, while i denotes the intensity of the current, we shall have, $i = \frac{E}{R}$, and this is how Ohm expresed his law.

Thus if we double the electro-motive force without altering the resistance, the intensity of the current will be doubled; and again, if we double the resistance without altering the electro-motive force, the intensity will be reduced to one-half.

403. We have now to ascertain how we may estimate the electric resistance of substances. This is found to depend on three things.

1st. *The electric resistance of a conductor depends upon the nature of its substance.*

2nd. *Its resistance varies inversely at its cross section;* that is to say, a wire with a large cross section offers much less resistance to the passage of a current than one with a small section.

3rd. *Its resistance is proportional to its length;* that is to say, if a current has to pass through two miles of wire it will be twice as much resisted as if it had to pass through one mile.

404. A battery is generally composed of two parts: 1st, the internal or other liquid conductors which are essential to its action; 2nd, the outer and metallic conductors. The resistance offered by the former may be called the internal resistance, and that offered by the latter, the external resistance of the battery.

Now let us denote by E the electro-motive force of one cell, and by R the essential or internal resistance of one cell of a battery, while r denotes the external resistance, which may be increased or diminished at will. Then we shall have by Ohm's law for a single cell in this circuit—

$$i = \frac{E}{R + r}$$

Next let there be 10 cells, then we shall have the electro-motive force and the internal resistance both increased tenfold, so that now $i = \dfrac{10\,E}{10\,R + r}$

405. These formulæ will enable us to determine the intensity of the current obtained by any arrangement of a voltaic battery.

Thus let the external resistance sensibly vanish, then the intensity will be the same in both the cases mentioned above; for although in the one case the electro-motive force is increased ten times, the resistance is also unavoidably increased in the same proportion, and hence both numerator and denominator of the fraction representing intensity are multiplied by the same number. If therefore the external resistance be very small, we do not gain much by increasing the number of cells. But suppose that while the essential resistance, or R, is equal to 10, the external resistance is equal to 100, then we shall have, for one cell, $i = \dfrac{E}{110}$, and for 10 cells, $i = \dfrac{10\,E}{200} = \dfrac{E}{20}$; if therefore the external resistance be great compared to the internal or essential resistance, a considerable increase in the intensity of the current is obtained by increasing the number of cells. Thus, in producing the electric light, it is necessary that the discharge should pass between charcoal points with an air-space between. This implies a great resistance, and it is therefore necessary that there should be a large number of cells.

Again, the thermo-electric current is one in which the external resistance is generally much greater than the internal or essential resistance. For in this case, the whole arrangement being metallic without any interposed fluids, the essential resistance is extremely small; but in order to make use of the current, it is generally necessary to have a coil of wire constituting an external resistance much greater than the internal one. It is therefore advantageous to multiply the number of couples. Generally not fewer than 25 couples are used in order to form a thermo-pile.

406. Let us now, instead of increasing the number of cells, adhere to one cell, but increase the size of the plates. In this case the electro-motive force will be unaltered, remaining equal to E, but the internal resistance will be diminished, since the cross section of the conductor is increased. If the area of each plate be increased 10 times, we shall have, $i = \frac{E}{\frac{R}{10} + r} = \frac{10E}{R + 10r}$. Now if the external resistance be small compared to the internal, we see that the intensity will vary nearly as the area of the plate. Thus if $r = \frac{1}{10}$, while $R = 10$, we shall have for the large plates, $i = \frac{10E}{11}$, while for the small plates, $i = \frac{E}{10 \cdot 1}$, the former of these being nearly ten times as great as the latter.

Hence when the external resistance is small we gain most by increasing the size of the plates. This is the case when the battery is used to produce thermal effects. Thus if we wish to melt an iron wire, it is more advantageous to have a few cells of large size than a great number of small cells.

407. We thus perceive how the intensity of the current due to any arrangement of cells may be determined. Ohm likewise showed that the intensity is the same in all parts of the circuit; that is to say, *the same quantity of electricity passes through all cross sections of a battery in the same time, whether the cross section be that of the cell or of the conducting wire.* This has also been verified by experiment.

408. Suppose now that we have a galvanometer inserted in a voltaic circuit, and that the intensity of the current, as determined by its influence upon the needle, is i. Suppose also that 12·36 metres of tin wire of the cross section of one square millimetre form part of this circuit, and that we take away the tin wire and replace it with silver wire of the same thickness, but of 100 metres in length. It will be found that the intensity of the current is unaltered by this substitution; but since $i = \frac{E}{R}$, it follows that the resistance of the whole current is the same in both cases, and hence (since the other parts of the circuit were common to both) that the resistance of the silver wire is equal to that of the tin wire. Now if the resistance of 100 metres of silver wire is equal to that of 12·36 metres of such tin wire, it follows (Art. 403) that the resistance of equal lengths of such silver and tin wire may be represented by $\frac{1}{100}$ and $\frac{1}{12·36}$ and hence the conductivities of the two metals, which are reciprocal to these resistances (Art. 402), will be represented by 100 and 12·36.

409. Electric Conductivity.—By this, or by some other similar method, we may obtain the electric conductivity of the various metals. The following results are those of Dr. A. Matthiessen and M. von Bose:—

Name of Metal.	Electric conductivity at 0° C. (Silver at 0° C. = 100)	at 100° C. (Silver at 0° C. = 100)
Silver (hard-drawn)	100·00	71·56
Copper (hard-drawn)	99·95	70·27
Gold (hard-drawn)	77·96	55·90
Zinc	29·02	20·67
Cadmium	23·72	16·77
Tin	12·36	8·67
Lead	8·32	5·86
Arsenic	4·76	3·33
Antimony	4·62	3·26
Bismuth	1·245	0·878

It has been remarked by Principal Forbes that metals follow one another in the same order, whether as conductors of heat or of electricity, and this is borne out by comparing the above table with that of Art. 219. Tait has furthermore lately shown that if two specimens of the same metal vary in their electrical conductivity, they vary in the same manner as regards their thermal conductivity.

Finally, it would appear that both the electric and the thermal conductivity of metals are diminished by increasing their temperature, in such a manner that the conductivity varies inversely as the absolute temperature (Art. 247).

LESSON XLVI.—EFFECTS OF THE ELECTRIC CURRENT.

410. Physiological Effects.—The discharge of a Leyden jar battery (Art. 351) may perhaps be likened to that of a cannon-ball from a field-piece, while a voltaic battery may be likened to a machine which keeps perpetually discharging enormous quantities of excessively small shot.

The one effect is sudden and awe-inspiring, the other is continuous and of a comparatively quiet nature.

There is great *tension, i.e.* electro-motive force, in the Leyden jar battery, but the *quantity* of electricity which passes is not great.

On the other hand, the tension of voltaic electricity is so small that it is only a very powerful battery that can send its spark across an appreciable thickness of air. But the quantity of electricity which passes in a voltaic battery is very great; and a battery of this kind which has been in silent action for a few minutes may probably have accomplished as much work as could be done by a flash of lightning, in which phenomenon the greatest possible effect is produced with the smallest possible means as regards quantity of electricity.

The destructive effect of the voltaic current upon animal life is not therefore so great as that of a Leyden jar battery. With a single cell the shock produced is hardly perceptible, but with 100 or 150 cells it is very great, and would be dangerous if continued for any length of time.

411. Thermal Effects.—When the electric current is made to pass through a circuit it heats this circuit, and the heating effect is proportional to the resistance (Art. 402) which the circuit interposes to the passage of the current. By means of this resistance, that species of energy which we term electricity in motion is converted into that other species of energy which we term heat, and the heat so produced is proportional to the resistance offered to the current.

Now if we diminish in any proportion the cross section of a wire we increase its resistance in the same proportion (Art. 403), and it therefore follows that by reducing to one-half the cross section of a wire, we double the amount of heat generated in it by the passage of the same quantity of electricity. Again, since this double amount of heat has only half the amount of metal to influence, it follows that the initial rise of temperature will be increased fourfold ; that is to say, the initial *increase of temperature produced in one second by the passage of the same quantity of electricity will vary inversely as the square of the cross section.*

412. In the next place, *the heat generated in a given time is proportional to the square of the intensity of the current.* We may deduce this from the previous law by supposing that we have two wires of single thickness close together, while single currents are made to pass through each simultaneously, so that we may imagine one current to go through the one wire and one through the other.

Therefore by means of this double current going through a double wire, twice as much heat will be generated in a given time as by a single current going through a single wire ; but we have just seen that when a double current goes through a single wire, twice as much heat is generated as when it goes through a double wire ; that is to say, four times as much as when a single current goes through a single wire.

413. We have previously seen (Art. 407) that the quantity of electricity which passes in unit of time through every cross section of a closed circuit is the same ; and we have also seen (Art. 402) that the resistance is inversely proportional to the conductivity ; we can therefore, if we know the electric conductivity of the various materials of which the circuit is composed, find the distribution of heat in the various parts of the circuit. Thus let one part be composed of a metre of silver wire two square millimetres in cross section, and another of five metres of zinc wire four square millimeters of cross section, what will be the relative heating effects of the current on these two wires? If we call the heat produced in the first wire unity, that produced in the second will be $1 \times \frac{100}{29} \times 5 \times \frac{2}{4} = 8{\cdot}62$, in which expression the second factor is on account of conductivity (see table, Art. 409), the third on account of length (Art. 403), and the last on account of cross section.

414. We can thus tell the relative distribution of heat in the various parts of a battery; but in order to tell the whole heating effect produced from first to last, we must bear in mind the origin of the heat. This is, in fact, the burning of the fuel zinc, the potential energy of which is converted in the first instance into electricity in motion, and ultimately (let us suppose) into heat. Now a certain quantity of zinc consumed will give us a certain definite quantity of heat, neither more or less; and it has been shown by Joule that if the same quantity of zinc be combined with acid in an ordinary vessel, it will give out the same amount of heat as if it were consumed by means of the voltaic arrangement.

Thus the difference between dissolving zinc by acid in an ordinary vessel, and doing so by the voltaic arrangement, is not in the quantity of heat which it gives out, but in the distribution of this heat. For in the voltaic arrangement heat may be developed many miles from the cells in which the combustion takes place, but in the ordinary case the heat is produced in the vessel in which the zinc is dissolved.

415. Electric Light.—When a voltaic battery is very powerful, it is not always necessary to bring the poles into actual contact with each other, for the current will pass through a small interval of air. This current will give out a continuous light, the nature of which will depend upon the nature of the substances which form the terminals; the light in fact consists of small particles of the terminals and of the intervening air, intensely luminous, and often in a state of vapour. When the poles are formed of carbon, this light is the most intense which we can produce by any means, and almost rivals in lustre the light of the sun; it is called, by way of distinction, the electric light.

416. Chemical Effects.—The electric current is capable of decomposing certain compound bodies into their constituent elements; thus water is decomposed into the gases oxygen and hydrogen. Faraday was the first to discover the laws which regulate this action of the current, and he has termed decomposition by the battery **electrolysis,** while the term **electrolyte** has been applied to any substance which is capable of being so decomposed.

The voltaic battery first enabled us to demonstrate the compound nature of certain substances that had previously been considered elements.

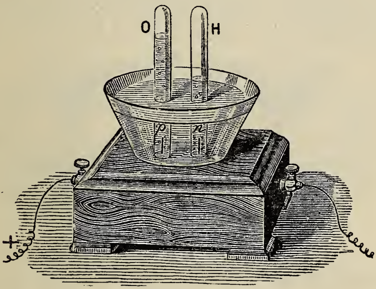

Fig. 136.

Thus Davy, by a battery of 250 cells, decomposed potass and soda, and showed that they were the oxides of the metals potassium and sodium.

When a battery is used to decompose water, an arrangement like the above (Fig. 136) is used. At the left hand we have the positive pole, and at the right hand the negative pole of our battery, so that the progress of the current is from left to right.

Now if we have two vessels, O and H, both filled with water, and if *p* and *n* be two platinum terminals of the battery entering these vessels, it will be found that if the battery is strong enough, the current will decompose the water, and that oxygen gas will appear in the vessel O, while hydrogen gas appears in H, the volume of the hydrogen being about twice as great as that of the oxygen.

The elements which appear at the positive pole of a battery are called **electro-negative** elements, and those which appear at the negative pole **electro-positive.** Oxygen is the most electro-negative element, and potassium the most electro-positive.

417. If we study the chemical action of the current, as represented in Fig. 136, we naturally ask, how is it that the oxygen appears at the one wire, and the hydrogen at the other?

Is the oxygen of each particle decomposed carried bodily to the one pole and the hydrogen to the other? In order to test to what extent this will hold, Davy performed the following experiment. He took three glasses, A B C, into the first of which he put a solution of sulphate of soda, into the second syrup of violets, while the third contained pure water. These three vessels he connected together by moistened threads of asbestos.

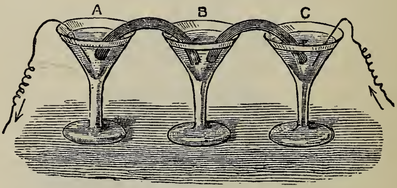

FIG. 137.

The current was then made to go, as in the figure, from C to A. The consequence was that in time the sulphate of soda was decomposed, the soda being left in A, while the acid was found in C. When the current was reversed, the acid was found in A, and the soda in C; but in neither case was the syrup of violets affected by the passage through it of the acid or the alkali.

418. Grotthuss has imagined an hypothesis which may explain this peculiar action of the current. In order to simplify conception, let us suppose that we are decomposing water, and that *p* denotes the positive, and *n* the negative pole. Then he supposes that the oxygen of the molecule of water next the positive pole will place itself next that pole, as in the figure, and that the whole row of particles between the two poles will follow, so that in fact the oxygen of each particle will point to the positive, and the hydrogen of each to the negative pole. This is the first step.

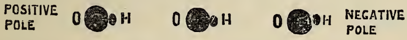

FIG. 138.

The next is the separation of the oxygen at the positive pole, while the hydrogen of that atom combines with the oxygen of the next atom, and the hydrogen of the second with the oxygen of the third, and so on, until we come to the negative pole, when the last atom of the hydrogen is set free.

The intervening particles are then twisted round, so that the oxygen atoms face the positive, and the hydrogen atoms the negative pole, and the same process is repeated.

419. This hypothesis of Grotthuss is in accordance with the laws of electrolytic action which were discovered by Faraday.

These are as follows :—

1st. *The liquid electrolyte must be a conductor in order that electrolysis may take place.*

2nd. *The current decomposes quantities of the various electrolytes which it traverses in the proportion of their chemical equivalents*, so that if PbI_2 is decomposed at one part of a circuit, and $SnCl_2$ at another, we shall obtain for 207 parts by weight of lead, 254 parts by weight of iodine, 118 of tin, and 71 of chlorine, as the results of the decomposition.

3rd. *The quantity of a body decomposed in a given time is proportional to the intensity of the current;* that is to say, to the quantity of electricity which passes in that time.

420. Electrotype Processes.—The voltaic battery may be made the means of slowly depositing metals from their solutions. Thus copper may be deposited on an engraved copper plate, and when detached, the copper deposit will represent faithfully the engraved plate; only the lines which are depressions in the original will appear in relief upon the copy. If now a new deposit be made upon the copy thus obtained, we shall have an exact impression of the original copper plate.

A body may also be silvered or gilt by means of the current. The covering of an inferior metal with a coating of silver is very much practised, and plates may thus be produced at a comparatively small expense, which will serve all the purposes of solid silver.

421. Miscellaneous Effects.—If a piece of heavy glass be subjected to the action of a powerful electro-magnet, and if a ray of polarised light be made to traverse the glass in the line of the magnetic poles, the plane of polarisation will be twisted round to the right or left, according to the direction of the current.

Another peculiarity of the current is the stratification of the light which is given out when it traverses a gas or vapour of very small pressure. We have a series of zones alternately light and dark, which occasionally present a display of colours. These stratifications have been much studied by Gassiot and others, and are found to depend upon the nature of the substance in the tube. If, however, the vacuum be a perfect one, Gassiot has found that the most powerful current is unable to pass through any considerable length of such a tube. Another effect produced by the passage of electricity is the production of **ozone.** This substance is supposed to be a peculiar modification of oxygen, into which ordinary oxygen is converted by the passage of the current. It is a powerful bleaching agent, and has a very peculiar smell, which may be noticed when an electric machine is in action.

421a. Recent Applications of Electricity. The Telephone.—Quite recently Professor Graham Bell has produced an instrument called the telephone, by means of which an insulated wire is made the medium of conveying speech. It operates in this way: we have in the first place an iron membrane or plate placed symmetrically in front of one pole of a bar magnet, which is covered with insulated wire. One operator talks immediately in front of the iron membrane, which is consequently thrown into a complicated state of vibration. This complicated state of vibration represents the three elements of the speaker's voice, namely, pitch, intensity, quality. But in each vibration of the iron membrane we have a portion of the iron alternately approaching towards or receding from the magnet, and the same result will be produced as when a piece of soft iron is made rapidly to approach or recede from a magnet covered with insulated wire, namely, that if the circuit of insulated wire be complete, secondary currents will be produced in the insulated wire, these being in one direction for the approach, and in the opposite for the recession, of the soft iron.

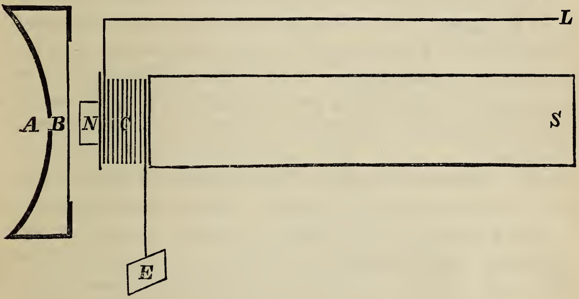

FIG. 139.

When the iron membrane vibrates, therefore, in front of the magnet, it produces in the insulated wire, which is arranged so as to form a complete circuit, a series of secondary currents, which in fact register all the peculiarities of the speaker's voice, so far as currents can be said to do so. At the other end of the circuit, which may be many miles distant, we have another instrument precisely similar to that now described, with the difference that the operator now places his ear instead of his mouth to the iron membrane. Now the currents passing along the insulated wire, which wire is wrapped round the magnet at this end, just as it was round the other magnet, produce magnetic changes in the magnetism of the magnet, and of the iron membrane in front of it. These magnetic changes cause a state of vibration of this second iron membrane precisely similar to that of the membrane at the other end. The consequence is that an operator at one end placing his ear to the iron membrane will hear the very sounds given out by a speaker at the other end of the circuit.

In Fig. 139 we have the various parts of such a telephone. Here A denotes the mouth-piece, B the iron membrane, N the north pole of the magnet, whose other pole is S. C is the insulated wire connected at E with the earth, and at L with the telegraphic line which goes to, and is similarly connected with, a similar instrument at the other end, the whole forming one circuit.

421b. The Microphone.—We are indebted for this instrument to Professor D. E. Hughes. It was found by him that if, in the circuit of an electric current, a tube be placed containing several pieces of carbon in contact with each other, any sonorous vibration, by affecting the closeness and nature of the contact between these various pieces of carbon, will affect the resistance of the circuit, and hence also the strength of the current.

If the circuit, which may be a long one, have at the one end a microphone and at the other a telephone, sonorous vibrations taking place near the microphone will produce variable or intermittent currents, which at the other end will, by means of the telephone, reproduce sonorous vibrations similar to those which originally excited them.

So great is the delicacy of this arrangement, that sounds quite imperceptible to the human ear, as, for instance, the noise made by a fly when walking, may be rendered distinctly audible at the distance of many miles.

421c. The Induction Balance.—Professor Hughes is likewise the inventor of this instrument. Its principle of action may be explained as follows :—

If two separate secondary coils (which ought to be at least one metre apart) are joined together in such a manner that the current induced by a primary in the one shall exactly neutralise that induced by the same primary in the other, then we have a perfect balance, and there will be no current in the secondary coils, whatever changes are taking place meanwhile in the primary.

Now let the primary current have in circuit a microphone with a seconds clock as the source of sound. We know from the last article that every second alterations in the strength of the primary current will by these means be produced, nevertheless these alterations will not induce any current in the balanced secondary circuit, so that if a telephone be attached to this circuit we shall hear no noise. If, however, a coin be placed alongside or inside one of the secondary coils, it will be electrically influenced by the primary, and will in its turn interfere with the secondary to which it is attached, so that the two secondaries will now no longer balance each other. Alternating currents will therefore now pass through the secondaries, and these will, by means of the telephone, reproduce the source of sound; that is to say, the beating of the clock. If two exactly similar coins be similarly placed in the two coils of the induction balance, there will, of course, be no current, and hence no noise; but so delicate is this instrument, that if the one coin be made of false metal, or even if it be a trifle too light compared with the other, a sound will at once be produced.

421d. The Bolometer.—A method originally employed by Siemens has been adapted by Professor S. P. Langley to the construction of a very delicate measurer of heat. The instrument for this purpose is called a Bolometer. It will appear from Art. 409 that the electrical conductivity of a metal is diminished, and its electrical resistance increased, as it rises in temperature. Now, suppose that two circuits conveying equal and opposite currents meet in a galvanometer, the needle will of course remain at rest. Suppose, however, that a portion of one of these circuits is heated, the consequence will be that its resistance will be increased, and (by Ohm's law) the current passing through it will therefore be diminished. The two opposing currents will now no longer balance each other, but the one whose circuit has not been heated will prevail, and the galvanometer will be deflected. Now, suppose that these circuits each contain a sheet of extremely thin platinum foil—if both sheets be at exactly the same temperature the balance of currents will be perfect, and the galvanometer will not be deflected. If, however, one of these sheets be heated, this will produce a current in one direction, and if the other, this will produce a current in the opposite direction. If the sheets be exceedingly thin, then a very small quantity of heat may suffice to produce a considerable result. The sheets may thus be compared to the two faces of a thermopile (Art. 291), and indeed this instrument is found to be capable of giving us even more delicate and trustworthy measurements of heat than can be obtained by means of the thermopile. Professor Langley has by its means determined with much precision the exact distribution of energy in the solar spectrum (see Art. 299).

421e. Secondary Batteries.—A recent modification by Faure of the battery of Planté will best serve to illustrate these secondary batteries, or accumulators, as they are sometimes called. The two plates, or electrodes, of this battery consist of two pieces of sheet-lead rolled up without actual contact, and coated with red lead : these are immersed in dilute sulphuric acid. If now this battery be charged by a powerful current, the coating of the one plate will become converted into peroxide of lead, while that of the other plate will be deoxidised. The plates are now in a condition, if connected together, to furnish a current by themselves, which will continue until both are once more reduced to the same state of oxidation.

421f. Electric Lighting.—We have seen (Art. 415) that when a powerful electric current passes between carbon poles not in actual contact it produces a very intense light. As this light, besides being exceedingly brilliant, can now be rendered very steady, it has been much used for the illumination of lighthouses and large public places. It is, however, unsuitable for smaller rooms, but in these the place of gas can be advantageously supplied by another species of light, also electrical. This consists of a small glass globe rendered a perfect vacuum, and containing sealed into it two electric poles or terminals, connected together by a delicate thread of prepared carbon. When a powerful current is made to pass through the terminals of such a globe, the thread of carbon is rendered intensely brilliant, and from the globe being a vacuum the thread does not waste away as it would otherwise do. In this arrangement we have the maximum of light with the minimum of heat, and no bad air whatever; the lamp will, therefore, in all probability be largely used in the course of time. Currents sufficiently powerful and continuous to serve for lighting purposes are best supplied by magneto-electric machines (Art. 398), and it has been suggested that the introduction of secondary batteries would serve to obviate the inconvenience occasioned by a temporary stoppage of the machinery, inasmuch as these when charged would continue to give out a current on their own account for some time.

CHAPTER IX.

Lesson XLVII.

ENERGY OF CHEMICAL SEPARATION.—CONCLUDING REMARKS.

422. We have pointed out, in Art. 103, that in the efforts to separate an atom of carbon from one of oxygen the energy which we employ is transmuted into a species of potential molecular energy, just as when a stone is separated from the earth and carried to the top of a house, the energy employed in doing this is transmuted into potential energy.

Further, we have seen that when this carbon is burned in the fire, this molecular potential energy is converted into molecular energy of motion; or, in other words, heat is generated.

It is natural, therefore, to expect that a definite quantity of carbon will, when burned, always furnish a definite quantity of heat.

423. Andrews in this country, and Favre and Silbermann in France, have investigated the quantity of heat given out by chemical combination, and from their researches the following table has been derived :—

UNITS OF HEAT DEVELOPED DURING COMBUSTION IN OXYGEN.

Substance burned.	Kilogrammes of water raised 1° C. by the combustion of one kilogramme of each substance.	Compound formed.
Hydrogen	34135	H_2O
Carbon	7990	CO_2
Sulphur	2263	SO_2
Phosphorus	5747	P_2O_5
Zinc	1301	ZnO
Iron	1576	Fe_3O_4
Carbonic oxide	2417	CO_2
Marsh gas	13085	
Olefiant gas	11900	
Alcohol	7016	

424. Andrews has likewise studied the heat given out during the mutual action of metals, and has been led to the following result :—

If there be three metals, A, B, C, such that A will displace B and C from their combinations, while B will displace C, then the heat developed by substituting A for C will be equal to that produced by substituting A for B, **plus** *that produced by substituting B for C.*

This law is similar to that obtained for electro-motive force (Art. 374), and this leads us to believe that the electro-motive forces are really those which cause heat when chemical combination takes place. This conjecture is confirmed by the fact, that if the metals be classed according to the amount of heat which they give out when displacing one another, we reproduce the electro-motive series of Art. 375.

425. Dissipation of Energy.—We have seen (Art. 110) that the law of the conservation of energy is nothing more than an intelligent and well-supported denial of the chimera of perpetual motion, and that a machine can no more create work than it can create matter. Nevertheless a champion of perpetual motion might assent to all this without absolutely giving up his cause.

"I acknowledge," he might say, "that perpetual motion in one sense of the word, is quite impossible, for no machine can *create* energy, but yet I do not see that a machine might not be constructed that would produce work for ever. Allowing that heat is a species of molecular motion, and hence that all substances are full of a kind of invisible energy, may we not suppose a machine to exist which converts this molecular motion into ordinary work, drawing first of all the heat from the walls, then from the adjacent air; cooling down, in fact, the surrounding universe, and transforming the energy of heat so abstracted into substantial work? There is no doubt that work can be converted into heat—as, for instance, by the blow of a hammer on an anvil—why, therefore, cannot this heat be converted back again into work?"

We reply to such a one by quoting the laws discovered by Carnot, Thomson, Clausius, and Rankine, who have all from different points of view been led to the same conclusion, fatal to all hopes of perpetual motion. We may, they tell us, with the greatest ease convert mechanical work into heat, but we cannot by any means convert all the energy of heat back again into mechanical work. In the steam-engine we do what can be done in this way; but it is a small proportion of the whole energy of the heat that is there converted into work; for a large portion is dissipated, and will continue to be dissipated, however perfect our engine may become. Let the greatest care be taken in the construction and working of a steam-engine, yet we shall not succeed in converting one-fourth of the whole energy of the heat of the coals into mechanical effect.

In fact, the process by which work can be converted into heat is not a completely reversible process, and Sir W. Thomson has worked out the consequences of this fact in his theory of the dissipation of energy.

As far as human convenience is concerned, the different kinds of energy do not stand on the same footing, for we can make great use of a head of water, or of the wind, or of mechanical motion of any kind, but we can make no use whatever of the energy represented by equally diffused heat. If one body is hotter than another, as the boiler of a steam-engine is hotter than its condenser, then we can make use of this difference of temperature to convert some of the heat into work; but if two substances are equally hot, even although their particles contain an enormous amount of molecular energy, they will not yield us a single unit of work.

Energy is thus of different *qualities*, mechanical energy being the best, and universal heat the worst; in fact, this latter description of energy may be compared to the waste heap of the universe, in which the *effete* forms of energy are suffered to accumulate, and this waste heap is always continuing to increase. But before attempting to discuss the probable effect of this process of deterioration upon the present system of things, let us look around us and endeavour to estimate the various sources of energy that have been placed at our disposal.

426. Sources of Energy.—To begin with our own frames. We all of us possess a certain amount of energy in our systems, a certain capacity for doing work. By an effort of his muscles the blacksmith imparts a formidable velocity to the massive hammer which he wields;—now what is consumed in order to produce this? We reply, the tissues of his body are consumed. If he continues working for a long time, he will wear out these tissues and nature will call for food and rest:—for the former in order to procure the materials out of which new and energetic tissues may be constructed;—for the latter, in order to furnish time and leisure for repairing the waste. Ultimately, therefore, the energy of the man is derived from the food which he eats; and if he works much, that is to say, spends a great deal of energy, he will require to eat more than if he hardly works at all. Hence it is well understood that the diet of a man sentenced to imprisonment with hard labour, must be more generous than that of one who is merely imprisoned, and that the allowance of food to a soldier in time of war must be greater than in time of peace.

In fact, food is to the animal what fuel is to the engine, only an animal is a much more economical producer of work than an engine. Rumford justly observed that we shall get more work out of a ton of hay if we give it as food to a horse, than if we burn it as fuel in an engine. It is in truth the combustion of our food that furnishes our frames with energy, and there is no food capable of nourishing our bodies which, if well dried, is not also capable of being burned in the fire. Having thus traced the energy of our frames to the food which we eat, we next ask, whence does food derive its energy? If we are vegetarians, we need not go further back; but if we have eaten animal food, and have transferred part of the energy of an ox, or of a sheep into our own systems, we may ask, whence has the ox or the sheep derived its energy? The reply will be, undoubtedly, from the food which it consumes, this food being a vegetable. Ultimately, then, we are led to look to the vegetable kingdom as the source of that great energy which our frames possess in common with those of the inferior animals, and we have now only to go back one step further and ask, whence vegetables derive the energy which they possess?

In answering this question, let us endeavour to ascertain what really takes place in the leaves of vegetables. A leaf is, in fact, a laboratory, in which the active agent is the sun's rays. A certain species of the solar ray enters this laboratory and immediately commences to decompose carbonic acid into its constituents, oxygen and carbon, allowing the oxygen to escape into the air, while the carbon is, in some shape, worked up and assimilated. Thus, first of all, we have a quantity of carbonic acid drawn in from the air: that is the raw material. Next, we have the source of energy, the active agent: that is, light. Thirdly, we have the useful product: that is, the assimilated carbon. Fourthly, we have the product dismissed into the air again, and that is oxygen.

We thus perceive that the action which takes place in a leaf is the very reverse of that which takes place in an ordinary fire. In a fire we burn carbon, and make it unite with oxygen in order to form carbonic acid, and in so doing we change the energy of position derived from the separation of two substances having so great an attraction for each other as oxygen and carbon, into the energy of heat. In a leaf, on the other hand, these two strongly attractive substances are forced asunder, the powerful agent which accomplishes this being the sun's rays, so that it is the energy of these rays which is transformed into the potential energy or energy of position, represented by the chemical separation of this oxygen and carbon. The carbon, or rather the woody fibre into which the carbon enters, is thus a form of potential energy; and when made to combine again with oxygen, either by direct combustion or otherwise, it will in the process give out a great deal of energy. When we burn wood in our fires we convert this energy into heat, and when we eat vegetables we assimilate this energy into our systems, where it ultimately produces both heat and work. We are thus enabled to trace every step of this wonderful process: we have, first of all, the sun's rays building up vegetable food; in the next place we have the ox or sheep fed by means of this food; and lastly, we have the tissue of the ox or sheep entering into and sustaining our own frames.

We have not, however, quite done yet with vegetable fibre, for that part of it which does not enter into our frames may, notwithstanding, serve as fuel for our engines, and by this means be converted into useful work. And Nature, as if anticipating the wants of our age, has provided an almost limitless store of such fuel in the vast deposits of coal, by means of which so large a portion of the useful work of the world is done. In geological ages this coal was the fibre of a species of plant, and it has been stored up as if for the very benefit of generations like the present.

But there are other products of the sun's rays besides food and fuel. The miller who makes use of water-power or of wind power to grind his corn, the navigator who spreads his sail to catch the breeze, are both indebted to our luminary equally with the man who eats meat or who drives an engine. For it is owing to the sun's rays that water is carried up into the atmosphere to be again precipitated so as to form what is called a head of water, and it is also owing to the sun's heat that winds agitate the air. With the trivial exception of tidal energy, all the work done in the world is due to the sun, so that we must look to our luminary as the great source of all our energy.

Intimately linked as we are to the sun, it is natural to ask the question, will the sun last for ever? or will it also die out?

Now there is no apparent reason why the sun should form an exception to the fate of all fires, its only difference being one of size and time. It is larger and hotter, and will last longer than an ordinary lamp, but it is nevertheless a lamp, or, to speak more correctly, a very large hot body.

In fine, the principle of degradation would appear to hold throughout; and if we regard not mere matter but useful energy, we are driven to contemplate the death of the universe.

Recapitulation.—It may be desirable, before concluding, to recapitulate the various transmutations of energy.

427. Visible Energy.—Visible Energy of Motion is transmuted into visible potential energy when a stone is projected upwards and lodged on the top of a house (Art. 111), and it is transmuted into heat when friction or percussion stops a body in motion (Art. 113).

It is transmuted into electrical separation when we work the electric machine (Art. 356), and into electricity in motion when a revolving conductor is brought between the poles of a powerful magnet (Art. 396).

Visible Potential Energy is generally converted into visible energy of motion, and through it into the other forms of energy.

428. Heat.—This species of energy is converted into visible motion in the heat engine (Art. 245). It is converted into radiant energy when a hot body radiates (Art. 334). It is converted into electrical separation when tourmalines and other gems are heated (Art. 356). It is converted into electricity in motion in the thermo-electric pile (Art. 378). Finally, it is converted into chemical separation when a body is decomposed by heat (Art. 215).

429. Radiant Energy.—This species of energy is converted into heat when radiant light or heat is absorbed by a body (Art. 334), and it is converted into chemical separation when a ray of sunlight decomposes chloride of silver in photography, or carbonic acid in the leaves of plants (Art. 425).

430. Electrical Separation.—The energy of electrical separation is transformed into visible motion when two oppositely electrified bodies approach each other (Art. 356), and it is transformed into the energy of electricity in motion when two such bodies are connected together by means of a wire (Art. 356).

431. Electricity in Motion.—This form of energy is converted into visible motion when currents act on one another, as in Art. 393 ; it is converted into absorbed heat when a current meets with resistance (Art. 411) ; and into chemical separation when a current decomposes a compound body (Art. 416).

432. Chemical Separation.—This form of energy is transmuted into heat when a substance burns, or when combustion takes place (Art. 422); into electrical separation when two dissimilar metals are brought into contact (Art. 371); and into electricity in motion in the voltaic battery.

These form some of the chief transmutations of the various forms of energy into one another, but it ought to be borne in mind that the classification of energy into various forms is simply one of convenience, and represents the present state of our knowledge of the subject.

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